EE461 Homework3, 2009

Note:

For 3.7, you should get the same result regardless of different definitions of geometric PMF.

For 3.5 and 3.9, using different definitions of geometric PMF, the answer will be different. Here I only use Type0 geometric PMF.

For geometric and uniform 3.9 (c), you can either use a numerical method or solve by hand. Closed-form expressions for the parameters of geometric and uniform can be obtained. Also note there is no assumption that m=0 for uniform PMF.

For Poisson 3.9 (c), an inequality is first obtained and solved numerically by root finding algorithm.

3.7

Pr[error occurs in first 5 bytes]

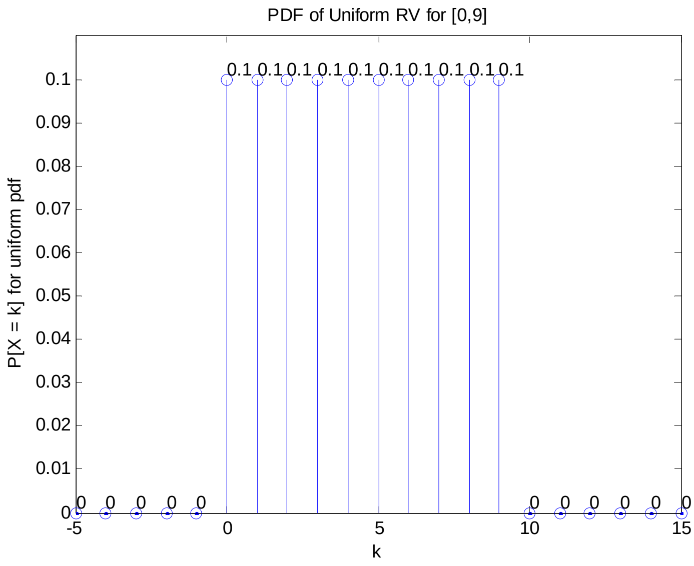
=1- Pr[no error occurs in first 5 bytes]

= $1-(8/9)^5 \approx 0.4451$

Besides the above method, you can use geometric PMF as Example 3.4 does in the textbook.

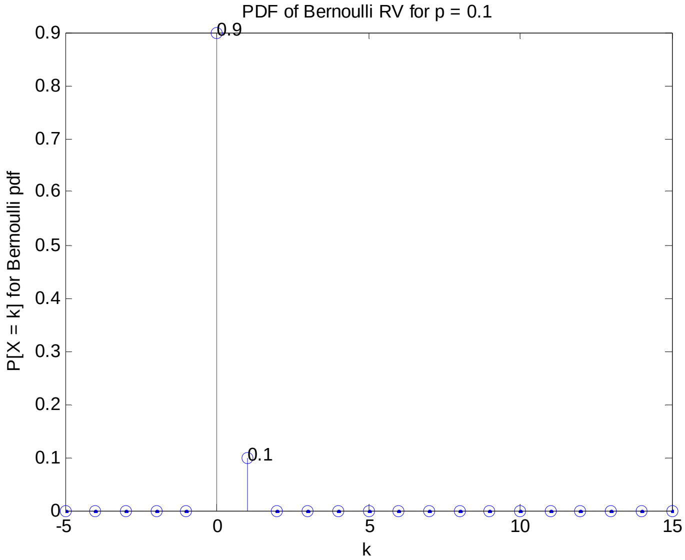
3.5

(a)
```
% Solution to Problem 3.5 part (a)
% P[X=k] = 1/(n-m+1) = 0.1 for m<=k<=n
m = 0;  n = 9;
k = -5:15;  %some zero values in the neighborhood of non-zero values
y = zeros(1,length(k));    %Making the array y the same length as k
index = find( (k>=m) & (k<=n) ); %find the index for the interval [m,n]
y(index)= 1/(n-m+1);    %uniform distribution
stem(k,y)
ylim([0,0.11])
title('PDF of Uniform RV for [0,9]')
xlabel('k')
ylabel('P[X = k] for uniform pdf')
for i=1:length(k) %labeling the values
    text(k(i)+0.002,y(i)+0.002,num2str(y(i)));
end
sum(y(index(1:4)))  %corresponds to k=1~4
```



