

Activity Answer No. 8

February 11, 2012

In this setup you have three data points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , which are known. They may have been measured or taken from a table, for example. Your goal is to interpolate them with the cubic polynomial

$$p(x_i) = y_i = a + bx_i + cx_i^2$$

$$p(x_1) = y_1 = a + bx_1 + cx_1^2$$

$$p(x_2) = y_2 = a + bx_2 + cx_2^2$$

$$p(x_3) = y_3 = a + bx_3 + cx_3^2$$

Assume we will use the methods of linear systems of equations to solve these equations. That is we will cast this in the form: $\mathbf{Az}=\mathbf{b}$.

- a) How many equations are there (rows of \mathbf{A})? And, how many unknowns (cols of \mathbf{A})?
three rows and three columns.
- b) What are the unknowns? Are the equations linear in these unknowns? Write the \mathbf{z} vector in terms of these unknowns.

the three unknowns: a, b, c . the equations are linear. $\mathbf{z} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

- c) What is the right hand side or \mathbf{b} vector? Write \mathbf{b} in terms of the constant terms in the three equations.

$$\mathbf{b} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

- d) Create the \mathbf{A} matrix. Remember, each row must contain those factors that multiply the unknowns in the \mathbf{z} vector. The elements of \mathbf{A} can be complex, but must be known for each equation prior to solving the system. Write the matrix row by row in terms of the parameters of the problem:

$$\mathbf{A} = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix}$$

See reverse side