and including the ρ-meson by means of the "bipion" amplitude of De Tollis et al. [3]. The upper, middle and lower curves at each gamma ray energy are for Λ = - 0.5, 0 and 0.5, respectively, where Λ is the γ - π - ρ coupling constant in units of the electronic charge e. The square of the ρ-meson mass was taken to be 22.5 m_π^2.

Robinson et al. [14] have found Λ = - 1.2 ±0.4 by comparison of the McKinley theory with π^+ photoproduction, mainly at backward angles for photon energies of 220 to 250 MeV. From the dashed curves of fig. 1, it is apparent that a comparison of R with this theory leads to a quite different estimate of Λ. From this discrepancy we conclude that the calculation of $\mathcal{F}^-$ is most likely in error by more than has been supposed.

In order to estimate Λ most reliably from R, we have taken account of the corrections to the isoscalar amplitude, which have been calculated by Warburton and Gourdin [16]. The solid curves show the McKinley theory as modified in this way. The corrections are approximately angle independent and consist of a Λ-independent change in R plus a contribution closely equal to - 0.15 Λ for the energy range of interest here. When the ρ-meson effects are calculated in the bipion approximation a subtraction constant disappears which is present in a more detailed approach [1,2]. The Λ-dependent correction of Warburton and Gourdin represents part of this constant.

Comparing the corrected theory with experiment, an energy-dependent discrepancy somewhat outside the estimated uncertainty of the coulomb corrections is seen to exist. Considering the average fit between theory and experiment at the three energies, we estimate:

$$\Lambda = + 0.1 \pm 0.3 .$$

The error includes the uncertainty in the coulomb corrections as well as a moderate uncertainty in the theory. The theoretical uncertainty can only be guessed. With a completely trustworthy theory, the existing data seem capable of defining Λ to ± 0.15.

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CORRELATION STUDIES OF THE α-PARTICLE BREAKUP OF THE 9.6 MeV 3⁻ STATE IN C^{12} INDUCED BY (α, α') SCATTERING *

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Recent work on the theory of angular correlations has shown [1] that when an even-even nucleus breaks up by alpha particle emission to a 0^+ final state following excitation by a nuclear reaction, the correlation function of the breakup alpha particles has the angular dependence:

$$W_j(\Theta, \Phi) \sim \left| \sum_m (-1)^m p_m^j \, e^{-im\Phi} \, d_{mo}^j(\Theta) \right|^2 , \qquad (1)$$

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where the angles Θ and Φ are the polar and azimuthal angles measured in spherical polar coordinates with respect to the quantisation or z axis, j and m are the spin and spin-projection of the decaying nucleus, $p_m{}^j$ is an element of the polarisation tensor which describes the polarisation of the decaying nucleus and is a complex number characterizing the population of the mth substate, and $d_{mm'}{}^j(\Theta)$ is an element of the reduced rotation matrix, using the definition of Rose [2)] and of Brink and Satchler [3)].

The rotation matrix elements have the symmetry property that $d_{m\bar{o}}{}^j = (-1)^m d_{mo}{}^j$. It follows from this relationship and from (1) that the correlation function will have a particularly simple Φ-dependence at values of Θ where elements of the rotation matrix vanish. At such values of Θ the contributions from a particular pair of substates $\pm m$ eliminated from the correlation function, and an experimental determination of the Φ-dependence of the correlation function provides both a critical test of the theory mentioned above and an unambiguous measurement of the spin of the decaying state, and, in addition, provides information about the polarisation of that state.

The 9.6 MeV 3^- state in C^{12}, which is unbound to alpha decay, was selected to test this prediction because carbon is a convenient target nucleus, and because the 9.6 MeV state emits fairly energetic breakup alphas and is well separated from adjacent α-emitting levels. In this situation $j = 3$ and the values of Θ which are of interest are those where the matrix elements $d^3_{\pm 10}$ and $d^3_{\pm 30}$ vanish. (The even substates have not been included here, because, by choosing the quantisation axis perpendicular to the reaction plane and by using alpha particle inelastic scattering to excite the state in question, the reflectional symmetry theorem of Bohr [4)] may be employed to exclude population of these even substates.) It is found that the matrix elements $d^3_{\pm 10}$ vanish when $\Theta \cong 63.5^o$ and 116.5^o and that all the elements of interest vanish when $\Theta = 0^o$ and 180^o. Thus, at 63.5^o the correlation function should result only from coherent interference between the $m = \pm 3$ substates. Evaluation of (1) at this value of Θ gives:

$$W_3(63.5^o, \Phi) \sim [(1-a_3)^2 + 4a_3 \sin^2 3(\Phi - \tfrac{1}{6}\delta_3)] , \qquad (2)$$

where $a_3 = |p^3_{-3}/p^3_3|$ and $e^{i\delta_3} = p^3_{-3}/a_3 p^3_3$, that is, a_3 and δ_3 are real numbers corresponding to the relative amplitude and the relative phase of the $m = -3$ substate with respect to the $m = 3$ substate. In other words, at $\Theta = 63.5^o$ the correlation function is predicted to have a Φ-dependence described by an isotropic term plus a sinusoid which repeats every 60^o. The relative magnitudes of the isotropic and sinusoidal terms are related to the relative populations of the $m = 3$ and -3 substates, and the phase of the sinusoid is related to the relative phase of the substates.

The measurement of the predicted correlation function must be carried out while maintaining Θ, an angle measured in the centre of mass of the recoiling carbon nucleus, at a fixed value. This requirement creates experimental difficulties, for it means that θ and φ, the polar and azimuthal angles as measured in the laboratory system, must *both* be varied to keep Θ constant. Moreover, the necessary transformations of angles are not directly obtainable from the two-dimensional kinematical equations usually employed, and three-dimensional kinematics must be used. Fig. 1 shows a diagram of the three-dimensional kinematical system. Here v_{cm} is the laboratory velocity of the centre of mass, v is the particle laboratory velocity, and V is the particle centre of mass velocity. Since V is a constant, it traces out a sphere, upon which the locus of velocities with a given value of Θ traces a circle. In this geometry the angles are related by the equations:

$$\sin(\Phi - \varphi) = (v_{cm}/V)(\sin\varphi/\sin\Theta) , \qquad (3a)$$

$$\tan\theta = \tan\Theta\,(\sin\Phi/\sin\varphi) . \qquad (3b)$$

In the situation where the reaction is the breakup of a nucleus with laboratory kinetic energy E

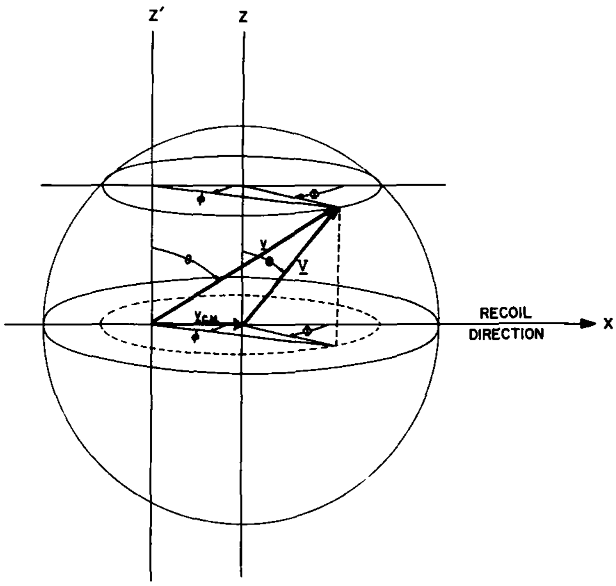

Fig. 1. Three dimensional kinematics diagram indicating relation between laboratory (lab.) system and the recoil centre of mass system for breakup reactions. The lab. velocity of the recoiling excited nucleus before the breakup occurs is denoted by v_{cm}. The angles θ and φ are the lab. polar and azimuthal angular coordinates of a breakup particle having a lab. velocity v. The angles Θ and Φ and velocity V are the corresponding coordinates and velocity of the same particle in the recoil centre of mass system.

and breakup energy Q into an alpha particle with mass M_α and a residual nucleus with mass M_r, the velocity ratio in (3a) is:

$$v_{cm}/V \cong [(M_\alpha/M_r)(E/Q)]^{\frac{1}{2}} . \qquad (3c)$$

A Fortran program was written for the Indiana University 709 computer which employed these relationships to calculate a set of kinematics tables which, for a given nuclear reaction, inelastic alpha scattering angle, and value of Θ, tabulated θ, Φ, the break-up alpha energy, and the solid angle correction corresponding to even increments of φ. These tables made it possible to maintain a constant value of Θ while measuring the correlation function, and also facilitated solid angle corrections and the other necessary transformations.

