EE 416 Homework Assignment 1

DUE: Next Wednesday. Always include matlab printout on white and not black background.

1. Let A, B be finite sets. Which is smaller in cardinality: $A \cap B$ or $A \cup B$? When are they equal?

2. If $P(A) = a$, $P(B) = b$, then $P(A \mid B) \geq (a + b - 1)/b$. Prove using basic properties.

3. We have a probability space with events $A_1, A_2, \ldots$ We know that

$$P[A_1 \cup A_2] \leq P[A_1] + P[A_2]$$

Recall the proof. State conditions for an equality. Extend this to 3 sets. Prove that

$$P[A_1 \cup A_2 \cup A_3] \leq P[A_1] + P[A_2] + P[A_3]$$

For any $N = 1, 2, 3, \ldots$, justify the "Union Bound,"

$$P(A_1 \cup A_2 \cup \ldots A_N) \leq \sum_{k=1}^{N} P(A_k).$$

No formal induction proof is required. This inequality is a fundamental method for estimating the probability of a union of events. It overbounds the probability by assuming the events are mutually exclusive -disjoint.

4. This is a problem in Geometric Probability. For this problem you can use the following data. I didn't measure it.

Coin	Diameter
Roosevelt Dime	17.9 mm
Lincoln cent	19.0 mm
Jefferson Nickel	21.2 mm
Washington Quarter	24.3 mm
Kennedy Half Dollar	30.6 mm
Eisenhower Dollar	38.1 mm

A US dime, penny and quarter have the above diameters. The radius R is half of that. We are playing a game where we toss a coin randomly onto a huge (infinite) *square grid* (graph paper). The lines are of width $W = 5mm$ and are separated by distance D, as measured from the inside the lines.

Sketch a picture and label it! The game is simple. You toss a coin and win if you do not touch a grid line. If we want the odds to be 50:50, what is the right dimension for D with the coins listed above.

Write down a general formula, in terms of coin radius, edge width, and grid inner width, R, W, D, that gives the probability of winning (no touch). It should be intuitive and reflect the 2 dimensional geometry. Geometric probability usually reduces to the calculation of lengths, areas, volume, or hypervolumes. Since an "at random" assumption is made, the probability distribution is uniform.

5. Another geometric probability problem. It is equally likely that two signals arrive at a receiver at any time $0 < t < T$, where T is known. Assume that the receiver is jammed if both signals arrive within time difference τ. Computing areas on a $T \times T$ square, (i) find a general formula for the probability of jamming, P_J, and (ii) the probability that both signals are received clearly, P_C. Show (iii) that for very small $\gamma := \tau/T$, $P_J \approx 2\gamma$. As $T \to \infty$, $P_J \to 0$. Explain (iv) why this is true.

6. A communication link succeeds half the time it is used. To increase reliability, a set of n parallel links is constructed. Each operates independently of all the others, and the system succeeds when at least 1 link of the n succeeds. What is the smallest n that gives a system success probability of 3 9's, 0.999. Does this about double when at least 2 links must succeed?

Networks A communications link connects the source, S, to the receiver, R, through connected links. Follwing S, it consist of N identical series elements a, followed by M parallel elements b, as depicted. Each element fails independently of the others. Let A_k be the event that a_k is working, with probability $P(A_k) = p_k$, and similarly for B_k, with $P(B_k) = q_k$. Using unions and intersections of appropiate events,
(1) find a general expression for the event O, that the link is out (outage).
Then (2) find P_O, the corresponding probability. If $p_k = 0.99$, $q_k = 0.50$, $1 \leq N \leq 10$, (3) how large must M be to assure that $P_O \leq 0.1$? (4) What happens if $N > 10$? Solve numerically with MATLAB$^{\text{tm}}$.

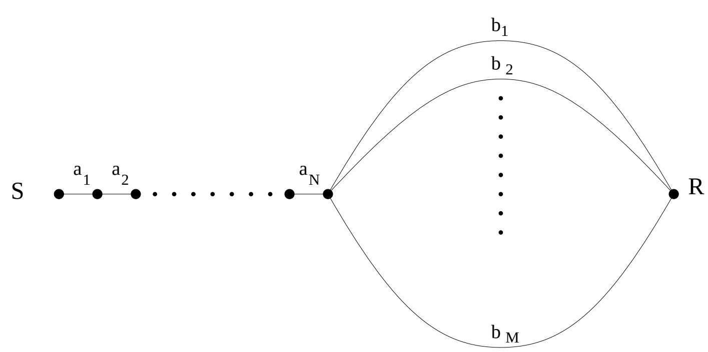

Figure 1: Series-Parallel Connection