EE416 Random Signals for Communications and Signal Processing

Solutions for Homework 4

3.12

(a)

Because the integral of PDF from minus infinity to infinity must be 1,

$$\int_{-\infty}^{\infty} f_X(x)dx = \int_0^2 Cdx + \int_4^6 2Cdx + \int_7^9 Cdx = 1$$

So

$$C = \frac{1}{8}$$

(b)

$$P[1 < x \leq 8] = 1 - \int_8^9 \frac{1}{8}dx - \int_0^1 \frac{1}{8}dx = \frac{3}{4}$$

(c)

Solution 1:

Because

$$\int_0^2 \frac{1}{8}dx + \int_4^5 \frac{1}{4}dx = \frac{1}{2}$$

We have **M=5**

Solution2:

Because of the symmetry property of this distribution,

We have M=(4+6)/2=5

3.13

In all the problems here, we use the property: the integral of PDF from minus infinity to infinity must be 1.

(a)

$$\int_0^{2\pi} (B + Asinx)dx = 1$$

So we have $B = \frac{1}{2\pi}$

Also, PDF needs to be greater than 0 at all values.

So we have $|A| \leq \frac{1}{2\pi}$

(b)

$$\int_0^2 Adx + \int_2^3 \frac{A}{2}dx = 1$$

So we have $A = \frac{2}{5}$

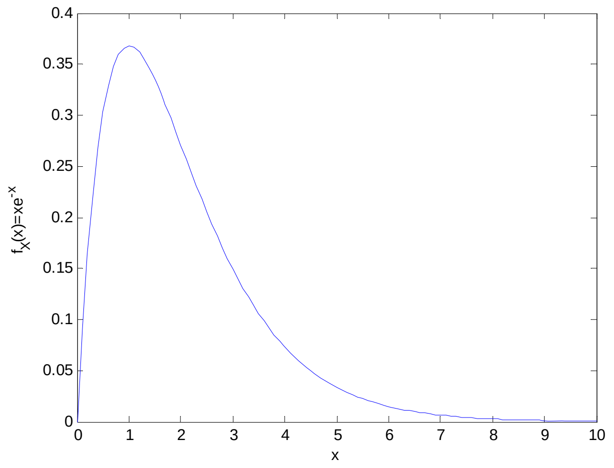
(c)

$$\int_{-1}^{1} (\frac{A}{2}x + \frac{A}{2})dx + \int_{1}^{3} (-\frac{A}{2}x + \frac{3}{2}A)dx = 1$$

So we have $A = \frac{1}{2}$.

3.18

(a) $\int_0^\infty f_X(x)\,\mathrm{d}x = 1$, C=1



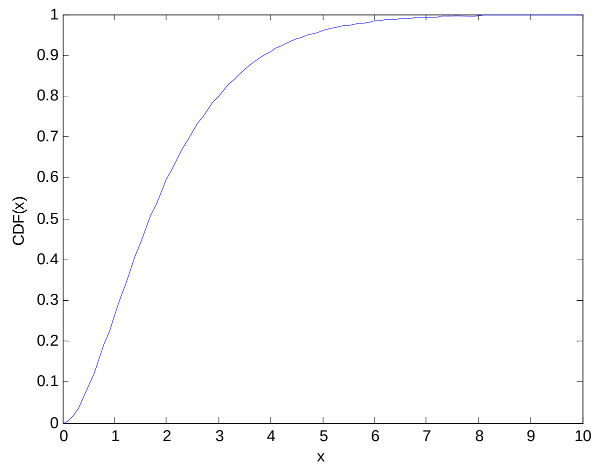
(b) $\Pr[X \le 1] = \int_0^1 f_X(x)\,\mathrm{d}x = 0.2642$

$\Pr[X > 2] = \int_2^\infty f_X(x)\,\mathrm{d}x = 0.4060$

(c) $\Pr[x_0 < X \le x_0 + 0.001] = \int_{x_0}^{x_0+0.001} f_X(x)\,\mathrm{d}x = \begin{cases} 3.0342*10^{-4}, \text{ when } x_0 = 1/2 \\ 2.7060*10^{-4}, \text{ when } x_0 = 2 \end{cases}$

At peak of the PDF, the integral would be largest. The peak of the PDF is at $x = 1$, which is given by solving $\frac{\mathrm{d}f_X(x)}{\mathrm{d}x} = 0$. Therefore $x_{0,\max} = 1$.

