CSS 455 Winter 2012 C. Jackels
Activity No. 10 February 27, 2012

Part I. Divided Differences and Newton Coefficients.

In a a previous activity, you found the Newton form of the polynomial interpolant

Example

- TheNewton interpolant for:

i	1	2	3
x_i	-2	0	1
y_i or $f(x_i)$	-27	-1	0

$$\varphi_1(x) = 1$$
$$\varphi_2(x) = (x - x_1)$$
$$\varphi_3(x) = (x - x_1)(x - x_2)$$

$$p(x) = -27 + 13(x - x_1) - 4(x - x_1)(x - x_2)$$

Consider the following two arrangements of the same data and carry out in parallel the divided difference calculations suggested:

i	0	1	2		***i***	0	1	2
x_i	-2	0	1		x_i	0	-2	1
y_i	-27	-1	0		y_i	-1	-27	0

i	0	1	2		***i***	0	1	2

See reverse side

x_i	-2	0	1		x_i	0	-2	1
y_i	-27	-1	0		y_i	-1	-27	0
$f[x_i]$	-27	-1	0		$f[x_i]$	-1	-27	0
$f[x_i,x_{i+1}]$	13	1			$f[x_i,x_{i+1}]$	13	9	
$f[x_i,x_{i+1},x_{i+2}]$	-4				$f[x_i,x_{i+1},x_{i+2}]$	-4		

In both tables above , enter the zero-order divided differences: ***$f[x_i]$ (= $f(x_i) = y_i$).***

In the tables above, enter the 1st divided **differences:** $f[x_i, x_{i+1}] = \dfrac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$

(Note there is one less entry in this row.)

In the tables above, enter the 2nd divided differences $f[x_i, x_{i+1}, x_{i+2}] = \dfrac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$

(Note there is one less entry in this row.)

Using the coefficients in the 2nd column (under *i=0)* of each table, write quadratic polynomials of the general Newton form, one for each table:

$$f[x_0](1) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

Left side Table polynomial: (-27)(1) +13(x-x_0) -4(x-x_0)(x-x_1)

Right side Table Polynomial: (-1)(1) +13(x-x_0) -4(x-x_0)(x-x_1)

See reverse side

Substitute the values of the x_0 and x_1 for each table and arrive at a polynomial in simplified form:

Left Side Polynomial: **$(-27)(1) +13(x-x_0) -4(x-x_0)(x-x_1)= -27 +13(x+2)-4(x+2)(x)=$**
$= -27 +26 +13x -4x^2 -8x = -1 + 5x -4x^2$

Right Side Polynomial: $(-1)(1) +13(x-x_0) -4(x-x_0)(x-x_1) = -1 + 13(x)-4(x)(x+2) =$
$=-1 +13x -4x^2 -8x = -1 +5x -4x^2$.

Answer the following questions:

1. Does the divided difference method produce the Newton polynomial interpolant of the problem?
   Yes it does – same as on page 1.
2. Does the order of the data points in the divided difference approach affect the interpolant obtained? What is your evidence?
   No it does not – the points have been interchanged above. While not a general proof, it is suggestive.
3. Suppose you wanted to add a fourth data point to this data set.

   a. What changes in the above results would be necessary? Describe the process you would carry out in terms of modifying the divided difference tables and their results. *Do not actually carry this out.*
   You would add a fourth column of data and carrying out the divided difference calculations would push the table down one more row to include the 3rd divided difference $f[x_0,x_1, x_2,x_3]$

   b. Does it make any difference if the 4th point extends the range of *x* or is within the above range? How do you know? Not as far as the procedure goes, since they can be in any order. Place it at the end of the table in either case.

See reverse side