CSS 455 Winter 2012 C. Jackels
Activity No. 9 Answers February 14, 2012

Part I. Consider the following set of three data points:

x_i	-2	0	1
$y_i = f(x_i)$	-27	-1	0

The goal is to find the interpolant in terms of monomials.

1. What is the order of the interpolating polynomial? What are the monomial functions to be used? The order will be 2nd order (quadratic). The functions used are 1,x, and x^2. An n-1 order polynomial fits n-points exactly.

2. If the coefficients of the terms are designated a_i, write down the system of linear equations in the form of the Vandermonde matrix for this system. Put in numeric values for all except the unknown coefficients.

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$$

1 = [1 1 1] x = [-2 0 1] and x^2= [4 0 1] and y = [-27 -1 0]. Ax = b is give as

$$\begin{pmatrix} 1 & -2 & 4 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -27 \\ -1 \\ 0 \end{pmatrix}$$

3. At this point, you can solve the 3 x 3 system either using matlab or by hand. Having done that, write down the polynomial interpolant *p(x)* for the data and evaluate it at *x=-1*

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ -4 \end{pmatrix}$$ $p(x) = -1 + 5x - 4x^2$

Part II. Consider the following set of three data points:

x_i	-2	0	1
$y_i = f(x_i)$	-27	-1	0

The goal is to find the interpolant in terms of Lagrangian polynomials.

1. What is the order of the interpolating polynomial? 2nd order

See reverse side

2. Write down the three Lagrangian basis functions for this problem from the slide:

$$l_0(x) = \frac{(x)(x-1)}{6}$$

$$l_1(x) = \frac{(x+2)(x-1)}{-2}$$

$$l_2(x) = \frac{(x+2)(x)}{3}$$

3. Obtain the ℓ_1 function above from the general formula: $l_j(x) = \prod_{k \neq j} \frac{(x - x_k)}{(x_j - x_k)}$

$$l_1(x) = \prod_{k \neq 1} \frac{(x - x_k)}{(x_j - x_k)} = \frac{(x = x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x+2)(x-1)}{(0+2)(0-1)} = -\frac{(x+2)(x-1)}{2}$$

4. If the coefficients of the terms are designated a_i, write down the system of linear equations in the form below. The lagrangian functions above play the roles of the ϕ*'s* below. Put in numeric values for all except the unknown coefficients.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -27 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_n(x_n) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

5. At this point, you can solve the 3 x 3 system either using matlab or by hand. Having done that, write down the polynomial interpolant *p(x)* for the data and evaluate it at *x=-1. By inspection:*

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -27 \\ -1 \\ 0 \end{pmatrix}$$

$$p(x) = -27\frac{x(x-1)}{6} - 1\frac{(x+2)(x-1)}{-2} + 0$$

$$p(x) = \tfrac{-27}{6}x(x-1) + \tfrac{1}{2}(x+2)(x-1)$$

$$p(x) = \tfrac{-9}{2}x(x-1) + \tfrac{1}{2}(x+2)(x-1)$$

$$p(-1) = -9 - 1 = -10$$

See reverse side

Part III. Consider the following set of three data points:

x_i	-2	0	1
$y_i = f(x_i)$	-27	-1	0

The goal is to find the interpolant in terms of Newton polynomials.

1. What is the order of the interpolating polynomial?
   2nd order

2. Write down the three Newton basis functions for this problem from the slide:

$$\varphi_1(x) = 1$$
$$\varphi_2(x) = (x - x_1)$$
$$\varphi_3(x) = (x - x_1)(x - x_2)$$

3. Obtain the $\phi_2(x)$ function above from the general formula: $\varphi_j(x) = \prod_{k=1}^{j-1}(x - x_k)$

$$\varphi_2(x) = \prod_{k=1}^{1}(x - x_2) = \underline{x - x_1}$$

4. If the coefficients of the terms are designated a_i, write down the system of linear equations in the form below. The Newton functions above play the roles of the ϕ's below. Put in numeric values for all except the unknown coefficients.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -27 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_n(x_n) \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

5. At this point, you can solve the 3 x 3 system either using matlab or by hand. Having done that, write down the polynomial interpolant *p(x)* for the data and evaluate it at *x=-1*.

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -27 \\ 13 \\ -4 \end{pmatrix}$$ p(x) = -27 +13(x+2) +4 (x+2)(x)

P(-1) = -27 + 13(1) -4(1)(-1) = -10

See reverse side