Application of Austern-Blair theory to the interference between Coulomb and nuclear excitation in the inelastic scattering of heavy ions*

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The connection between elastic and inelastic scattering of heavy ions in the combined Coulomb and nuclear fields is discussed in terms of the Austern-Blair theory. It is shown that the first order distorted-wave Born-approximation inelastic scattering amplitudes may be obtained from the elastic scattering amplitudes to a good degree of accuracy. Consequently they contain essentially the same information about the nuclear potentials as the elastic scattering S-matrix elements.

[NUCLEAR REACTIONS Calculations of HI inelastic scattering with Coulomb-nuclear interference; comparison of DWBA with Austern-Blair approximation; cases studied $^{56}Fe(^{16}O,^{16}O')$ and $^{26}Mg(^{16}O,^{16}O')$; both $\sigma(\theta)$ and $\sigma(E)$.]

I. INTRODUCTION

For the inelastic scattering of heavy ions above the Coulomb barrier both the long range Coulomb field and the short range nuclear field give important contributions to the scattering amplitude. While the inelastic scattering in the Coulomb field is well understood,[1] the situation is less clear for inelastic scattering in the nuclear field. One of the most successful approaches to describing the latter has been the collective model[2] which assumes the nuclear excitation to be due to a nonspherical optical potential.[3] By coherently adding the transition amplitudes due to the Coulomb and nuclear fields, a good description of a variety of heavy ion inelastic scattering data has been obtained.[4-10]

Both semiclassical theories[11-13] and the distorted-wave Born approximation[4-10] (DWBA) have been used for the interpretation of the experimental data, which usually exhibit a characteristic dip in the angular distributions (or excitation functions) at an angle (or energy) where the elastic scattering starts to deviate from Rutherford scattering. This dip may be understood to be due to a destructive interference between Coulomb excitation and nuclear inelastic scattering. Measurement and analysis of cross sections in the vicinity of this interference minimum have been stated to result in a greater sensitivity of the angular distributions to the optical model parameters.[5]

On the other hand, it had already been pointed out by Austern and Blair[14] that the inelastic scattering amplitude for strongly absorbing particles is closely related to the elastic scattering amplitude, and a formalism was developed which allows for the calculation of the inelastic scattering amplitude in the nuclear field from the S matrix for elastic scattering. This approach is particularly attractive since it allows for a test of the collective model without specific assumptions about the deformed shape of the heavy ion optical potential which leads to the form factor for nuclear inelastic scattering.

The theory of Austern and Blair has not previously been applied to the interference of Coulomb and nuclear inelastic scattering and the question has remained open as to whether the Austern-Blair connection between elastic and inelastic scattering amplitudes is accurate enough to allow for quantitative calculations or whether a DWBA treatment is necessary. In particular, the applicability of the Austern-Blair (AB) theory would give a natural explanation of the observation that the DWBA treatment of heavy ion inelastic scattering is sensitive to the optical potential in essentially the same region of internuclear distances as is the elastic scattering of heavy ions if measured to a sufficient degree of accuracy.[15] Furthermore, the AB theory would provide a simple potential-independent way of determining if the data can be interpreted in terms of single step nuclear and Coulomb excitation processes or if coupled channel effects are important. The present paper gives a numerical comparison between the AB theory (slightly generalized[16] to include the interference between nuclear and Coulomb excitation) and the standard DWBA treatment of this problem. The Coulomb excitation part of these comparisons is treated exactly and also using a more approximate treatment.

The formalism is presented in Sec. II. Numerical calculations are discussed in Sec. III. A summary and conclusion is given in Sec. IV. Some details about the calculations are given in the Appendix.

II. THEORY

For inelastic scattering from the initial state i to the final state f the transition amplitude may be written in the DWBA as[3]

$$T_{fi}^{\mathrm{DW}} = \int d^3r \chi^{(-)}(\vec{k}_f, \vec{r}) \langle \phi_f | V_f | \phi_i \rangle \chi^{(+)}(\vec{k}_i, \vec{r}). \quad (1)$$

In Eq. (1) the initial and final momenta are denoted by $\vec{k}_i$ and $\vec{k}_f$, the wave functions of relative motion calculated from a spherical optical potential $V_S(r)$ are $\chi^{(-)}(\vec{k}_f, \vec{r})$ and $\chi^{(+)}(\vec{k}_i, \vec{r})$, the intrinsic wave functions of the initial and final states are ϕ_i and ϕ_f, and the (nonspherical) effective interaction in the final channel is V_f. For our calculations a spherical optical potential of Woods-Saxon shape has been used:

$$V_S(r) = V_C(r) + U(r). \quad (2)$$

The Coulomb part is given by

$$V_C = \begin{cases} Z_1 Z_2 e^2/r, & \text{for } r \geq R_c \\ [Z_1 Z_2 e^2/2R_c][3 - (r/R_c)^2], & \text{for } r \leq R_c \end{cases} \quad (3)$$

with

$$R_c = 1.25 A_2^{1/3}. \quad (4)$$

The short range nuclear part is given by

$$U(r) = -Vf(R_R, a_R; r) - iWf(R_I, a_I; r) - 4iW_S f(R_S, a_S; r)[1 - f(R_S, a_S; r)], \quad (5)$$

where $f(R, a; r) = \{1 + \exp[(r - R)/a]\}^{-1}$ is a Woods-Saxon function, and

$$R_R = r_R(A_1^{1/3} + A_2^{1/3}), \quad R_I = r_I(A_1^{1/3} + A_2^{1/3}), \quad R_S = r_S(A_1^{1/3} + A_2^{1/3}). \quad (6)$$

