

Part 1.

$$y_i = \sum_{k=1}^n a_{ik} x_k$$

- Given the matrix-vector product  $y = Ax$  where  $y$  is an  $(r \times 1)$  column vector,  $x$  is an  $(n \times 1)$  and  $A$  is an  $(r \times n)$  matrix, write pseudo code for a suitable algorithm.
- Estimate in terms of  $r$  and  $n$  the number of floating point operations (multiplies, additions and subtractions) in this algorithm.

```

For i from 1 to r
  Sum = 0
  For k from 1 to n
    Sum = sum + a(i,k)*x(k)
  End
  y(i) = sum
end
    
```

The inner loop is run  $rxn$  times (an  $n^2$  operation) and executes one add and one multiply each time:  $2r n$  flops.

Part II

Given the algorithm for the row-ordered matrix vector product:

```

%compute the Ax product
y = zeros(m,1);
for i = 1:m
  y(i) = 0;
  for j = 1:n
    y(i) = y(i) + A(i,j)*x(j);
  end
end
    
```

$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$	$a_{16}$
0	$a_{22}$	$a_{23}$	$a_{24}$	$a_{25}$	$a_{26}$
0	0	$a_{33}$	$a_{34}$	$a_{35}$	$a_{36}$
0	0	0	$a_{44}$	$a_{45}$	$a_{46}$
0	0	0	0	$a_{55}$	$a_{56}$
0	0	0	0	0	$a_{66}$

Modify as needed for the case where  $A$  is upper triangular

```

%compute the Ax product
y = zeros(m,1);
for i = 1:m
  y(i) = 0;
  for j = i:n
    y(i) = y(i) + A(i,j)*x(j);
  end
end
    
```

Each run of the j-loop (across a row) now need only begin with the diagonal element to skip multiplies involving zeros. You could write the inner loop as a dot product with the active vectors being (i:n) in length.

### Part III

$$C_{kj} = \sum_{i=1}^r A_{ki} B_{ij}$$

- Given the matrix-matrix product  
 $\mathbf{C} = \mathbf{A}\mathbf{B}$   
where  $\mathbf{A}$  is an  $(m \times r)$  matrix,  $\mathbf{B}$  is an  $(r \times n)$  matrix and  $\mathbf{C}$  is an  $(m \times n)$  matrix write pseudo code for a suitable algorithm.
- Estimate in terms of  $m$ ,  $n$ , and  $r$  the number of floating point operations (multiplies, additions and subtractions) in this algorithm.

```
For j from 1 to n
  For k from 1 to m
    Sum = 0
    For i from 1 to r
      Sum = sum + A(k,i)*B(i,j)
    End
    C(k,j) = sum
  End
end
```

The inner loop is run  $rxn$  times (an  $n^3$  operation) and executes one add and one multiply each time:  $2nm$  flops.

See reverse side