Solitary wave-based strain measurements in one-dimensional granular crystals

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Abstract
We investigate the transmission and backscattering of solitary waves in granular crystals to assess their initial strains under various levels of compression. We assemble a one-dimensional granular chain that includes a pair of heavy impurities to scatter incident solitary waves. Using a scanning laser Doppler vibrometer and an instrumented sensor particle, we experimentally show that the speed of solitary waves backscattered in the region of impurities is highly sensitive to the applied strains. Based on this pulse-echo mechanism of solitary waves, we briefly demonstrate the feasibility of localized strain sensing via multiple scatterers distributed in a granular chain. We find that numerical results obtained from a discrete element model are in excellent agreement with experimental results. This study forms a foundation for constructing a solitary wave-based sensor system to measure distributed strains in ordered granular systems.

1. Introduction
Granular crystals—defined as assemblies of ordered, tightly-packed elastic particles—are receiving increasing attention due to their versatile waveguide modes [1-7]. The focus has been placed particularly on their capability to support nonlinear waves resulting from the balance between dispersion and nonlinearity [1, 7]. The simplest form of such granular crystals is a chain of spherical particles under the Hertzian contact law (i.e. \( F \sim \delta^{3/2} \), where \( F \) is the compressive force and \( \delta \) is the approach between the particles) [8]. Under the external excitations, the one-dimensional (1D) granular crystal can efficiently generate stable nonlinear waves with compact support. These highly nonlinear waves are called solitary waves, physically analogous to fluidic and optical solitary waves [1, 9]. By leveraging the controllable formation and propagation of solitary waves in granular crystals, researchers have proposed new types of engineering devices, such as acoustic lenses, impact mitigators and sensing and actuation devices [1, 10-12].

Compared with linear elastic waves in continuum media, highly nonlinear solitary waves in granular crystals exhibit unique physical properties, such as high robustness, compact-supportedness and distinctive superposing and scattering characteristics [1, 7]. Notable among these is an extremely slow and amplitude-dependent propagation speed of solitary wave. According to Nesterenko’s pioneering work, the speed of solitary wave \( V_s \) can be analytically expressed as a function of the normalized strain \( \xi_r \):

\[
V_s = \frac{c_0}{\xi_r - 1} \sqrt{\frac{4(3 + 2^{5/2} - 5\xi_r)}{15}}
\]

(1)

in the continuum approximation of a 1D granular chain composed of homogeneous spherical particles [1]. Here the normalized strain \( \xi_r \) is the ratio of maximum dynamic strain \( \xi_m \) to the static strain \( \xi_0 \) induced by the pre-compression of the chain. The coefficient \( c_0 \) is the sound speed in the granular chain under the application of \( \xi_0 \):

\[
c_0 = \frac{3E}{\pi \rho (1 - v^2)^{1/4} \xi_0^{1/4}}
\]

(2)

where \( E \), \( v \) and \( \rho \) are the elastic modulus, Poisson’s ratio and density of the spheres in the chain, respectively. Researchers have shown experimentally and numerically the dependence of solitary waves’ speed on the maximum dynamic strains, thereby verifying tunable transmission of solitary waves in granular crystals [1, 2, 13]. Also reported in the field of nonlinear granular dynamics are the physical phenomena such as scattering and localization of solitary waves when...
granular crystals embed impurity particles [3, 4]. Due to such unique mechanisms, solitary waves have shown great potential as information carriers to discern impurities or defects in granular media in a nondestructive manner [5]. Furthermore, it has been demonstrated that granular chains with optimized arrangements of distinctive particles can be used as impact absorbing media by means of localization and thermalization of solitary waves [14, 15].

