TABLE I. The $l=0$ phase shifts for dineutron-dineutron scattering as a function of the relative kinetic energy. The Brink and Boeker potential $B1$ is used and $b=1.5$ fm. Phase shifts on the first and second lines correspond to $N-\nu_l=4$ and 8, respectively.

E (MeV)	0.25	3	5	10	20
η_0	2.88	2.26	2.02	1.59	1.60
η_0	2.89	2.27	2.02	1.60	1.05

accurate as wanted and allows for f_l the traditional boundary conditions on the scattering wave g_l. In this representation the removal of the trivial states forbidden by the Pauli principle and the numerical treatment of the Hill-Wheeler equation are quite straightforward. The difficult problem of formulating a matching condition for f_l has been reduced to a similar but much more familar problem for g_l.

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Spin Determinations with α–Heavy-Ion Angular Correlations*

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A new reaction-mechanism–independent method is proposed for determining spins of nuclear levels from angular correlations between α particles and heavy ions. This is an alternative approach to the well-known Method II of Litherland and Ferguson, permitting geometry other than 0° or 180° for the detection of the reaction particle.

Recently it has been found[1] that certain heavy-ion reactions have large cross sections for populating continuum states with relatively large α-decay widths, via the transfer of four or eight nucleons. This has produced a revival of interest in using particle-α angular correlations to determine the spins of such states. The usual method employed in performing such spin determinations has been the Litherland-Ferguson Method II,[2] which involves a coincidence angular correlation between the decay-particle (α) detector and the reaction-particle (e.g., heavy ion) detector located at 0° or 180°, the latter employing an annular detector, a magnetic analyzer, or a beam-stopping foil in front of the counter. However, in some experiments, particle detection at 0° or 180° is either not possible or highly undesirable.

The present work proposes an alternative method of spin determination which is independent of reaction mechanism. This approach does not require any specific position for the reaction-particle detector, so it may be placed at any convenient, arbitrary angle, e.g., the angle of maximum cross section. The price to be paid for this advantage is that the decay-particle detector must be moved in a precalculated trajectory out of the reaction plane, i.e., its angular position must be moved in both polar and azimuthal coordinates. However, when this is done the correlation function is a simple sinusoid with a frequency characteristic of the spin of the state. This simple form arises from a fortuitous grouping of the zeros of the reduced-rotation-matrix element which determine the form of the correlation functions in three dimensions, as will be seen below.

It has been shown in a previous paper[3] that the correlation function for a reaction leading to a state of spin J, which then emits a particle of helicity σ to a spin-zero final state, can be written as

$$W_J(\theta,\varphi)=\sum_\sigma|\sum_M(-1)^M p_M{}^J e^{-iM\varphi} d_{M\sigma}{}^J(\theta)|^2, \quad (1)$$

where M is the spin projection of J on the quantization or z axis, $p_M{}^J$ is a complex coefficient which characterizes the nuclear polarization of the state J produced by the nuclear reaction, θ and φ are the angular coordinates of the decay particle in spherical polar coordinates, and $d_{M\sigma}{}^J(\theta)$ is a reduced-rotation-matrix element.[4]

We may make two simplifying restrictions for the case of interest here. First, since the decay particle is an α the spin and helicity are zero and the sum over σ can be dropped. Second, if we deal only with reactions populating the state of spin J in which the target, incident, and outgoing particles are spinless, and if we choose the z axis of our coordinate system perpendicular to the reaction plane, then we may invoke reflection symmetry[5] to show that $J+M$ is even. We may now drop the $(-1)^M$ factor and (1) becomes

$$W_J(\theta,\varphi)=|\sum_{M=-J}^{J} p_M{}^J e^{-iM\varphi} d_{M0}{}^J(\theta)|^2, \quad (2)$$

where $J+M$ is even. This relation is deceptively simple in appearance; for the general case of a nuclear reaction producing state J, the nucleus will be polarized in some way which depends on the reaction mechanism and is characterized by a set of strongly M-dependent,[6] complex coefficients, $p_M{}^J$. In general, this correlation function will be fairly complicated and difficult to use for spin determinations. However, as it happens there is a particular value of θ for each value of J for which most of the elements of the reduced rotation matrix in (2) are essentially zero, and the correlation function is greatly simplified. This can be seen from Table I, which gives the central zeros (i.e., those nearest 90°) of the rotation-matrix elements which would appear in Eq. (2) for various values of J. Only natural-parity states are considered here since only those would have an allowed α transition to a 0^+ final state, as we have assumed. We see that for all the J values which we have considered there are zeros within a few degrees of each other, in all of the d's, except for the "stretched" element $d_{J0}{}^J$. The last column in Table I gives the average of the root angles (θ_{av}) for the various d's, weighted with the derivative of d with θ as it crosses zero. It is seen that these average angles are within about two degrees of the zeros of all the d's considered.