Here A_1, Z_1 and A_2, Z_2 are the mass and charge of the projectile and target nuclei, respectively. These definitions are consistent with those of Ref. 5. The matrix element for the 2^λ excitation reads in the collective model[3,5]

$$\langle \phi_f | V_f | \phi_i \rangle_{\lambda\mu} = \left\{ \delta_\lambda \frac{dU(r)}{dR} + \frac{4\pi Z_1 e}{2\lambda + 1} \frac{[B(E\lambda, i \to f)]^{1/2}}{r^{\lambda+1}} \right\} Y_\lambda^{\mu *}(\hat{r}), \quad (7)$$

where we have introduced the deformation length δ_λ which may be connected to the deformation parameter β_λ as

$$\delta_\lambda = \beta_\lambda R. \quad (8)$$

We have chosen the standard method of connecting the deformation parameter for a quadrupole interaction ($\lambda = 2$) in first order to the reduced transition probability $B(E2, i \to f)$ by assuming a homogeneous charge distribution with a sharp surface, according to[5]

$$B(E2, i \to f) = \left(\frac{3}{4\pi} Z_2 e R_c \right)^2 (\beta_2 R_c)^2. \quad (9)$$

Following the formalism of Satchler[3,17] we introduce the reduced amplitudes $\beta_{\lambda\mu}$ defined by

$$\beta_{\lambda\mu} = \frac{-i^{-\lambda}}{(2\lambda + 1)^{1/2}} \times \int \chi^{(-)*}(\vec{k}_f, \vec{r}) \langle \phi_f | V_f | \phi_i \rangle_{\lambda\mu} \chi^{(+)}(\vec{k}_i, \vec{r}) d^3r \quad (10)$$

in terms of which the unpolarized cross section for a pure 2^λ excitation is given by

$$\frac{d\sigma}{d\Omega} = \left[\frac{A_1 A_2}{(A_1 + A_2) 2\pi\hbar^2} \right]^2 \frac{k_f}{k_i} \sum_\mu |\beta_{\lambda\mu}|^2. \quad (11)$$

The reduced amplitudes may be expressed in terms of the radial integrals by integrating over the angular variables

$$\beta_{\lambda\mu} = \frac{4\pi}{k_i k_f} \sum_{l_i, l_f} i^{l_i - l_f - \lambda} \exp[i(\sigma_{l_i}^{(i)} + \sigma_{l_f}^{(f)})](2l_f + 1)^{1/2} \times (\mathcal{I}_{l_f, l_i}^N + \mathcal{I}_{l_f, l_i}^C)(l_f 0, \lambda 0 | 00) \times (l_f - \mu, \lambda\mu | l_i 0) Y_{l_f}^{-\mu}(\theta, 0), \quad (12)$$

where

$$\mathcal{I}_{l_f, l_i}^N = \int_0^\infty f_{l_f}(k_f, r) \left[\delta_\lambda \frac{dU(r)}{dR} \right] f_{l_i}(k_i, r) dr \quad (13)$$

and

$$\mathcal{I}_{l_f, l_i}^C = \int_0^\infty f_{l_f}(k_f, r) \left\{ \frac{4\pi Z_1 e}{2\lambda + 1} \frac{[B(E\lambda, i \to f)]^{1/2}}{r^{\lambda+1}} \right\} \times f_{l_i}(k_i, r) dr. \quad (14)$$

Here, the z axis has been chosen in the incident beam direction. The angular momenta of relative motion in the initial and final channels are denoted by l_i and l_f, the corresponding Coulomb phase shifts are $\sigma_{l_i}^{(i)}$ and $\sigma_{l_f}^{(f)}$ and the radial wave functions of relative motion are $f_{l_f}(k_f, r)$ and $f_{l_i}(k_i, r)$. For the sake of convenience, the radial integrals have been written as the sum of the nuclear radial integrals $\mathcal{I}_{l_f, l_i}^N$ and the Coulomb radial integrals $\mathcal{I}_{l_f, l_i}^C$. Here we also define the diagonal integrals:

$$\mathcal{I}_l \equiv \int_0^\infty f_l(k, r) \frac{\partial U(r)}{\partial R} f_l(k, r) dr \quad (15)$$

and in the adiabatic limit ($k_i = k_f$) we observe that $\mathcal{I}_{l,l}^N = \delta_\lambda \mathcal{I}_l$. Our calculations with the Austern-Blair approximation will involve approximations to these diagonal integrals.

The off-diagonal nuclear radial integrals, which are the quantities of interest in inelastic scattering, have been approximated[3,14,18] in three ways:

$$\mathcal{J}^N_{l_f,l_i} \simeq \delta_\lambda \mathcal{I}_{\bar{l}}(E_i) \equiv \mathcal{J}^{(1)}_{l_f,l_i} \tag{16}$$

or

$$\mathcal{J}^N_{l_f,l_i} \simeq \delta_\lambda [\mathcal{I}_{l_f}(E_f)\mathcal{I}_{l_i}(E_i)]^{1/2} \equiv \mathcal{J}^{(2)}_{l_f,l_i} \tag{17}$$

or

$$\mathcal{J}^N_{l_f,l_i} \simeq \delta_\lambda \mathcal{I}_{\bar{l}}(\bar{E}) \equiv \mathcal{J}^{(3)}_{l_f,l_i}, \tag{18}$$

where

$$\bar{l} = \tfrac{1}{2}(l_i + l_f), \quad \bar{E} = \tfrac{1}{2}(E_i + E_f) \tag{19}$$

and E_i and E_f denote the center of mass energies of the initial and final system, respectively. For the sake of brevity we will denote the approximations given by Eqs. (16), (17), and (18) as (1), (2), and (3), respectively. Approximation (2) was suggested by Hahne[18] and is the only one that emerges from the formal structure of Eq. (13). It should be emphasized that the $\mathcal{I}_l$'s are complex quantities and that great care must be taken to insure that the phase is in the proper quadrant after the square root. Hahne's approximation is known to be the best of the three considered here particularly for the scattering of strongly absorbed particles[3] and has been extensively tested for α scattering.[18] This is also borne out in our calculations for the case of heavy ion scattering in the Coulomb-nuclear interference region. Approximation (1) uses the average l value in the diagonal integral but otherwise rather unsymmetrically emphasizes the entrance channel. Approximation (3) is more symmetrical than (1) and is somewhat better in that respect. The latter two approximations, although less accurate, offer the advantage that they require only the solution of the Schrödinger equation (or a knowledge of the elastic scattering S matrix) at a single energy, and it is instructive to see the rather remarkable degree of accuracy of even these simple approximations (see Secs. III B and C below).