In this paper, we study the propagation and backscattering of solitary waves in the vicinity of impurities to assess initial strain fields (ε0) applied to granular crystals in an efficient and controllable manner. We assemble a 1D granular crystal that includes a pair of impurities heavier than the ordinary particles in the chain. These impurities are positioned in specific locations of a granular crystal such that they act as scatterers to echo a portion of incoming wave packets while transmitting the rest of solitary waves. To measure strains in the granular chain, we apply external excitations at one end of the chain and record the incoming and backscattered packets of solitary waves using an instrumented sensor bead that we embed in a selected particle spot. By analysing the difference in time of arrival between solitary wave packets backscattered from a pair of impurities, we can back-calculate the static strains initially applied to the granular chain. In principle, this mechanism is similar to optical fibre Bragg grating sensors for strain measurements, except that we employ mechanical solitary waves as information carriers instead of optical signals [9].

For experimental verifications, we visualize the propagation and backscattering of solitary waves in granular crystals using full-field particles’ velocities measured by a scanning laser Doppler vibrometer. The temporal force profiles of solitary waves are also recorded by an instrumented sensor particle that embeds a calibrated piezoelectric disc to verify the sensitive responses of solitary waves to strain variations. We then compare the experimental results with numerical simulations obtained from a discrete particle model. Using the combined experimental and numerical approaches, we demonstrate that the propagation speed of backscattered solitary waves is sensitive to static strains applied to a uniformly compressed granular chain. Based on such high responsiveness of solitary waves, we also attempt to show the feasibility of localized strain sensing via multiple scatterers distributed in a granular chain. To the best of the author’s knowledge, the sensitive ‘pulse-echo’ mechanism of solitary waves has not been exploited for strain measurements in association with strategically positioned granular impurities. This constitutes the focus of this study in conjunction with the visualization technique of solitary waves via a laser Doppler vibrometer.

The rest of this paper is structured as follows: we first describe an experimental setup for measuring solitary waves in a granular chain in section 2.1. We then give a brief overview of the discrete element method in section 2.2. The experimental and numerical results based on the laser Doppler vibrometer are compared and discussed in section 3.1. Based on the solitary waves’ force profiles measured by an instrumented sensor particle, we analyse and discuss the effect of strains on solitary wave propagation in section 3.2. We briefly demonstrate the feasibility of distributed sensing in section 3.3 using multiple scattering objects embedded in the granular chain. Finally, in section 4, we summarize our findings and present our conclusions.

2. Experimental and numerical approaches

2.1. Experimental and numerical approaches

Figure 1 shows the digital image and the schematic of the experimental setup consisting of a granular crystal and a scanning laser Doppler vibrometer (SLDV, Polytec PSV400). The laser vibrometer measures the velocity profiles of the particles in the granular crystal. We assemble a one-dimensional granular chain composed of 30 chrome steel (AISI 52100) spheres and two heavy tungsten carbide impurities placed at the 14th and the 19th particle sites from the right end of the chain (i.e. excitation side). The radius, density, Young’s modulus and Poisson’s ratio of the chrome steel spheres are 9.525 mm, 7780 kg m\(^{-3}\), 200 GPa and 0.28, respectively. The density, Young’s modulus and Poisson’s ratio of the tungsten carbide particles are 15 800 kg m\(^{-3}\), 668 GPa and 0.24, while their radii are kept identical to those of the chrome steel beads. The ratio of the mass of the impurity to the regular particle is approximately two. All the spherical particles are supported by four polished, stainless-steel rods (12.7 mm diameter) to restrict their lateral motions, while allowing free axial vibrations of particles (see the inset of figure 1(a)).

The granular chain is excited by a piezoelectric actuator, which is connected to an external function generator and an electrical amplifier synchronized with the SLDV. The SLDV scans 325 points along the granular chain, corresponding to about 10 measurement points along the perimeter of each particle. At each measurement point, 20 measurements are taken and then averaged in order to enhance the signal-to-noise ratio. To enhance the reflection of laser, we apply aerosol sprays on the surface of granules, avoiding contamination of the granular contact interfaces. It should be noted that the SLDV is sensitive only to the particles’ velocity components in the direction of the laser beam towards the equipment (υ in figure 1(b)). Thus, the axial velocity of each granule (υx) is obtained using a trigonometry rule (i.e. \(υ_x = υ/\sin θ\), where θ is the scanning angle of the laser beam). The distance between the SLDV and the granular chain is 1.4 m.

