Discrete Random Variables

Bernoulli PMF (p. 3-4)

$$f_K[k] = \begin{cases} p & k = 1 \\ 1-p & k = 0 \end{cases}$$

$$\mathcal{E}\{K\} = p\,; \quad \text{Var}[K] = p(1-p)$$

$$G_K(z) = (1-p) + pz$$

Binomial PMF (p. 3-4)

$$f_K[k] = \binom{N}{k} p^k (1-p)^{(N-k)} \qquad 0 \le k \le N$$

$$\mathcal{E}\{K\} = Np\,; \quad \text{Var}[K] = Np(1-p)$$

$$G_K(z) = ((1-p) + pz)^N$$

Geometric PMF (type 1) (p. 3-5)

$$f_K[k] = p(1-p)^{(k-1)} \qquad 1 \le k < \infty$$

$$\mathcal{E}\{K\} = 1/p\,; \quad \text{Var}[K] = (1-p)/p^2$$

$$G_K(z) = \frac{pz}{1-(1-p)z}$$

Geometric PMF (type 0) (p. 3-6)

$$f_K[k] = p(1-p)^k \qquad 0 \le k < \infty$$

$$\mathcal{E}\{K\} = (1-p)/p\,; \quad \text{Var}[K] = (1-p)/p^2$$

$$G_K(z) = \frac{p}{1-(1-p)z}$$

Pascal PMF (p. 5-30)

$$f_K[k] = \binom{k-1}{n-1} p^n (1-p)^{(k-n)} \qquad k \ge n$$

$$\mathcal{E}\{K\} = n/p\,; \quad \text{Var}[K] = n(1-p)/p^2$$

$$G_K(z) = \frac{p^n z^n}{[1-(1-p)z]^n}$$

Poisson PMF (p. 3-7)

$$f_K[k] = \frac{\alpha^k}{k!} e^{-\alpha} \qquad 0 \le k < \infty$$

$$\mathcal{E}\{K\} = \alpha\,; \quad \text{Var}[K] = \alpha$$

$$G_K(z) = e^{\alpha(z-1)}$$

(Discrete) Uniform PMF (p. 3-8)

$$f_K[k] = \frac{1}{n-m+1} \qquad m \le k \le n$$

$$\mathcal{E}\{K\} = (n+m)/2\,; \quad \text{Var}[K] = [(n-m+1)^2 - 1]/12$$

$$G_K(z) = \frac{1}{n-m+1} \frac{z^m - z^{n+1}}{1-z}$$

Continuous Random Variables

Chi-Square PDF (p. 5-32)

$$f_X(x) = \frac{x^{(n-2)/2} e^{-x/2}}{2^{n/2}\Gamma(n/2)} \qquad x > 0$$

$$\mathcal{E}\{X\} = n\,; \quad \mathrm{Var}[X] = 2n$$

$$M_X(s) = \frac{1}{(1-2s)^{n/2}}$$

Exponential PDF (p. 3-15)

$$f_X(x) = \lambda e^{-\lambda x} \qquad x \geq 0$$

$$\mathcal{E}\{X\} = 1/\lambda\,; \quad \mathrm{Var}[X] = 1/\lambda^2$$

$$M_X(s) = \frac{\lambda}{\lambda - s}$$

Gamma & Erlang PDF (p. 5-31)

$$f_X(x) = \frac{\lambda^a x^{a-1}}{\Gamma(a)} e^{-\lambda x} \qquad x \geq 0$$

$$\mathcal{E}\{X\} = a/\lambda\,; \quad \mathrm{Var}[X] = a/\lambda^2$$

$$M_X(s) = \frac{\lambda^a}{(\lambda - s)^a}$$

The Erlang density occurs when $a = n$, a positive integer; then $\Gamma(n) = (n-1)!$.

Gaussian PDF (p. 3-16)

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$$\mathcal{E}\{X\} = \mu\,; \quad \mathrm{Var}[X] = \sigma^2$$

$$M_X(s) = e^{\mu s + \sigma^2 s^2/2}$$

Laplace PDF (p. 4-19)

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x-\mu|}$$

$$\mathcal{E}\{X\} = \mu\,; \quad \mathrm{Var}[X] = 2/\alpha^2$$

$$M_X(s) = \frac{\alpha^2}{\alpha^2 - s^2}\, e^{\mu s}$$

Rayleigh PDF (p. 5-40)

$$f_X(x) = \alpha^2 x e^{-\alpha^2 x^2/2}\,; \qquad x \geq 0$$

$$\mathcal{E}\{X\} = \sqrt{\pi/2}\Big/\alpha\,; \quad \mathrm{Var}[X] = (2 - \pi/2)/\alpha^2$$

$M_X(s)$ does not exist.

Uniform PDF (p. 3-14)

$$f_X(x) = \frac{1}{b-a} \qquad a \leq x \leq b$$

$$\mathcal{E}\{X\} = \frac{b+a}{2}\,; \quad \mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

$$M_X(s) = \frac{e^{bs} - e^{as}}{(b-a)s}$$