

1. Find all conditions on a, b, c, d so that

$$p(x) = a e^{-cx} + b e^{-dx}, \quad x \geq 0.$$

is a pdf.

2. Toss a fair coin 10 times, and count the number of heads, c . The tosses are iid. We play a game. You put in \$1, I put in \$ w . You guess the number of heads, c .

What is your best guess?

What should w be so that the game is fair.

Fair is usually taken that your expected gain is \$0.

Comment: No calculators on test. You Pascal's triangle or otherwise.

3. You poll the class and find that of the $N=30$ people, F like football, B like baseball. Estimate by an upper bound the probability that a person likes football or baseball. Compute for $N=30$ and

(a) $F=15, B=25$

(b) $F=5, B=7$

Just leave the fraction as result.

4. Let $\tilde{t}$ be an exponential RV with mean 1.
Based on a coin toss ($P(H) = p = 1 - P(T)$),
you are given $y = 2\tilde{t}$ if H occurs,
otherwise $y = \tilde{t}$.

Find $E(y)$ without knowledge of the result of
the toss. How big can $E(y)$ be?

5. Let $\tilde{x} \geq 0$ be a positive RV $P(\tilde{x} < 0) = 0$.
We know that
- $$P(\tilde{x} > x) = e^{-x^2} \quad x \geq 0$$

(1) find $P(\tilde{x} < 1)$

(2) find $p(x)$, the pdf of $\tilde{x}$.