MP2 Test Maze

Solve:
1) Sparse Gauss Seidel
2) Explicit Ax=B

4 x 4 Maze: 4 unknowns

P_{11}	$P_{12}=0$	$P_{13}=0$	P_{14}
$P_{21}=1$	P_{22}	P_{23}	$P_{24}=0$
$P_{31}=1$	P_{32}	P_{33}	$P_{34}=0$
P_{41}	$P_{42}=0$	$P_{43}=0$	P_{44}

At Solution

P_{11}	$P_{12}=0$	$P_{13}=0$	P_{14}
$P_{21}=1$	P_{22}	P_{23}	$P_{24}=0$
$P_{31}=1$	P_{32}	P_{33}	$P_{34}=0$
P_{41}	$P_{42}=0$	$P_{43}=0$	P_{44}

$$P_{2,2}=\left(\tfrac{1}{4}\right)\left(P_{2,3}+P_{2,1}+P_{3,2}+P_{1,2}\right)$$
$$P_{2,3}=\left(\tfrac{1}{4}\right)\left(P_{2,4}+P_{2,2}+P_{3,3}+P_{1,3}\right)$$
$$P_{3,2}=\left(\tfrac{1}{4}\right)\left(P_{3,3}+P_{3,1}+P_{4,2}+P_{2,2}\right)$$
$$P_{3,3}=\left(\tfrac{1}{4}\right)\left(P_{3,4}+P_{3,2}+P_{4,3}+P_{2,3}\right)$$

$$P_{i,j}=\left(\tfrac{1}{4}\right)\left(P_{i,j+1}+P_{i,j-1}+P_{i+1,j}+P_{i-1,j}\right)$$

$$P_{i,j} = \left(\tfrac{1}{4}\right)\left(P_{i,j+1} + P_{i,j-1} + P_{i+1,j} + P_{i-1,j}\right)$$

P_{11}	P_{12} =0	P_{13} =0	P_{14}
P_{21} =1	P_{22}	P_{23}	P_{24} =0
P_{31} =1	P_{32}	P_{33}	P_{34} =0
P_{41}	P_{42} =0	P_{43} =0	P_{44}

$$P_{2,2} = \left(\tfrac{1}{4}\right)\left(P_{2,3} + P_{2,1} + P_{3,2} + P_{1,2}\right)$$
$$P_{2,3} = \left(\tfrac{1}{4}\right)\left(P_{2,4} + P_{2,2} + P_{3,3} + P_{1,3}\right)$$
$$P_{3,2} = \left(\tfrac{1}{4}\right)\left(P_{3,3} + P_{3,1} + P_{4,2} + P_{2,2}\right)$$
$$P_{3,3} = \left(\tfrac{1}{4}\right)\left(P_{3,4} + P_{3,2} + P_{4,3} + P_{2,3}\right)$$

To solve:
Iterate with i and j running over the interior cells.
Use prior values on right hand side to calculate updated ones on left
Initial guess: use 0.25 for four interior cells (or other choices).
Constants enter as any other cell value on the right, but are never updated on left.

Matlab Implementation

```
P(:,:) = 0.25;
P(1,1) = NaN;
P(4,4) = NaN;
P(1,4) = NaN;
P(4,1) = NaN;
P(2,1) = 1;
P(3,1) = 1;
P(1,2) = 0;
P(1,3) = 0;
P(4,2) = 0;
P (4,3) = 0;
P(2,4)=0;
P(3,4) = 0;
```

```
while (~converged)
  Pold = P;
  for i = 2:3
    for j = 2:3
      P(i,j) = (0.25)*(P(i,j+1) + P(i,j-1) + P(i+1,j) + P(i-1,j));
    end
  end
  %check tolerance
  error = max(max (abs(P-Pold)));
  if (error < tol)
    converged = true;
    P
  end
end
```

```
P =

   NaN     0     0   NaN
1.0000  0.3750  0.1250   0
1.0000  0.3750  0.1250   0
   NaN     0     0   NaN
```

Same Problem as Ax = b

- 4 unkowns and 4 equations (move constants to right hand side

$$\mathbf{x} = \begin{pmatrix} P_{2,2} \\ P_{2,3} \\ P_{3,2} \\ P_{3,3} \end{pmatrix}$$

$$P_{2,2} - \left(\tfrac{1}{4}\right)\left(P_{2,3} + P_{3,2}\right) = \left(\tfrac{1}{4}\right)\left(P_{2,1} + P_{1,2}\right) = \tfrac{1}{4}$$
$$\left(\tfrac{1}{4}\right)P_{2,2} - \left(P_{2,3}\right) - \left(\tfrac{1}{4}\right)\left(P_{3,3}\right) = \left(\tfrac{1}{4}\right)\left(P_{2,4} + P_{1,3}\right) = 0$$
$$\left(\tfrac{-1}{4}\right)P_{2,2} + \left(P_{3,2}\right) - \left(\tfrac{1}{4}\right)\left(P_{3,3}\right) = \left(\tfrac{1}{4}\right)\left(P_{3,1} + P_{4,2}\right) = \tfrac{1}{4}$$
$$\left(\tfrac{-1}{4}\right)\left(P_{2,3} + P_{3,2}\right) + \left(P_{3,3}\right) = \left(\tfrac{1}{4}\right)\left(P_{3,4} + P_{4,3}\right) = 0$$