It has been shown by Austern and Blair[14] that there exists a relation between the diagonal integrals $\mathcal{I}_l(E)$ and the elastic scattering reflection coefficients $\eta_l(E)$ [see Eq. (2.17) of Ref. 14]:

$$\int_0^\infty f_l(k,r)\frac{\partial U}{\partial h} f_l(k,r)dr = \frac{iE}{2k}\frac{\partial \eta_l}{\partial h} \tag{20}$$

and therefore

$$\mathcal{I}_l(E) = \frac{iE}{2k}\frac{\partial \eta_l(E)}{\partial R} = \frac{iE}{2k}\frac{\partial \eta_l(E)}{\partial l}\frac{\partial l}{\partial R}, \tag{21}$$

where h is any parameter of the spherically symmetric local optical potential from which the reflection coefficients are calculated and R is the radius of the same potential. Furthermore, we may approximate the derivative $\partial l/\partial R$ by considering the motion of particles in semiclassical Coulomb trajectories. At the semiclassical turning radius r_t we have the relations:

$$l(l+1) = kr_t(kr_t - 2n)$$

or (22)

$$kr_t = n + [n^2 + l(l+1)]^{1/2},$$

where n is the Coulomb parameter defined by $n = Z_1 Z_2 e^2/\hbar v$. Differentiating this expression with respect to r_t and associating the turning radius with the potential radius R by making the derivative negative, we obtain the approximate relation:

$$\frac{\partial l}{\partial R} \simeq \frac{-2k}{2l+1}[n^2 + l(l+1)]^{1/2}. \tag{23}$$

This expression is equivalent to Eq. (4.3) of Ref. 14 in the limit $n \to 0$ and $l \gg 1$. We may now substitute this expression into Eq. (21) to obtain an expression for the diagonal integrals which can be obtained from numerical differentiation of the reflection coefficients:

$$\mathcal{I}_l(E) \simeq \frac{iE}{2l+1}[n^2 + l(l+1)]^{1/2}\frac{\partial \eta_l(E)}{\partial l} \equiv \mathcal{I}_l^{(\mathrm{AB})}(E). \tag{24}$$

The derivatives of the reflection coefficients can be estimated by taking numerical differences between those for adjacent l values, or to a better approximation by using the second difference relation given by Eq. (25.3.4) of Ref. 19:

$$\frac{\partial \eta_l}{\partial l} \simeq \tfrac{1}{2}[3\eta_l - 4\eta_{l-1} + \eta_{l-2}]. \tag{25}$$

This permits the derivatives and therefore the radial integrals to be calculated directly during an optical model calculation through the use of approximations (1) through (3) and Eq. (24) by proceeding from small to large l values using the reflection coefficients already calculated. Once the elastic scattering amplitudes are known, the inelastic scattering amplitudes are determined and do not depend on any details of the optical potential beyond those which determine the elastic scattering. However, the Coulomb radial integrals, Eq. (14), are integrated numerically using the radial wave function $f_l(k,r)$, and therefore they depend on the optical potential via those radial wave functions.

The influence of the optical potential on the Cou-

lomb radial integrals will depend on the angular momentum values l_i and l_f. For angular momenta which are large enough that the elastic scattering S matrix elements are essentially equal to unity (i.e., having an impact parameter large enough to give pure Rutherford scattering), the dependence of the Coulomb integrals on the optical potential is expected to be very weak. Therefore the distorted waves in the region of interest may be approximated by regular Coulomb functions.

In this case one may avoid the numerical integration of the wave functions and approximate the Coulomb radial integrals according to the Sopkovitch approximation[20]:

$$\mathcal{J}^{C}_{l_f,l_i} \simeq (\eta_{l_f}\eta_{l_i})^{1/2}\int_0^\infty F_{l_f}(n_f,k_f r)\,\frac{4\pi Z_1 e}{(2\lambda+1)}\,\frac{[B(E\lambda,i\to f)]^{1/2}}{r^{\lambda+1}}\,F_{l_i}(n_i,k_i r)dr, \tag{26}$$

where $F_l(n,\rho)$ is the regular Coulomb function.[19] By using approximations (16), (17) or (18), and (26) one may, in principle, calculate the inelastic scattering amplitude by using only the knowledge of the elastic scattering S matrix without further relying on any specific assumptions on the choice of the optical potential.

III. CALCULATIONS

In this section we will examine the various approximations introduced in Sec. II. We have chosen as an example the inelastic scattering of ^{16}O on ^{56}Fe leading to the first excited 2^+ state in ^{56}Fe at 0.847 keV excitation energy which has been previously investigated.[9] By this means we will seek some insight into the inelastic scattering of heavy ions. It will be shown that the Austern-Blair theory represents a very good approximation indeed to first-order DWBA calculations and that the connection between elastic and inelastic scattering amplitudes holds to a surprising degree of accuracy.

A. Application of the Austern-Blair approximation to diagonal radial integrals in the adiabatic limit

In many applications of interest, the reactions are well matched, i.e., the wave functions describing the relative motion of the colliding nuclei in the entrance and exit channels are quite similar to the interaction region. In order to obtain some insight into these reactions it is useful to discuss first the adiabatic limit of the AB theory, i.e., the approximation represented by Eq. (24).

Here and in the following we write the elastic reflection coefficients or radial integrals in the following form

$$\eta_l = |\eta_l| e^{i\phi_l} \tag{27}$$

and define the corresponding deflection functions by

$$\begin{aligned}\theta_l &= \frac{d}{dl}\phi_l + 2\,\frac{d}{dl}\,\sigma_l \\ &\simeq (\phi_l - \phi_{l-1}) + 2(\sigma_l - \sigma_{l-1}),\end{aligned} \tag{28}$$

where σ_l is the Coulomb phase shift.

