Setup of the linear equation matrix for the HW3 problems.

In problem 3-1, you coded the small maze problem using Gauss Seidel iteration. The formulas you used were written in the notation for the probability matrix that was suggestive of the physical maze you were modeling. Everyone or nearly everyone got that set up correctly. Note that each of those equations that you iterated over will become an equation in the Ap=b setup. That is, each equation will be represented by one row of the Ap=b system. All of the unkown P's (the four in the middle of the maze) are the variables to be determined. They are in the vector p. The A matrix contains the coefficients of those variables in each equation. In each case, one of the variables will have coefficient 1 and the others will have (1/4) just as in the Gauss Seidel setup. Some of the P's are fixed and known the ones on the perimeter. When you enter them into the equation, they become constants that we put on the right hand side. So, the b's have those numbers in them.

Remember the P's can be in any order. I am choosing them as follows. They simply the four unknowns in this problem – they could be labeled as x vector components to fit the book model.

$$\mathrm{p} = \begin{pmatrix} P_{22} \\ P_{32} \\ P_{23} \\ P_{33} \end{pmatrix}$$

We have the **p** vector. Now we need the equations. One of them is given for i=3 and j=2:

$P_{32} = (1/4)(P_{33} + P_{31} + P_{42} + P_{22})$ note that some of these P's are unknowns and in the **p** vector above and others are known constants. Substitute them:
$P_{32} = (1/4)(P_{33} + 1 + 0 + P_{22})$ Now rearrange this with all of the unknowns on the left side and all of the known values on the right:

$(1)P_{32} - (0.25)P_{33} - (0.25)P_{22} = (1/4) = 0.25$

This is one row of the matrix equation. I will write it as the second (order is irrelevant):

$$\mathrm{A} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ -0.25 & 1 & 0 & -0.25 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \qquad \mathrm{p} = \begin{pmatrix} P_{22} \\ P_{32} \\ P_{23} \\ P_{33} \end{pmatrix} \quad \mathrm{b} = \begin{pmatrix} \cdots \\ 0.25 \\ \cdots \\ \cdots \end{pmatrix}$$

To see that this is correct calculate the product **Ap** and the second row of that product should equal the left hand side of the explicit equation above. The second element of **b** will equal the right hand side. The equation written above is represented by the 2nd row of the **Ap=b** matrix equation. Now you complete the system by writing down the three other equations , starting with the other P's on the left hand side and use them to fill in the remaining rows of the **Ap=b** system.

Now you have a problem setup that looks like the ones in the book. The A matrix is the matrix of coefficients, the p vector is the vector of unknowns, and b vector is the vector of right hand sides (known). Solve the system by the methods of the book to obtain the vector **p**, the elements of which should be the same as the ones you obtained iteratively in Gauss Seidel.