(d) $\mathrm{CDF}(x) = \begin{cases} \int_0^x f_X(t)\,\mathrm{d}t = 1-(1+x)e^{-x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$



3.20

(a)

Assume the uniform distribution has parameters a and b.

According to the definition of median,

$$\int_a^M \frac{1}{b-a} dx = \int_M^b \frac{1}{b-a} dx$$

So we have $M = \frac{b+a}{2}$

(b)

Assume the exponential distribution has the parameter λ.

According to the definition of median,

$$\int_0^M \lambda e^{-\lambda x}\, dx = \int_M^\infty \lambda e^{-\lambda x} dx$$

So we have $M = \frac{1}{\lambda} ln2$

(c)

Assume the Gaussian distribution has parameters μ and σ.

Because of the symmetry property of Gaussian distribution,

We know that

$$\int_{-\infty}^{\mu} f_{Gaussian}\,(x)dx = \int_{\mu}^{\infty} f_{Gaussian}\,(x)dx$$

So we have M=μ.

3.31

Let random variable Y denote the output of the quantizer.

The quantization rule is:

If the input $X \in [-\infty, -d]$, then $Y = -1.5d$;

If the input $X \in [-d, 0]$, then $Y = 0.5d$;

If the input $X \in [0, d]$, then $Y = 0.5d$;

If the input $X \in [d, \infty]$, then $Y = 1.5d$.

(a)

$$\Pr\left[Y=-1.5d\right]=\Pr\left[X\le -d\right]=\int_{-\infty}^{-d} f_X(x)\,\mathrm{d}x=\frac{1}{2}e^{-2d}$$

$$\Pr\left[Y=-0.5d\right]=\Pr\left[-d<X\le 0\right]=\int_{0}^{-d} f_X(x)\,\mathrm{d}x=\frac{1}{2}\left(1-e^{-2d}\right)$$

$$\Pr\left[Y=0.5d\right]=\Pr\left[0<X\le d\right]=\frac{1}{2}\left(1-e^{-2d}\right)$$

$$\Pr\left[Y=1.5d\right]=\Pr\left[X>d\right]=\frac{1}{2}e^{-2d}$$

(b)

$$\Pr\left[X>2d\right]=\int_{2d}^{\infty} f_X(x)\,\mathrm{d}x=\frac{1}{2}e^{-4d}$$

$$\Pr\left[X<-2d\right]=\frac{1}{2}e^{-4d}$$

3.35

$$\Pr\left[A\right]=\int_{2}^{\infty}\lambda e^{-\lambda x}\mathrm{d}x=e^{-2\lambda}$$

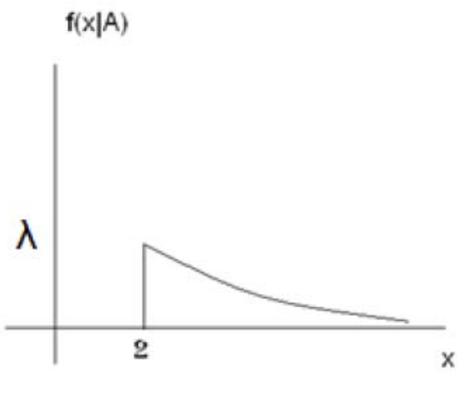
The conditional CDF is given by

$$F_{X|A}\left[x\,|\,A\right]=\frac{\Pr[X\le x, A]}{\Pr[A]}=\begin{cases}\dfrac{\Pr[2\le X\le x]}{\Pr[X\ge 2]}, & x\ge 2\\ 0, & x<2\end{cases}$$

$$=\begin{cases}\dfrac{e^{-2\lambda}-e^{-\lambda x}}{e^{-2\lambda}}, & x\ge 2\\ 0, & x<2\end{cases}$$

Taking derivative, the conditional pdf is

$$f_{X|A}\left[x\,|\,A\right]=\frac{\mathrm{d}F_{X|A}\left[x\,|\,A\right]}{\mathrm{d}x}=\begin{cases}\dfrac{\lambda e^{-\lambda x}}{e^{-2\lambda}}, & x\ge 2\\ 0, & x<2\end{cases}$$



Grading:
15 pts for 3.12, where 5 for each question;
20 pts for 3.13, where 10 for a), 5 for b) and 5 for c);
20 pts for 3.18, where 5 for each;
15 pts for 3.20, where 5 for each;
15 pts for 3.31, where 10 for a) and 5 for b);
15 pts for 3.35.