In the classical limit, these quantities have a very simple physical interpretation[21]: The transition probability for a given partial wave with orbital angular momentum l is given by $|\eta_l|^2$ and the classical deflection angle is given by θ_l. See, however, the more general discussions of Ref. 22.

Figure 1 gives a comparison in magnitude and deflection function (i.e., phase) of the radial integrals $\mathcal{J}_l$ calculated from the left hand side of Eq. (16) and the corresponding values $\mathcal{J}_l^{(AB)}$ of the Austern-Blair theory obtained from the right hand side of Eq. (24) for the case of ^{16}O on ^{56}Fe at 43 MeV laboratory energy. The potential parameters have been taken from Ref. 9 and are listed on Table I. Also included in Fig. 1 are the elastic reflection coefficients η_l and the deflection function θ_l(Coulomb) corresponding to pure Coulomb scattering.

The differences of the absolute phases between $\mathcal{J}_l$ and $\mathcal{J}_l^{(AB)}$ are shown in Fig. 2. It is evident from the figures that the relation given by Eq. (24) between the elastic scattering S matrix and the diagonal radial integrals taken in the adiabatic limit holds to a good degree of accuracy. Characteristic features of the radial integrals which have been found also for transfer reactions with nonvanishing Q values[23] are seen from Fig. 1 to be implicit in the elastic scattering reflection coefficients alone: The radial integrals are localized in angular momentum and the inelastic deflection functions exhibit a characteristic dip which has previously been attributed to the dominance of one or a few Regge poles in the description of the wave function of relative motions close to the nuclear surface.[24] Note that the corresponding dip in the elastic deflection function is much smaller.

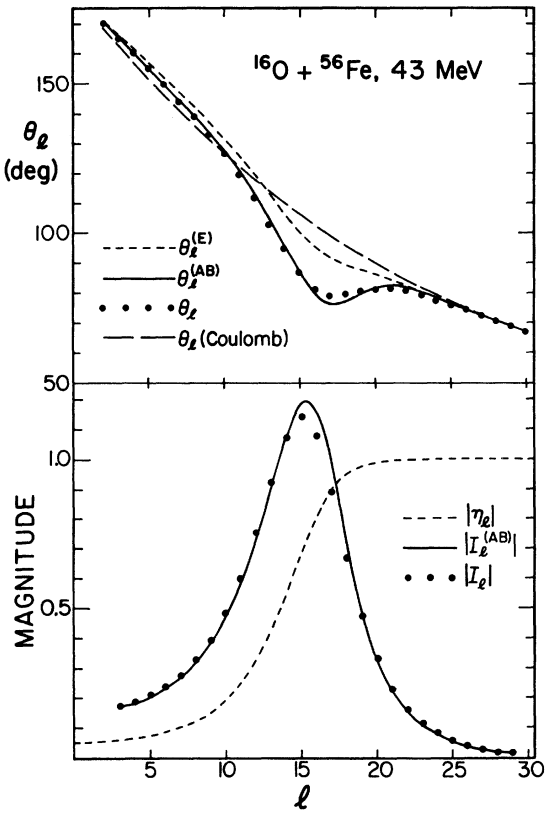

FIG. 1. Moduli and deflection functions of elastic scattering reflection coefficients η_l, adiabatic diagonal radial integrals $\mathcal{I}_l$, and the corresponding AB approximation $\mathcal{I}_l^{(AB)}$, for the scattering of ^{16}O from ^{56}Fe at 43 MeV laboratory energy. The calculations have been performed using the potential parameters of Ref. 9. Also shown is the deflection function corresponding to pure Coulomb scattering.

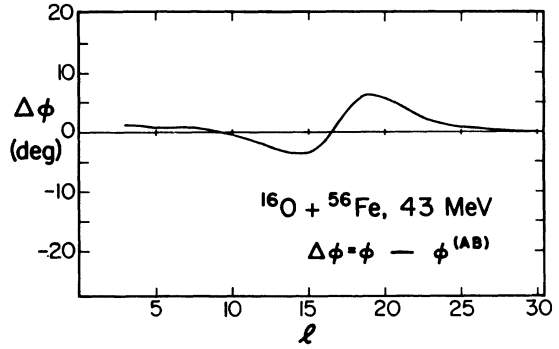

FIG. 2. Phase difference between adiabatic diagonal radial integrals and the corresponding AB approximation for the scattering of ^{16}O from ^{56}Fe at 43 MeV.

For all angular momenta the elastic scattering deflection function follows quite closely that of pure Coulomb scattering.[25] This may be taken as an *a posteriori* justification of the assumption in the derivation of Eq. (24) that the particles move on Rutherford trajectories.

As a second illustration, Eq. (24) is applied to the scattering of ^{16}O on ^{26}Mg at 40 MeV using the potential parameters of Ref. 26 which had been used for the interpretation of the two-proton transfer reaction $^{26}Mg(^{16}O,^{14}C)^{28}Si$. This is a "surface-transparent" potential using the parameterization of Ref. 27. The deflection functions and the values η_l, $\mathcal{I}_l$, and $\mathcal{I}_l^{(AB)}$ are shown in Fig. 3 and the phase difference between $\mathcal{I}_l$ and $\mathcal{I}_l^{(AB)}$ is shown in Fig. 4. Again, Eq. (24) holds to a good degree of accuracy and the conclusions given above apply here also. Since there is a steeper transition of the elastic scattering reflection coefficients from $\eta_l \sim 0$ to $\eta_l \sim 1$, there is a sharp localization of the radial integral $\mathcal{I}_l$ in angular momentum space. As before, the deflection function θ_l has a pronounced minimum at the "grazing" angular momentum, which is not present in the elastic deflection function $\theta_l^{(E)}$. These general features agree with the characteristics of the radial integrals discussed in Ref. 26.