Constants

Solve as Ax = b

- 4 unkowns and 4 equations

$$P_{2,2}-\left(\tfrac{1}{4}\right)\left(P_{2,3}+P_{3,2}\right)=\left(\tfrac{1}{4}\right)\left(P_{2,1}+P_{1,2}\right)=\tfrac{1}{4}$$
$$\left(-\tfrac{1}{4}\right)P_{2,2}+\left(P_{2,3}\right)-\left(\tfrac{1}{4}\right)\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{2,4}+P_{1,3}\right)=0$$
$$\left(\tfrac{-1}{4}\right)P_{2,2}+\left(P_{3,2}\right)-\left(\tfrac{1}{4}\right)\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{3,1}+P_{4,2}\right)=\tfrac{1}{4}$$
$$\left(\tfrac{-1}{4}\right)\left(P_{2,3}+P_{3,2}\right)+\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{3,4}+P_{4,3}\right)=0$$

$$\mathbf{b}=\begin{pmatrix}0.25\\0\\0.25\\0\end{pmatrix}$$

$$A=\begin{pmatrix}1&-.25&-.25&0\\-.25&1&0&-.25\\-.25&0&1&-.25\\0&-.25&-.25&1\end{pmatrix}\quad \mathbf{x}=\begin{pmatrix}P_{2,2}\\P_{2,3}\\P_{3,2}\\P_{3,3}\end{pmatrix}$$

```
% solve by matrix methods
A = [1 -.25 -.25 0; -.25 1 0 -.25; -.25 0 1 -.25; 0 -.25 -.25 1];
b = [0.25; 0; 0.25; 0];
%Solve for x
X = A\b
```

$$P_{2,2}-\left(\tfrac{1}{4}\right)\left(P_{2,3}+P_{3,2}\right)=\left(\tfrac{1}{4}\right)\left(P_{2,1}+P_{1,2}\right)=\tfrac{1}{4}$$
$$\left(-\tfrac{1}{4}\right)P_{2,2}+\left(P_{2,3}\right)-\left(\tfrac{1}{4}\right)\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{2,4}+P_{1,3}\right)=0$$
$$\left(\tfrac{-1}{4}\right)P_{2,2}+\left(P_{3,2}\right)-\left(\tfrac{1}{4}\right)\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{3,1}+P_{4,2}\right)=\tfrac{1}{4}$$
$$\left(\tfrac{-1}{4}\right)\left(P_{2,3}+P_{3,2}\right)+\left(P_{3,3}\right)=\left(\tfrac{1}{4}\right)\left(P_{3,4}+P_{4,3}\right)=0$$

$$\mathbf{b}=\begin{pmatrix}0.25\\0\\0.25\\0\end{pmatrix}$$

$$A=\begin{pmatrix}1&-.25&-.25&0\\-.25&1&0&-.25\\-.25&0&1&-.25\\0&-.25&-.25&1\end{pmatrix}\quad \mathbf{x}=\begin{pmatrix}P_{2,2}\\P_{2,3}\\P_{3,2}\\P_{3,3}\end{pmatrix}$$

```
X =

    0.3750
    0.1250
    0.3750
    0.1250
```

By Iterative Jacobi (p.236)

```
%solve by iterative Jacobi method (p. 236 of
turner)
%A and b from above
D = diag(A);
n = size(A,1);
A_D = A - diag(D);
x = zeros(n,1);
nits = 15;
for k = 1:nits
   x = (b - A_D*x)./D;
   S(:,k)=x;
end
S
```

```
S =

  Columns 1 through 9

    0.2500    0.3125    0.3438    0.3594    0.3672    0.3711    0.3730    0.3740    0.3745
         0    0.0625    0.0938    0.1094    0.1172    0.1211    0.1230    0.1240    0.1245
    0.2500    0.3125    0.3438    0.3594    0.3672    0.3711    0.3730    0.3740    0.3745
         0    0.0625    0.0938    0.1094    0.1172    0.1211    0.1230    0.1240    0.1245

  Columns 10 through 15

    0.3748    0.3749    0.3749    0.3750    0.3750    0.3750
    0.1248    0.1249    0.1249    0.1250    0.1250    0.1250
    0.3748    0.3749    0.3749    0.3750    0.3750    0.3750
    0.1248    0.1249    0.1249    0.1250    0.1250    0.1250
```