IkkatFunctions

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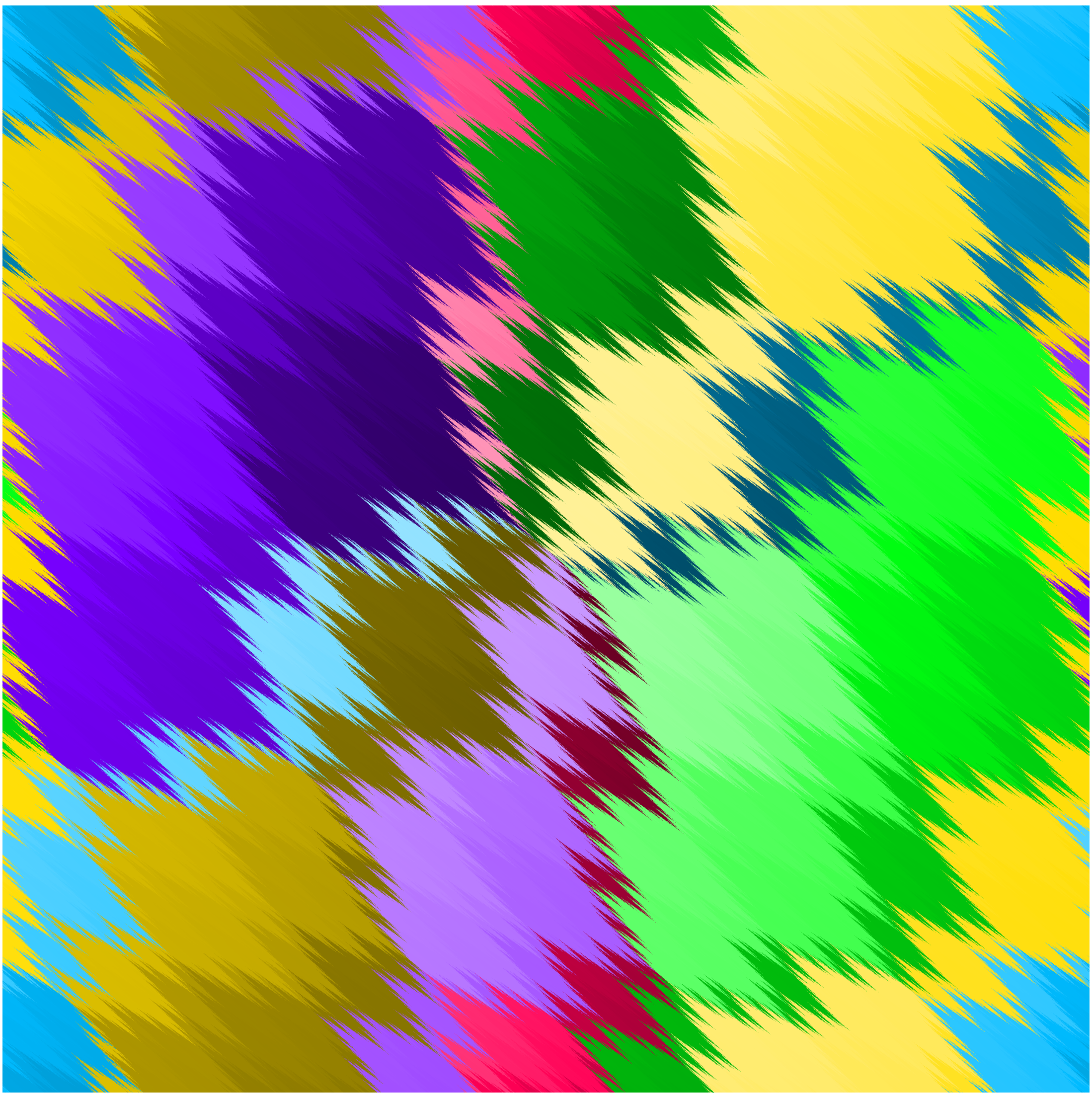

Slow Chaos

The boundary between structure and randomness is famously fluid: in one moment a flock of birds appears to be a disorganized rabble, in the next, it coalesces into a highly ordered winged squadron. The central theme of my research has been understanding this boundary: studying structure in seemingly random systems, and vice-versa, detecting randomness in structured systems. A motivating class of problems I study are *polygonal billiards*: the motion of a ball on a walled table shaped like a polygon. The ball starts rolling in a fixed direction, and we make two important simplifying assumptions: the table has no friction; and when the ball hits a wall, it bounces off at the same angle as it hits, and doesn't slow down. When a ball starts moving it continues at its initial speed for all time, unless it ends up in a corner. Polygonal billiards are examples of *slow chaos*: they sit on the boundary between very disordered and highly ordered.

Renormalization is the idea of *speeding up* the system in order to observe long-term behavior of the original system at shorter time scales. Imagine replacing the billiard ball by one moving faster, and as the speed increases, by the movement of a focused beam of light in a mirrored polygonal room. Billiards and related dynamical systems have been the subject of active research for more than one hundred years because they are wonderfully flexible. The techniques used to study billiards are used to study the movement of electrons (in the presence of a magnetic field) on Fermi surfaces of metals; potential flows in heterogeneous materials; and even supersymmetric quantum field theories.

An eigenfunction is a kind of heat map which behaves in a predictable way if we follow the trajectories of the billiard ball in a particular direction, in which we understand the renormalization very well. An *invariant function* would be a heat map where the temperature didn't change at all along trajectories, and it is known that for the direction we're studying here, any such function would just be constant. Eigenfunctions, while not invariant, change in a predictable way, and in general don't exist for most directions for the billiard. For this *particular* direction, we have many non-trivial eigenfunctions, leading to these mysterious fractal pictures. The title *IkkatFunctions* is inspired by the resemblance of these pictures to textiles woven using the ikkat technique, which is used across the world.

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