B. Application of the Austern-Blair approximation to the radial integrals of nuclear inelastic scattering for $Q \neq 0$

We will now discuss the Austern-Blair approximation for the radial integrals $\mathcal{I}^N_{l_f,l_i}$ applied to the inelastic scattering of ^{16}O on ^{56}Fe ($2^+, E_x = 0.847$ keV) at 43 MeV incident energy. In Figs. 5–7 the exact radial integrals $\mathcal{J}^N_{l_f,l_i}$ obtained from the numerical integration of Eq. (13) are compared with the radial integrals $\mathcal{J}^{(1)}_{l_f,l_i}$, $\mathcal{J}^{(2)}_{l_f,l_i}$, and $\mathcal{J}^{(3)}_{l_f,l_i}$ obtained by using the approximations of Eqs. (16)–(18). Also shown in the figures are the phase differences $\Delta\phi^{(k)}_{l_f,l_i}$ between the phase of the exact radial integrals $\mathcal{J}^N_{l_f,l_i}$ and the approximation $\mathcal{J}^{(k)}_{l_f,l_i}$ ($k = 1,2,3$). The phase differences are of special interest for the calculation of the interference between Coulomb and nuclear inelastic scattering.

TABLE I. Parameters used in the present calculations.

	V (MeV)	r_R (fm)	a_R (fm)	W (MeV)	r_I (fm)	a_I (fm)	W_s (MeV)	r_s (fm)	a_s (fm)	r_c (fm)	δ_2	$\frac{B(E2,i\to f)}{(e^2\,\mathrm{fm}^4)}$
$^{16}O+^{56}Fe$	30.0	1.3	0.533	7.6	1.3	0.37	0	...	...	1.25	0.23	974
$^{16}O+^{26}Mg$	100	1.22	0.5	65	1.17	0.05	3.34	1.17	0.5	1.25	...	...

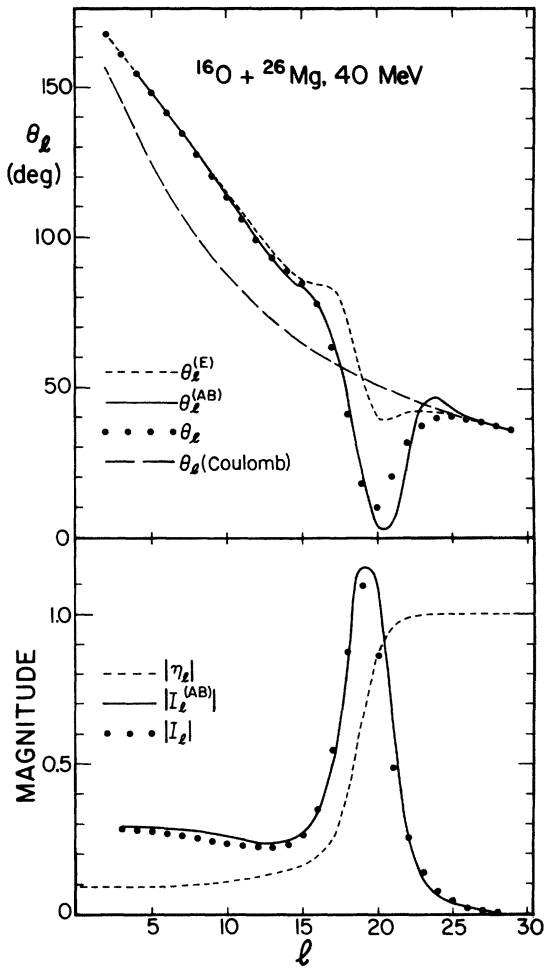

FIG. 3. Moduli and deflection functions of elastic scattering reflection coefficients η_l, adiabatic diagonal radial integrals $\mathcal{I}_l$, and the corresponding AB approximation, $\mathcal{I}_l^{(AB)}$, for the scattering of ^{16}O from ^{26}Mg at 40 MeV laboratory energy. The surface transparent optical potential of Ref. 26 has been used. Also shown is the deflection function for pure Coulomb scattering.

We note that Fig. 5 corresponds to the most semiclassical of the radial integrals in that $l_i > l_f$. Thus the collision results in a loss of energy ($Q < 0$), a loss of momentum, and a loss of angular momentum. For this case we find the best agreement of all the approximations, particularly (2) and (3), with the exact calculation. This can probably be attributed to the semiclassical nature of the approximation given in Eq. (22). In Figs. 6 and 7 we go progressively away from the semiclassical situation with no loss of angular momentum or an actual gain in angular momentum in the exit channel, and the approximations used become progressively worse. However, we find that approximation (2), which takes the geometric mean

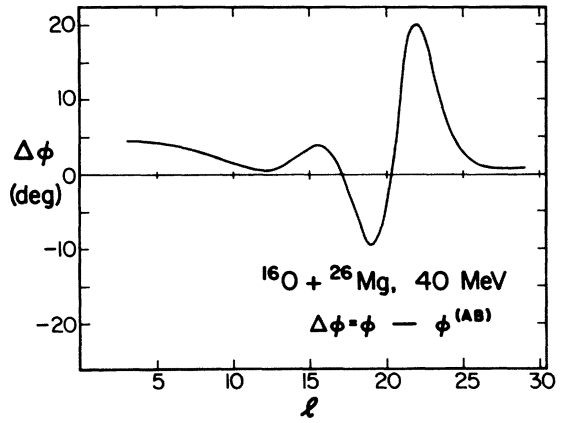

FIG. 4. Phase difference between adiabatic diagonal radial integrals and the corresponding AB approximation for the scattering of ^{16}O from ^{26}Mg at 40 MeV laboratory energy.

of the diagonal integrals of the entrance and exit channels, does remarkably well in reproducing the behavior of the exact radial integrals, even in the nonsemiclassical cases. The phase is reproduced to within 5° and the moduli are in good agreement with the exact calculations.