To assess the behaviour of backscattered solitary waves under various strains, we apply 17 different levels of pre-compression (F0) from 0 to 148 N to the first particle of the chain. For the fine control of F0, a linear control stage with coil springs is employed (figure 1(b)). In addition to the SLDV, the temporal force profiles of solitary waves are measured by an instrumented sensor particle positioned in the 7th particle spot. The piezoelectric disc in the sensor particle converts dynamic forces into voltage signals when solitary waves pass...
by the sensor site (see the inset of figure 1(b)). The details of fabrication and calibration processes of the sensor particle are discussed in [16]. While we employ a piezoelectric actuator for synchronized excitations of the granular chain in SLDV tests, we use mechanical impact by a striker (identical to the spheres composing the chain) for the force profile measurements via the instrumented sensor particle. This is to observe the responsiveness of solitary waves more evidently, by generating orders of magnitude larger solitary waves compared with the piezoelectric actuator [17]. The striker is released from the top of a ramp using a solenoid switch, and the impact speed is measured optically to be 1.66 m s\(^{-1}\). The sensor particle is connected to an oscilloscope with the sampling frequency of 10 MHz, and the acquired signals are analysed in Matlab.

2.2. Numerical setup

We conduct numerical simulations of solitary waves’ interactions with impurities using a discrete element model (DEM). The DEM method approximates a 1D granular chain into an assembly of point masses connected by nonlinear springs. Previous studies have shown that it can successfully simulate the formation and propagation of solitary waves in granular crystals [1, 13, 17, 18]. In this approach, the equation of motion of \(n\)th particle in the granular chain can be expressed mathematically as

\[
m_n \frac{\partial^2 u_n}{\partial t^2} = A[u_{n-1} - u_n]^3/2 - A[u_n - u_{n+1}]^3/2,\]

where \(A = E \sqrt{2R}/3(1 - v^2)\). (3)

Here \(m_n\) and \(u_n\) are the mass and displacement of the particle from its uncompressed position, and the bracket \([x]_+\) takes only positive values, denoting that the granular interactions involve only compressive forces. The contact coefficient \(A\) is given by the Hertzian contact law [8]. A modified equation of motion is used for the first and the last particles in the chain to impose unique boundary conditions, such as striker impact and half-spaced wall constraint. In this study, we solve this ordinary differential equation for all the particles using the Runge–Kutta–4 integration scheme implemented in Matlab [19].

3. Results and discussion

3.1. Laser Doppler vibrometer measurements

The full-field measurements of the particles’ velocities are shown in the surface plot in figure 2(a), where no static pre-compression is applied (i.e. \(\zeta_0 = 0\)). The colour intensity corresponds to the magnitude of normalized velocities. The highlighted line from the origin represents the incoming solitary wave propagating in the positive \(x\)-direction. The slope of this line is 487 m s\(^{-1}\), representing an extremely slow speed of the incident solitary wave that is typically on an order of magnitude smaller than the speed of dilatational waves within the particles [1]. When the incident solitary wave reaches the fixed wall at approximately \(x = 0.61\) m, it is reflected back towards the opposite direction. Notably, we also observe two packets of solitary waves propagating in the negative \(x\)-direction, which are branched off from the incident solitary wave at \(x = 0.25\) and \(x = 0.34\) m positions as indicated by a dashed red box in figure 2(a). These positions correspond to the two impurity sites, denoting the backscattering phenomenon of solitary waves in the region of impurities. The simulation results of the particles’ temporal velocity profiles are provided in figure 2(b). The numerical results are in good agreement with the experimental measurement and confirm the formation of backscattered solitary waves in the impurity sites, as indicated by a dashed red box.

The mechanism of solitary wave’s backscattering against heavy impurities can be explained by the particle-like properties of solitary waves [6, 17]. A packet of solitary waves propagating along the granular chain naturally perturbs...
Figure 2. Full-field velocity profiles of granular particles. (a) Experimental measurements of particles' velocities using scanning laser Doppler vibrometry. (b) Numerical simulation results of particles’ velocities based on DEM.