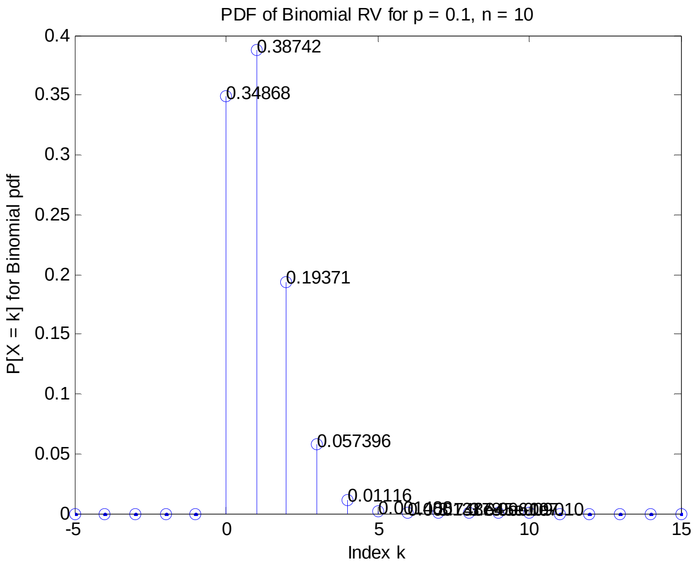
$P_{\text{uniform}}[1 \le k \le 4] = 0.4$

(b)
```
%Solution to Prob 3.5 part (b)
% P(X=1)=0.1, P(X=0)=0.9
k=-5:15;%index k
y=zeros(1,length(k));%making array y the same length as k
index = find( (k>=0) & (k<=1) ); %find the index for the interval [0,1]
y(index(1))=0.9;%bernoulli distribution
y(index(2))=0.1;
stem(k,y)
title('PDF of Bernoulli RV for p = 0.1')
xlabel('k')
ylabel('P[X = k] for Bernoulli pdf')
Bern_sum = sum(y(7:10))    %Adding up probabilities from k = 1 to 4
for i=1:length(index) %labeling the values
   text(k(index(i))+0.003,y(index(i))+0.003,num2str(y(index(i))));
end
```



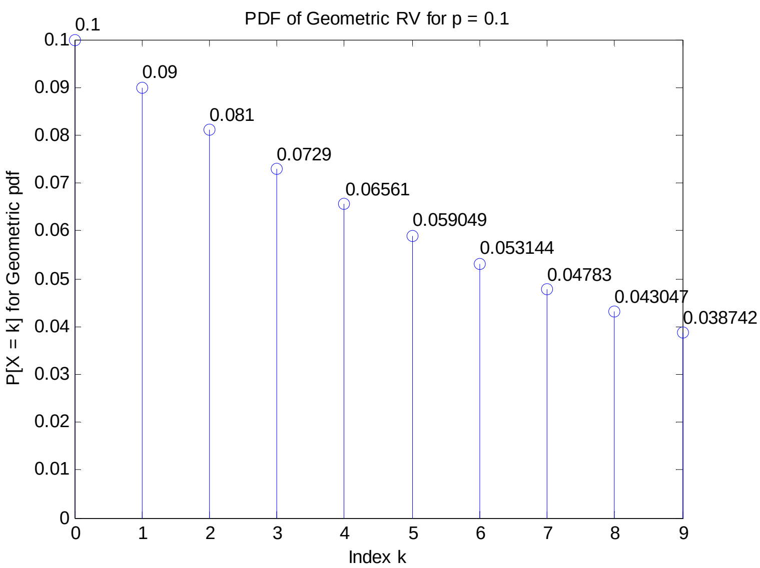
$P_{\text{Bernoulli}}[1 \le k \le 4] = 0.1$

(c)
```
% Solution to Prob 3.5 part (c)
% P(k)=(10 k)0.1^k*0.9^(10-k)
p = .1;
n = 10;
k=-5:15;
index = find( (k>=0) & (k<=10) ); % find the index for the interval [0,1]
y = binopdf(k,n,p);%get binomial distribution
stem(k,y)
title('PDF of Binomial RV for p = 0.1, n = 10')
xlabel('Index k')
ylabel('P[X = k] for Binomial pdf')
Bino_sum = sum(y(7:10))    %Adding up probabilities from k = 1 to 4
for i=1:length(index) %labeling the values
   text(k(index(i))+0.003,y(index(i))+0.003,num2str(y(index(i))));
end
```