By using an adiabatic approximation at the aver-

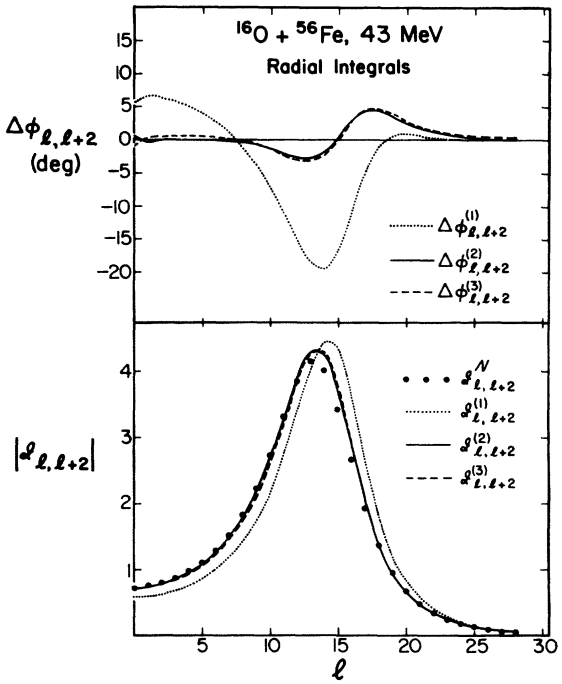

FIG. 5. Moduli of radial integrals $\mathcal{I}^N_{l,l+2}$ and AB approximations $\mathcal{I}^{(k)}_{l,l+2}$ ($k = 1, 2, 3$) in MeV fm, and phase differences $\Delta\phi^{(k)}_{l,l+2}$ between $\mathcal{I}^N_{l,l+2}$ and $\mathcal{I}^{(k)}_{l,l+2}$ in degrees for inelastic scattering of ^{16}O on ^{56}Fe at 43 MeV laboratory energy.

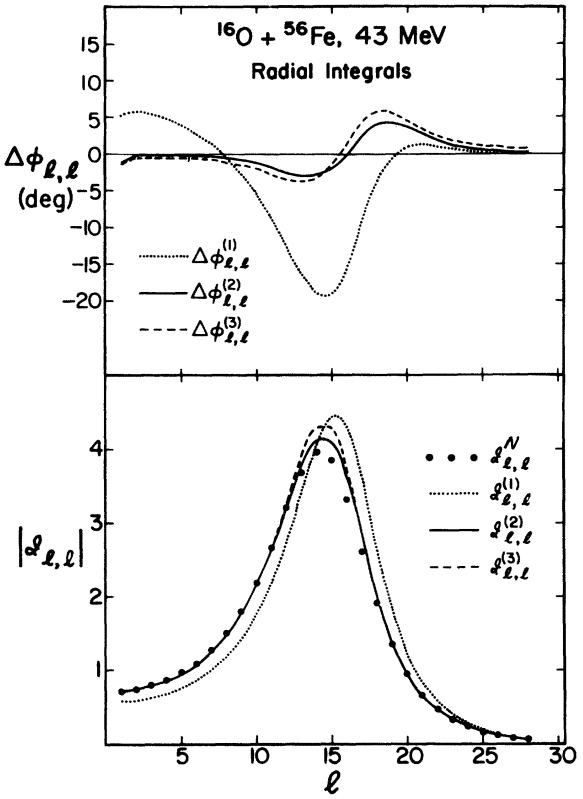

FIG. 6. Moduli of radial integrals $\mathcal{I}^{N}_{l,l}$ and AB approximations $\mathcal{I}^{(k)}_{l,l}$ ($k=1,2,3$) in MeV fm, and phase differences $\Delta\phi^{(k)}_{l,l}$ between $\mathcal{I}^{(N)}_{l,l}$ and $\mathcal{I}^{(k)}_{l,l}$ in degrees for inelastic scattering of ^{16}O on ^{56}Fe at 43 MeV laboratory energy.

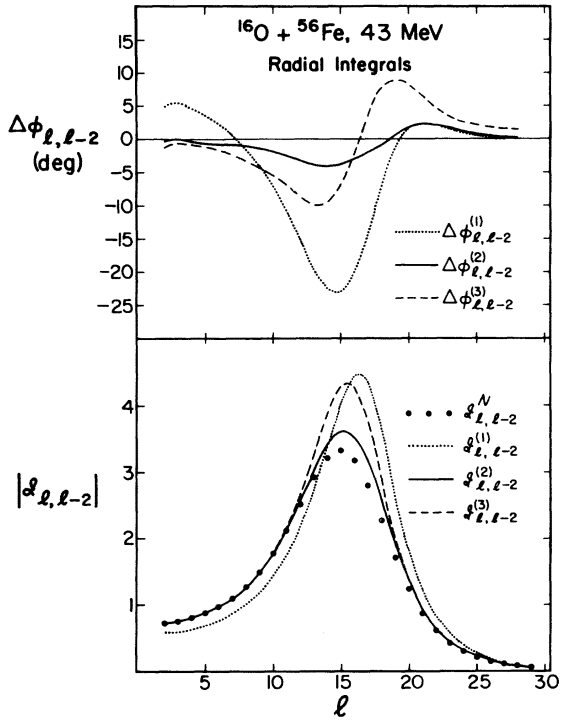

FIG. 7. Moduli of radial integrals $\mathcal{I}^{N}_{l,l-2}$ and AB approximations $\mathcal{I}^{(k)}_{l,l-2}$ ($k=1,2,3$) in MeV fm, and phase differences $\Delta\phi^{(k)}_{l,l-2}$ between $\mathcal{I}^{N}_{l,l-2}$ and $\mathcal{I}^{(k)}_{l,l-2}$ in degrees for inelastic scattering of ^{16}O on ^{56}Fe at 43 MeV.

age energy between entrance and exit channels, approximation (3) [Eq. (18)], one obtains an approximation which is significantly better than approximation (1), but not as good as approximation (2).

