CSS 455 Winter 2012 C. Jackels
Activity No. 6 January 31, 2012
Names (must be present):

Part 1. **Design an algorithm in pseudo code for solving the following system using forward substitution (start with $x_1=b_1/a_{11}$). Try to design a row-ordered dot-product type algorithm similar to the one for backward substitution:**

$$\mathbf{A} = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

- *First find the first value:*
  x(1) = b(1)/a(1,1)
- *Loop over the remaining rows of the matrix:*
  for i = 2:n
  *x(i) = (b(i) - a(i,1:i-1)*x(1:i-1))/a(i,i)*
  end

*Organized as dot product between the partial row of **A** and partial vector **x***

See reverse side

Part II

Given the matrices **M** and **A** below, calculate the product **MA,**

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{-4}{9} & 1 & 0 \\ \frac{-1}{9} & 0 & 1 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 9 & 3 & 2 \\ 4 & 1 & 6 \\ 1 & 7 & 2 \end{pmatrix}$$

*>> M*A*

ans =

```
    9.0000    3.0000    2.0000
         0   -0.3333    5.1111
         0    6.6667    1.7778
```

Given the matrix $\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{4}{9} & 1 & 0 \\ \frac{1}{9} & 0 & 1 \end{pmatrix}$ calculate the product **LM:**

>> L*M

ans =

```
     1     0     0
     0     1     0
     0     0     1
```

See reverse side

Part III

If the $\mathbf{L_1}$ is the inverse of $\mathbf{M_1}$, $\mathbf{L_2}$ is the inverse of $\mathbf{M_2}$, and $\mathbf{L_3}$ is the inverse of $\mathbf{M_3}$, then if $\mathbf{M=M_3M_2M_1}$, **and** $\mathbf{L=L_1L_2L_3}$, show that **L** is the inverse of **M,** that is, $\mathbf{L=M^{-1}}$.

$$\mathbf{LM} = (\mathbf{L}_1\mathbf{L}_2\mathbf{L}_3)(\mathbf{M}_3\mathbf{M}_2\mathbf{M}_1) = (\mathbf{L}_1\mathbf{L}_2(\mathbf{L}_3\mathbf{M}_3)\mathbf{M}_2\mathbf{M}_1) = (\mathbf{L}_1(\mathbf{L}_2\mathbf{M}_2)\mathbf{M}_1) = (\mathbf{L}_1\mathbf{M}_1) = \mathbf{I}$$

There L is the inverse of M. In each case above, the product of the matrix and its inverse yields the identity matrix, which commutes with the other matrixes and can be moved out of the expression. These intermediate identities were not shown above.

See reverse side