TABLE I. Central zeros of the reduced-rotation-matrix elements $d_{M0}{}^J(\theta)$, for $J+M$ even. θ_M(deg) is listed for each J^π.

J^π	$M=0/1$	2/3	4/5	6/7	θ_{av}
0^+	...	...	...	...	...
1^-	...	...	...	...	...
2^+	54.7	...	...	...	54.7
3^-	63.4	...	...	...	63.4
4^+	70.1	67.8	...	...	68.9
5^-	73.4	70.5	...	...	72.1
6^+	76.2	75.5	72.5	...	74.8
7^-	77.9	76.9	73.9	...	76.3
8^+	79.4	79.1	78.0	75.0	77.8

To an excellent approximation, tested numerically, the slope-weighted average of the root angles, θ_M, may be written as

$$\theta_{av}=\sum_M\theta_M(J^2-M^2)^{1/2}/\sum_M(J^2-M^2)^{1/2}, \quad (3)$$

where $M\neq J$. This means that the derivative weighting factor inhibits the contribution of the higher M values to θ_{av}, since these $d_{M0}{}^J(\theta)$ terms have the shallowest slopes at their root angles. This is fortunate, since Table I shows that those values of θ_M with the largest M values deviate the most from the mean.

Thus, for all the spins considered, and presumably for higher spins as well, there is a "magic" angle (θ_{av}) at which only the reduced-rotation-matrix elements corresponding to $M=\pm J$ are very different from zero. The latter all have roots only at 0° and 180° and are not far from their maximum values in the vicinity of θ_{av}. If one sets the d's with intermediate M values

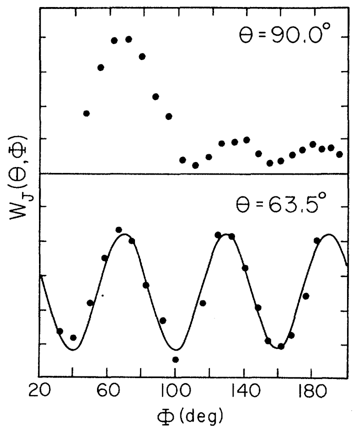

FIG. 1. Correlation functions for the reaction $^{12}C(\alpha, \alpha')^{12}C^*(\alpha_b)$. The reaction particle (α') is detected at 37.5° (c.m.) from the beam. The decay-particle (α_b) correlation functions are plotted versus φ for two different values of θ. Only the $d_{M0}{}^J(\theta)$ terms with $M=\pm 3$ contribute to the correlation function for $\theta=63.5°$, whereas both $M=\pm 1$ and $M=\pm 3$ terms contribute for $\theta=90°$.

identically equal to zero, then the correlation function can be written in the form

$$W_J(\theta_{\rm av}, \varphi) = A + B \sin^2(J\varphi - \varphi_0), \qquad (4)$$

where A, B, and φ_0 are related to the magnitudes and phases of the coefficients $p_J{}^J$ and $p_{-J}{}^J$, but for our purposes here can be treated as arbitrary constants. Thus, the correlation function reduces to a simple sinusoid-plus-constant form which is particularly easy to interpret and analyze.

It should be remembered that the angles (θ, φ) are center-of-mass angles and, in particular, are those for the center-of-mass system of the recoiling nucleus. The correlation experiment corresponds to holding θ fixed while varying φ. Since the measurements are performed in the laboratory, it is worth noting that this simple φ motion in the center-of-mass frame results in a complicated (θ, φ) trajectory in the laboratory frame. Because of this complication, one must prepare a table of (laboratory) polar and azimuthal angles which have the effect of varying φ while holding θ fixed (in the center-of-mass system).

Figure 1 shows the results of an experiment[7] in which a correlation of this form was measured for a 3^- state, both in the reaction plane (i.e., $\theta = 90°$) and at the average root angle, $\theta = \theta_{\rm av} = 63.4°$, of the reduced rotation matrix. The differences in the shapes of the two correlation functions and the simplicity of the correlation function at the root angles are quite striking. In this particular case, the state was populated by the reaction $^{12}C(\alpha, \alpha')^{12}C^*$(9.6 MeV) rather than by a heavy-ion transfer reaction, but the formalism is exactly the same because the target, beam, and coincident particles are all spinless.

In closing, it is worth noting that $\theta_{\rm av}$ does not change value very rapidly with J. This slow dependence of $\theta_{\rm av}$ on J permits the use of conventional plots of χ^2 (the variance of the best fit of W_J to the data) versus J. Even so, the experimental angle, θ, should be the $\theta_{\rm av}$ for the most likely value of the spin J. The first several data points should indicate, from the phasing, whether the expected J is close to the correct value. Final data, to be believable, should be taken at the $\theta_{\rm av}$ of the J assigned and the normalized χ^2 should be near unity.

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