Figure 3. Experimental (solid blue) and numerical (dashed red) profiles of solitary waves viewed from the instrumented sensor bead. (a) Profiles of solitary waves in the granular chain with zero strain. (b) Profiles of solitary waves when the applied strain is \( \xi_0 = 1.84 \times 10^{-4} \). (c) The temporal forces profiles of measured solitary waves under the variations of strains from \( \xi_0 = (0–2.92) \times 10^{-4} \). (d) The temporal force profiles of solitary waves simulated by DEM. The colour map shows the magnitude of dynamic forces.

particles with a spatial wavelength of approximately five particle diameters [1]. When the wave front reaches the interface between an ordinary particle and its adjacent heavy impurity, the colliding particle is reflected back in the opposite direction of solitary wave progression due to the inertia of the impurity larger than that of the colliding particle. This triggers the backscattering of solitary waves, while a portion of incident momentum and energy is carried by the transmitted solitary waves. The energy and momentum transmitted or backscattered by the presence of the defects are determined by the complex dynamic interaction of the defect particle with the neighboring particles. More details can be found in [6, 17].

3.2. Instrumented sensor particle measurements

To investigate the responses of backscattered solitary waves to various strains, we gradually increase pre-compression \( F_0 \) from 0 to 148N. This corresponds to the granular chain’s axial strain \( \xi_0 = 0 \) to \( 2.92 \times 10^{-4} \) according to the continuum expression of the granular chain: \( \xi_0 = \delta_0/2R \), where \( \delta_0 \) and \( R \) are the maximum static approach between neighboring spheres and their radii [1]. Here \( \delta_0 \) can be calculated by the Hertzian contact law \( \delta_0 = (F_0/A)^{2/3} \) with \( A \) defined in equation (3) [8]. The force profiles measured by an instrumented particle under \( \xi_0 = 0 \) and \( \xi_0 = 1.84 \times 10^{-4} \) are shown in figures 3(a)
and (b). The first impulses observed around 0.15 ms represent incident solitary waves, while the subsequent two packets are the backscattered solitary waves from the two impurities. In fact, the variation of the travelling speed of incident solitary waves depends on the pre-compression applied to the chain. However, the data acquisition in the experiments is triggered by the rising slope of the incident waves. That is, the incident solitary waves play a role as a triggering signal in this ‘pulse-echo’ approach that uses a single sensor. Therefore, the incident solitary waves are positioned in similar temporal spots between the compressed and noncompressed cases shown in figure 3.

The arrival time of the subsequent solitary waves contains information about the strains applied to the granular chain. We observe that when the static strain increases, the time-difference of arrival (TDOA) between two backscattered solitary waves is reduced accordingly. Compared with the zero-strain condition, the experimental TDOA under $\xi_0 = 1.84 \times 10^{-4}$ decreases by 28% from 0.478 to 0.335 ms. The numerical simulation results based on DEM (red dotted line in figures 3(a) and (b)) also confirm this observation.

The experimental and numerical results of solitary waves’ profiles over a range of applied strains are depicted in figures 3(c) and (d), respectively. In these surface maps, highlighted peaks around $t = 0.5$ and 1 ms correspond to the two solitary waves backscattered from the pair of impurities included in the chain. In both experimental and numerical results, we observe that the increase in strains results in narrower gaps between two peaks, thereby leading to the shorter TDOA.

The dependence of TDOA on external strains is more evident in figure 4. The error bars with circular marks represent the standard deviations of TDOA from five experimental measurements, while the numerical results are depicted in red square marks. From both experimental and numerical results, we observe that the TDOA decreases as the applied strains increase. This is attributed to the increasing stiffness between granules under the nonlinear Hertzian contact, which in turn results in the faster speed of both incoming and backscattered solitary waves. Such amplitude dependence of wave speed is a unique property of general types of nonlinear waves including solitary waves [20]. This is also predicted by the theoretical relationship between the static strain $\xi_0$ and solitary wave’s speed $V_s$ (see equation (1)), which is illustrated in the inset of figure 4. Considering that TDOA is inversely proportional to $V_s$, we conduct curve fitting of experimental (solid blue) and numerical (dotted red) data based on this analytical relationship (figure 4). We find that these fitted curves based on theoretical predictions corroborate the experimental and numerical data. The experimental results exhibit the slightly larger TDOA compared with simulation results, implying slower travelling speed of backscattered solitary waves in experiments. Such discrepancies are due to the dissipative effects in experiments.

