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CSS 455 Winter 2012 C. Jackels
Activity No. 13 March 2, 2012
Names (must be present):

Part I. Composite rules for numerical quadrature.

Download the script *CompositeNCCos.m* from the Set6 examples on the web site. Execute this script with Matlab. This program uses a composite Newton Cotes (m=3) rule to evaluate the integral at the right, which we have treated previously in class using the simple NC(m) rules:

$$\int_0^{\pi/2} \cos(x)dx = \sin(x)\Big|_0^{\pi/2} = 1$$

On each iteration, the program prints out the number of integration panels (N), the approximate value of the integral and the absolute error.

1. At least in this case, does the convergence appear to be monotonic?

2. Examine at least three pairs of the approximate integration errors (E_N and E_{2N}) to determine if they are approximately related by the expression with a common value of X:

$$E_{2N} \approx \left(\frac{1}{X}\right)E_N$$

   What is the average experimental value of X for the cases you chose?

3. Since there is a predictable relationship between successive errors (above), one may be able to use that to extrapolate to the correct answer.

   Consider the following proposed relationship, where S_N is the approximate integral with N panels and I_{exact} is the exact integral:

$$\left(I_{exact} - S_{2N}\right) \approx \left(S_{2N} - S_N\right)\Big/ Y$$

   Using two choices of N, calculate an average experimental of Y:

=======do not go on until after class discussion ===========================

Part II. We have seen that numerical integration of *humps* is much more challenging. Download the m-file *CompositeNCHumps.m* and execute it.

4. Using the "class value" for X above, determine at what value of *N* the integration errors for *humps* appear to be following the pattern we found above:

5. Using the "Class value" for Y above, estimate the exact value of the *humps* integral, extrapolating from *N=128.*

Part III.

6. Download and run *CompositeNCVariableM.m.* This run produces graphs for *cos* and *humps* functions that present integration error as a function of the number of panels and of the order of the NC integration. For each function, assume you have an approximate integral calculated with Simpson's Rule (m=3) at 30 integration panels. To improve the calculation, should you: a) double the number of panels or b) go to an m=5 NC method? Your discussion should be quantitative and provide a comparison of the two cases. How do the two cases differ?

Part IV

Using the approximation:

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

estimate the value of cos(π/2) and report the error for your assigned value of *h*. The exact answer is -0.1.