For the measurement, a self-supporting foil of natural carbon was bombarded with 22.5 MeV alpha particles from the Indiana University Cyclotron and inelastic and breakup alphas were detected using two Ortec surface barrier detectors which could be positioned independently. Coincidences were measured with a conventional fast-slow coincidence circuit and

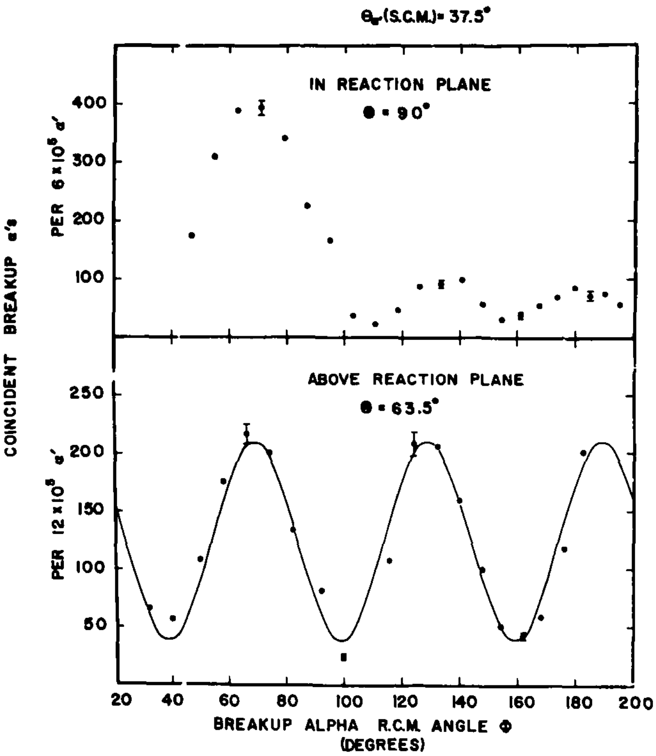

Fig. 2. Angular correlations α'-α_0 for the reaction $C^{12} + \alpha - \alpha' \rightarrow C^{12*}_{9.63} \rightarrow Be^8 + \alpha_0$ at a lab. bombarding energy of 22.5 MeV. Inelastic alpha particles were detected at a system centre of mass scattering angle of 37.5°. The coincidence yield of breakup alpha particles was measured as a function of the azimuthal recoil centre of mass angle Φ and at recoil centre of mass polar angles Θ of 90° and 63.5°. The solid curve is a least-squares fit to the experimental points using eq. (2).

recorded with a Nuclear Data 1024-channel two-parameter analyzer. The counter monitoring the breakup alphas was mounted on a specially-constructed arm which allowed variation of both θ and φ while maintaining a constant distance between the counter and the target.

The detector which monitored the inelastically scattered particles was set at a laboratory angle of 25° with respect to the beam (37.5° in the system centre of mass), and angular correlations were taken over a range of values of φ at 5° intervals, both in the reaction plane ($\Theta = \theta = 90°$), and out of the reaction plane at $\Theta = 63.5°$, setting θ in accordance with the tables described above.

Fig. 2 shows the results of these measurements after appropriate centre of mass transformations and solid angle corrections have been applied. The 63.5° data was least-squares-fitted with (2), and the solid line indicates this fit, which corresponds to the parameters $a_3 = 0.387 \pm 0.031$ and $\delta_3 = 233.7° \pm 3.0°$. The data taken in the reaction plane shows strong indications of interference arising from the $m = 1$ and -1 substates, and indicates that at least at this particular scattering angle all of the odd substates are populated fairly strongly. It should be mentioned that the angle Φ attains values in excess of 180° because of the transformation to the recoil centre of mass, even though the angle φ never ranged beyond 140°.

This result constitutes experimental verification of the theoretical prediction, and moreover, provides a confirmation, under very stringent conditions, that the 9.6 MeV state in C^{12} is indeed 3^-, as previous work has shown [5]. Thus, this technique is potentially a powerful tool for investigating the spins of states in even-even nuclei which break up by alpha particle emission. Moreover, a set of correlation measurements at different inelastic alpha scattering angles makes possible a determination of the polarisation of the decaying nucleus as a function of scattering angle. It is our intention to pursue this aspect of these measurements further and to measure such polarisations in C^{12} and other even-even nuclei.

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