3.3. Potential for distributed strain sensing

Although the behaviour of solitary waves in a uniformly compressed granular chain is investigated mainly in this study, the TDOA values introduced in the aforementioned sections are strongly affected by the localized strain between a pair of impurities. This is because the arrival time difference of two backscattered solitary waves is governed by the speed of solitary waves travelling between the two impurities. This means that each impurity behaves as a reflector, enabling the assessment of localized strains based on a ‘pulse-echo’ scheme. This is in contrast to the conventional ‘pitch-catch’ scheme, which requires at least two sensors positioned at both ends of a granular chain. In a sense, this mechanism is similar to the distributed strain measurements using optical fibre Bragg grating sensors and can therefore be used for distributed strain sensing.

To show the feasibility of strain sensing in multiple spots, a supplementary test using four impurities in a single granular chain is conducted. The experimental setup is identical to that described in section 2.1, except that the granular chain embeds four tungsten-carbide impurities at the 15th, 22nd, 29th and 36th particle spots. The PZT-embedded sensor particle is placed at the 11th particle site, and the end of granular chain is not restricted by the wall in order to avoid the generation of reflected waves from the wall. The DEM simulation results of solitary waves’ force profiles under no pre-compression are presented in figure 5(a) as a function of time and particle locations. The colour intensity represents the force magnitude of solitary waves when they pass by specific particle spots. The sensor and impurity locations are marked with dotted and dashed horizontal lines, respectively. Despite multiple scattering and localization of solitary waves, we find that each impurity generates a distinctive, backscattered solitary wave, which is directed towards the sensor spot. These solitary wave packets are observed around $t = 0.47, 1.0, 1.6$ and 2.3 ms, following the incident solitary wave around $t = 0.27$ ms. Due to the localization of solitary waves in the vicinity of impurities, the reflected solitary wave from the farther spot tends to exhibit the smaller force magnitude. The solitary waves’ arrival to the sensor site is postponed accordingly, caused by the attenuation of solitary waves. Due to the technical difficulties in imposing
Figure 5. (a) Numerical results of solitary waves propagating and backscattering in the vicinity of four impurities in a granular chain. (b) Experimental (solid blue curve) and numerical (dotted red curve) results of solitary wave’s force profiles when measured at the sensor spot \(n = 11\).

localized strains, we do not conduct testing under various combinations of distributed strains along the granular chain. However, the presented results demonstrate the feasibility of using multiple scatterers for localized strain sensing based on the ‘pulse-echo’ mechanism of solitary waves. Further studies will be pursued to investigate such effects under various compressions, in order to assess the localized strains accurately based on backscattered signals from multiple impurities.

4. Conclusion

In conclusion, we have investigated the effect of initial strains on the propagation of solitary waves in a granular chain that includes a pair of impurities. We experimentally visualized the transmission and backscattering of solitary waves in the region of impurities by means of a scanning laser Doppler vibrometer. Based on the solitary wave profiles measured from an instrumented sensor particle, we have found that the difference of travelling times between the two backscattered solitary waves is highly sensitive to the strains applied to the granular chain. The numerical results from a discrete element model agree well with experimental results and theoretical predictions. This approach based on ‘pulse-echo’ method is an improvement over the conventional ‘pitch-catch’ method, enabling localized strain measurements in multiple particle spots by strategically positioning impurities and measuring backscattered solitary waves using a single sensor. This work forms a foundation using highly nonlinear solitary waves as novel information carriers to measure strains in a granular system. The findings in this study can be further extended to real-time monitoring of distributed defects and/or external impacts in granular media by recording and analysing multiple packets of emitted solitary waves.

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References