$P_{\text{binomial}}[1 \le k \le 4] = 0.6497$

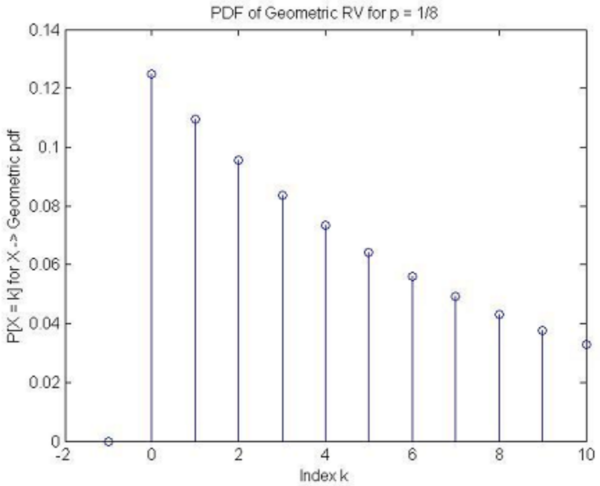
(d)
```
%Solution to Prob 3.5 part (d)
%geometric distribution type 0
%P(k)=0.1*0.9^k
p = .1;
y = geopdf(k,p);    % get geometric distribution
stem(k,y)
title('PDF of Geometric RV for p = 0.1')
xlabel('Index k')
ylabel('P[X = k] for Geometric pdf')
Geo_sum = sum(y(2:5)) %Adding up probabilities from k = 1 to 4
for i=1:length(k) %labeling the values
    text(k(i)+0.003,y(i)+0.003,num2str(y(i)));
end
```

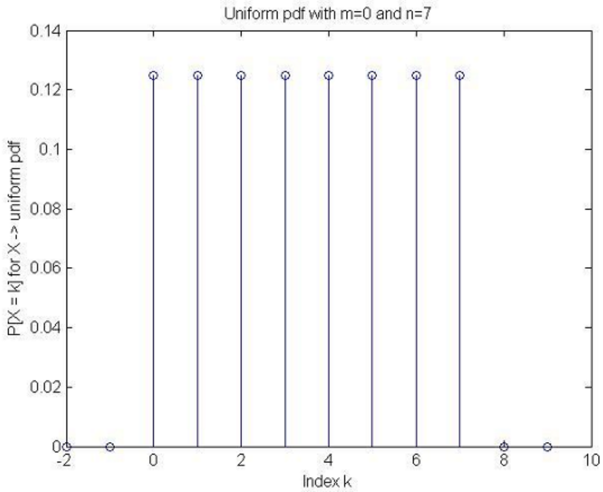


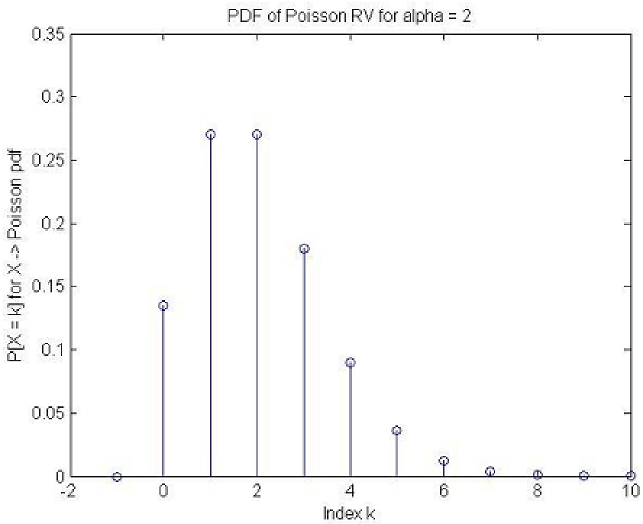
$P_{\text{geometic}}[1 \le k \le 4] = 0.3059$

3.9

(a) The matlab code for sketch is quite similar to Prob3.5, and thus omitted.







