QUIZ 1

Physics 325A Wed. Jan. 14, 2015

IN CLASS, OPEN BOOK & NOTES NO FAULT QUIZ. Feel free to discuss with others. Answer all parts, continuing on the back if necessary. For each part give an appropriate *formula* and when appropriate a numerical answer (1 significant digit suffices).

1) Electron States in a 1-dimensional wire of length L. Assuming the wire can be modeled as a free particle system in one dimension between $x = 0$ and $x = L$ with periodic boundary conditions $\psi(x+L) = \psi(x)$, the electron states are plane waves-spinors $\psi_{k,s}(x) = A\exp(ikx)\chi_s$ with $k = (2\pi m/L)$ where m is an integer.

(a) Assuming that there are N electrons in the wire, determine the Fermi-momentum k_F and the Fermi-energy ϵ_F in terms of the density of electrons per unit length $n = N/L$ at $T = 0$. Hint: first determine k_F by changing the sum $N = \sum_{ks} n_{ks}$ over k and s to a 1-dimensional integral with $n_{ks} = 1$ ($|k| \leq k_F$) and 0 ($|k| \geq k_F$).

$$N = \sum_{ks} n_{ks} = 2(L/2\pi)\int_{-k_F}^{k_F} dk = 2k_F L/\pi$$

$$k_F = (\pi/2)N/L$$

(b) Determine the mean energy per electron in this system E/N as a function of $n = N/L$ and ϵ_F. Hint: use the ratio method $E/N = \sum_{ks}\epsilon_k n_{ks}/\sum_{ks} n_{ks}$.

$$E/N = \sum_k \epsilon_k n_k \Big/ \sum_k n_k = (\hbar^2/2m)\int dk k^2 \Big/ \int dk = (\hbar^2/2m)k_F^3/3k_F = (1/3)\epsilon_F$$

(c) Determine the Fermi force exerted by electrons $F = -dE/dL$ on the wire. Hint: use the result of (b) for $E(N/L)$.

$$E = (1/3)N\epsilon_F = A/L^2, \qquad -\partial E/\partial L = 2E/L = (2/3)n\epsilon_F$$

(d) The density of states per unit length $g(\epsilon) = (1/L)\sum_{k,s}\delta(\epsilon - \epsilon_k) = A\epsilon^{\alpha}$. Show that $\alpha = -1/2$ and determine A by integrating $g(\epsilon)$ up to the Fermi energy.

$$g(\epsilon) = (1/L)\sum_{k,s}\delta(\epsilon - \epsilon_k) = (1/L)2(L/2\pi)\int (dk/d\epsilon)d\epsilon\delta(\epsilon - \epsilon_k) = (1/\pi)dk/d\epsilon \sim 1/k$$

$$g(\epsilon) = A\epsilon^{-1/2}, \qquad \int^{\epsilon_F} g(\epsilon) = N/L = A\epsilon_F^{1/2}/(1/2) \qquad A = 2/\epsilon_F^{1/2}$$