C. Application of the Austern-Blair approximation to Coulomb and nuclear inelastic scattering

From the previous sections it has become obvious that the Austern-Blair approximation can be used with good accuracy to calculate the scattering amplitude for the nuclear inelastic scattering of heavy ions. A more sensitive test of the approximation would be the calculation of the inelastic scattering in the combined Coulomb and nuclear potentials, since this interference will be sensitive not only to the absolute magnitudes of the corresponding transition amplitudes but also to the relative phases.

Figure 8 compares the angular distributions of the inelastic scattering of ^{16}O on ^{56}Fe at 43 MeV laboratory bombarding energy, predicted using the DWBA by numerical integration of the Coulomb and nuclear radial integrals [see Eqs. (13) and (14)] with those obtained by substituting the Austern-Blair approximations (1) and (2) [see Eqs. (16) and (17)] for the nuclear radial integrals. The adiabatic approximation (1) [denoted by AB(1) in the figure] gives qualitatively the same result

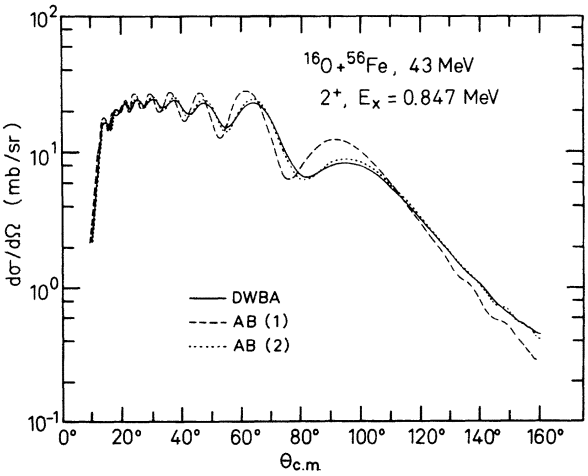

FIG. 8. Comparison of DWBA calculation and AB approximations (1) and (2) for inelastic scattering of ^{16}O on ^{56}Fe at 43 MeV laboratory energy.

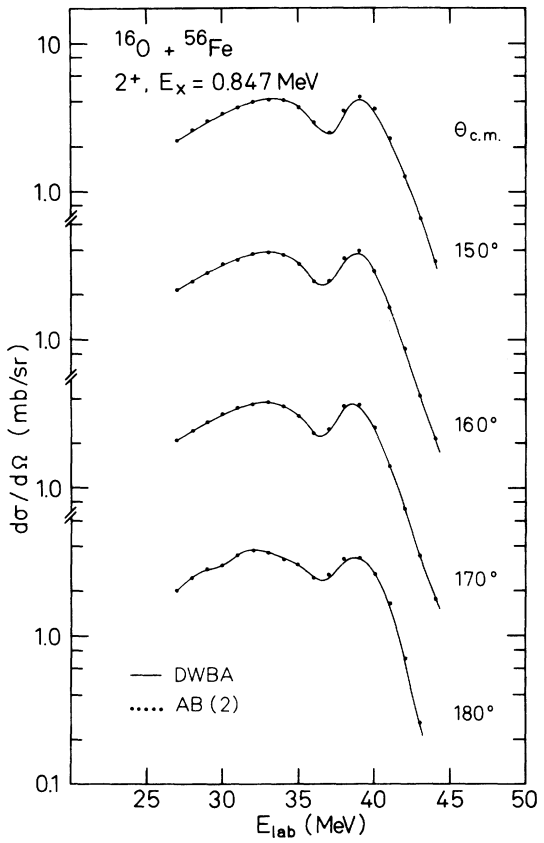

FIG. 9. Comparison of DWBA calculation and AB approximation (2) for inelastic scattering of ^{16}O on ^{56}Fe at 150°, 160°, 170°, and 180°.

as the DWBA. However, the numerical inaccuracies are significant enough to be experimentally observable. Hahne's approximation[18] [denoted by AB(2) in the figure] on the other hand, yields a remarkably good approximation to the exact calculation. This is also evident from Fig. 9 which compares the excitation functions at large scattering angles obtained from the DWBA and the Austern-Blair approximation (2).

D. On the connection between elastic and inelastic scattering

It has already been pointed out in Sec. II that the Coulomb radial integrals [Eq. (14)] depend on the optical potential via the radial wave functions. One may get an estimate of this implicit dependence on the optical potential by introducing an interior radial cutoff R_{cut} into the Coulomb radial integrals [as the lower limit of integration in Eq. (14)] and investigating the sensitivity of the inelastic scattering angular distributions on the cutoff radii. This has been done in the calculations presented in Fig. 10. It is evident from the figure that the calculations are insensitive to values of $R_{cut} \lesssim 9$

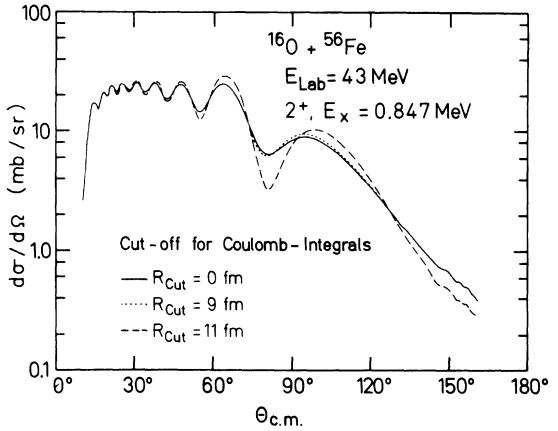

FIG. 10. Dependence of the angular distributions on the cutoff radius R_{cut} used in the evaluation of the Coulomb radial integrals.

fm, as compared with the real well radius of the optical potential of $R = 8.25$ fm. Even at a cutoff radius of 11 fm a reasonable approximation to the exact calculation is obtained. Thus, the calculations are only sensitive to the tail region of the optical potential.