(b.i) $P[K = k] = p(1-p)^k, \; k = 0,1,\cdots$

$P_{\text{geometric}}[K > 5] = p(1-p)^6 + p(1-p)^7 + \cdots = (1-p)^6 = 0.4488$

(b.ii) uniform on the interval [0,7]

$$P_{\text{uniform}}[K>5]=\frac{1}{8}+\frac{1}{8}=0.25$$

(b.iii) $P_{\text{Poisson}}[K>5]=1-\left\{e^{-\alpha}\left[1+\frac{\alpha}{1!}+\cdots+\frac{\alpha^5}{5!}\right]\right\}=0.0166$

(c.i) $P[K>5]=p(1-p)^6+p(1-p)^7+\cdots=(1-p)^6$

Solve for $(1-p)^6 \le 0.5 \quad \Rightarrow \quad p \ge 0.1091$

Therefore, the answer is $1 \ge p \ge 0.1091$, since p has to be less than or equal to 1 for binomial PMF.

(c.ii) We assume that $m \le n$. And m and n are integers, because we are talking PMF not PDF.

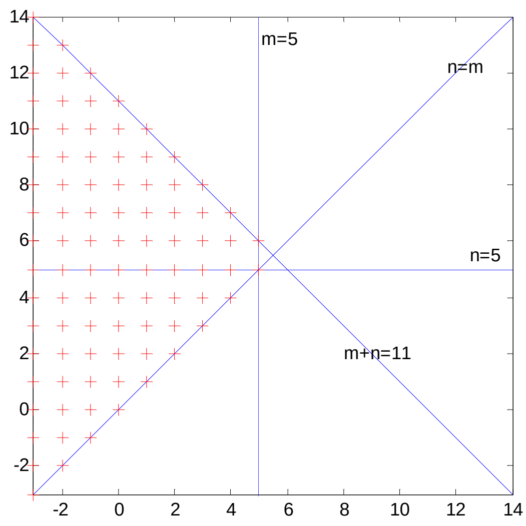
Three different cases: $5<m\le n$, $m\le 5\le n$, $m\le n\le 5$.

For the case of $5<m\le n$, Pr[K>5]=1.

For the case of $m\le 5<n$, Pr[K>5]= $\frac{n-5}{n-m+1}$. Solve for $\frac{n-5}{n-m+1}\le 0.5 \Rightarrow n+m\le 11$

For the case of $m\le n\le 5$, Pr[K>5]=0.

Therefore the solution is $\{m\le n\}\cap\{\{\{m\le 5<n\}\cap\{n+m\le 11\}\}\cup\{m\le n\le 5\}\}$ = $\{m\le n\}\cap\{m\le 5\}\cap\{n+m\le 11\}$ and m,n are integers, as shown in the following figure.



(c.iii) $g(\alpha)=P[K>5]=1-P[K\le 5]=1-\left\{e^{-\alpha}\left[1+\frac{\alpha}{1!}+\cdots+\frac{\alpha^5}{5!}\right]\right\}$

Since $\frac{dg}{d\alpha}=e^{-\alpha}\frac{\alpha^5}{5!}$ is always positive for $\alpha>0$, $g(\alpha)$ is a monotonic function.

1) Thus $g(\alpha)=0.5$ has a unique solution.

2) Since the gradient and the 2nd order gradient is known, $g(\alpha)=0.5$ can be solved by basic

numerical algorithms, like steepest descent method and Newton's method. Here I just use built-in matlab *fsolve*.

```
x0 = 1;          % Make a starting guess at the solution
[alpha,fval] = fsolve(@myfun,x0,options)  % Call optimizer

% myfun.m
function F = myfun(x)
F =(1-exp(-x).*sum( x.^(0:5)./factorial(0:5) ))-0.5;


%MATLAB returns
%alpha=5.6701.
```

Since $g(\alpha)$ is a monotonic increasing function, the solution to $g(\alpha) \le 0.5$ is $0 < \alpha \le 5.6701$, where $0 < \alpha$ is given by the definition of Poisson PMF.

Grading

15 for Problem 3.7;

40 for Problem 3.5;

10 for each distribution, where 5 for sketch, 3 for code and 2 for Pr[1<=k<=4].

45 for Problem 3.9

15 for each geometric, uniform and Poisson distribution, where 7 for sketch and matlab code, 3 for Pr[K>5], and 5 for determine the parameters.