CSS 455 Winter 2012 C. Jackels
Activity No. 3 (and 3b) January 9, 2012
Names (must be present):

The purpose of this activity is to explore formation of the Jacobian matrix and to use an iterative procedure to solve the catenary equation.

1. **The Jacobian** matrix arises in the solution of a system of equations in multiple unknowns. In Turner's example (p.45), he defines the functions to be solved as:

$$f_1(x,y) = 4x^2 + y^2 - 4 = 0$$

$$f_2(x,y) = x^2 y^3 - 1 = 0$$

To solve the system, you need the Jacobian, which consists of the full set of partial derivatives of both functions

The solution is:

$$\mathbf{J} = \begin{pmatrix} 8x & 2y \\ 2xy^3 & 3x^2y^2 \end{pmatrix} = \begin{pmatrix} f_{1x} & f_{1y} \\ f_{2x} & f_{2y} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

With you partner, verify that understand how each of these partial derivatives were obtained and that you understand all of the notation used above. *If you have any questions at all, have them answered now!*

In MP1, the following equations are presented:

$$\lambda \cosh\left(\frac{L_1}{\lambda}\right) = \lambda + s$$

$$\lambda \cosh\left(\frac{L - L_1}{\lambda}\right) = \lambda + s + H$$

Write them below in the form *$f_1(\lambda, L_1) = 0$ and $f_2(\lambda, L_1) = 0$.*

See reverse side

Find the four partial derivatives of these two functions $\partial f_1 / \partial \lambda$ $\partial f_1 / \partial L_1$ $\partial f_2 / \partial \lambda$ $\partial f_2 / \partial L_1$

If λ plays the role of x_1 and L1 that of x_2, indentify the derivatives as J_{11}, J_{12}, J_{21}, and J_{22}.

The following was "Part I of Activity 3b" in class

2. **Iterative solution to an equation.** The purpose of this exercise is to exhibit the fixed point iteration method of solution in problem with a single variable. Consider the catenary equation :

$$\lambda \cosh\left(L / 2\lambda \right) = \lambda + s$$

Write this equation in the form $f(\lambda) = 0$, with the constants $s=40$ *and* $L=300$.:

Rearrange the equation to provide at least *two different* equations of the form $\lambda = g(\lambda)$

Selecting one of them, use the initial guess $\lambda_0 = 270$ and evaluate the right hand side. This result is the next guess at the solution λ_1 . Now plug this new value into the right hand side to obtain the subsequent guess λ_3. Continue until it is clear whether or not the value is converging to the correct value of 287.68.

See reverse side