Due to the insensitivity of the calculations to the details of the optical potential one may calculate the inelastic scattering by using the approximation[18] given in Eq. (17) and the Sopkovich approximation[20] given in Eq. (26) for the nuclear and Coulomb radial integrals, respectively. By using this approximation [denoted by AB(2)-C] one may, in principle, obtain the inelastic scattering angular distributions by using only a knowledge of the elastic scattering S matrix, with no assumptions about the shape of the optical potential. The numerical comparison with exact calculation is shown in Fig. 11. The agreement is surprisingly

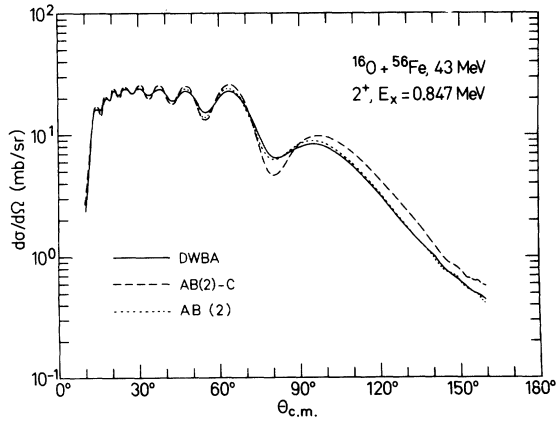

FIG. 11. Comparison of DWBA calculation and approximation AB(2)-C which uses only the elastic scattering reflection coefficients for the evaluation using the Sopkovitch approximation (Ref. 20) of the inelastic transition amplitude.

good (although not perfect). The phase of the interference pattern is well reproduced, and the amplitudes of the oscillations are only slightly enhanced as compared with the exact calculations.

IV. SUMMARY AND CONCLUSIONS

The connection between the elastic and inelastic scattering of heavy ions has been investigated by using the theory of Austern and Blair.[14] The theory is based on the collective model for nuclear excitations and is expected to apply for the case of strongly absorbed particles.

As a numerical example, the inelastic scattering of ^{16}O on ^{56}Fe leading to the first excited 2^+ state in ^{56}Fe has been chosen. It has been shown that the theory accounts for the qualitative features of DW radial integrals in the adiabatic limit, and reproduces the localization of the reaction amplitudes in angular momentum space and the occurrence of a minimum in the deflection function in the region of angular momenta where the radial integrals are peaked. For the more realistic case of nonvanishing Q values, the differences between the elastic S matrices in entrance and exit channel have been shown to be important. For our example, the approximation of Hahne[18] has been shown to give remarkably good results. The AB theory is accurate enough to account for the nuclear part of the inelastic scattering and to give essentially the same result as the DW theory. For the calculation of the inelastic scattering in the combined nuclear and Coulomb fields, the implicit dependence of the Coulomb radial integrals on the details of the optical potential has been shown to be very small. If the Coulomb radial integrals are approximated by pure Coulomb integrals multiplied by the elastic scattering reflection coefficients, one may completely avoid the reference to an explicit shape of the optical potential. Even with this crude approximation the qualitative features of the Coulomb-nuclear interference are well accounted for, with only slight deviations from DWBA predictions in the amplitude of the interference oscillations.

The success of the AB theory in predicting the inelastic scattering of heavy ions has several implications: The information content of elastic and inelastic scattering is, apart from the nuclear deformation, identical, since the inelastic scattering amplitudes may be expressed in terms of the elastic scattering amplitudes. Hence it is more a question of practicability and cross sections whether one measures the elastic or inelastic scattering to determine the potential parameters. Potentials giving identical elastic scattering amplitudes are expected to give also identical inelastic scattering amplitudes. In fact, it has been found empirically[28] that the potential ambiguities observed in the elastic scattering of heavy ions cannot be resolved by measuring angular distributions of single nucleon transfer reactions and interpreting them in terms of the DWBA. Since the structures of the radial integrals encountered in transfer reactions and in inelastic scattering are quite similar one may expect that the potential ambiguities encountered in the elastic and inelastic scattering will persist for transfer reactions, and it should be possible to obtain an understanding of the observations of Ref. 28 in terms of the AB theory.

Furthermore, the validity of the AB theory suggests a test of the single step reaction model generally used for the interpretation of Coulomb-nuclear inelastic scattering. By accurately measuring the elastic scattering and determining the elastic scattering S matrix, one should obtain a fit to the inelastic scattering without further variations of the optical potentials and by only adjusting the deformation parameter. Any failure or inconsistency of this procedure may be taken as evidence for a failure of the reaction model or for the importance of higher-order processes. In principle, the AB theory allows also for the inclusion of higher-order excitations.[14] The numerical accuracy of the theory has, however, not yet been investigated for the case of multiple excitations.

The authors are indebted to Professor John S. Blair for valuable suggestions on the discussion of the hierarchy of approximations developed in Sec. II.

APPENDIX

The calculations have been performed with the inelastic scattering code COULIT[29] which is based on a DWBA code written by Braithwaite.[30] The code may calculate the elastic scattering amplitude of entrance and exit channels for up to 51 partial waves. The Austern-Blair approximation $g_l^{(AB)}$, corresponding to Eq. (24), is calculated by using Eq. (25) for the derivatives of the elastic scattering reflection coefficients. The first 51 nuclear and Coulomb radial integrals may be either integrated numerically (with no limitation on the number of integration steps) or substituted by using the approximations corresponding to Eqs. (16)–(18) and (26), respectively. For the higher partial waves the nuclear radial integrals are neglected and the Coulomb radial integrals are calculated with the subroutine ITER of Samuel and Smilanksy[31] which calculates the electric quadrupole radial matrix elements for Coulomb excitation by means of an iteration procedure. In our calculations a total of 230 partial waves has been used and the first 51 Coulomb radial integrals have been integrated up to 150 fm.

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