CSS 455 Winter 2012 C. Jackels
Activity No. 10 February 14, 2012
Names (must be present):

Part I. Divided Differences and Newton Coefficients.

In a previous activity, you found the Newton form of the polynomial interpolant

Example

- TheNewton interpolant for:

i	1	2	3
x_i	-2	0	1
y_i or $f(x_i)$	-27	-1	0

$$\varphi_1(x) = 1$$
$$\varphi_2(x) = (x - x_1)$$
$$\varphi_3(x) = (x - x_1)(x - x_2)$$

$$p(x) = -27 + 13(x - x_1) - 4(x - x_1)(x - x_2)$$

Consider the following two arrangements of the same data and carry out in parallel the divided difference calculations suggested:

i	0	1	2		***i***	0	1	2
x_i	-2	0	1		x_i	0	-2	1
y_i	-27	-1	0		y_i	-1	-27	0

See reverse side

i	0	1	2		i	0	1	2
x_i	-2	0	1		x_i	0	-2	1
y_i	-27	-1	0		y_i	-1	-27	0
$f[x_i]$					$f[x_i]$			
$f[x_i,x_{i+1}]$					$f[x_i,x_{i+1}]$			
$f[x_i,x_{i+1},x_{i+2}]$					$f[x_i,x_{i+1},x_{i+2}]$			

In both tables above , enter the zero-order divided differences: $f[x_i]\ (= f(x_i) = y_i)$.

In both tables above, enter the 1st divided **differences:** $f[x_i, x_{i+1}] = \dfrac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$

(Note there is one less entry in this row.)

In both tables above, enter the 2nd divided differences $f[x_i, x_{i+1}, x_{i+2}] = \dfrac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$

(Note there is one more less entry in this row.)

Using the coefficients in the 2nd column (under *i=0)* of each table, write quadratic polynomials of the general Newton form, one for each table:

$$f[x_0](1) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

Left side Table polynomial:

Right side Table Polynomial:

See reverse side

Substitute the values of the x_0 and x_1 for each table and arrive at a polynomial in simplified form:

Left Side Polynomial:

Right Side Polynomial:

Answer the following questions:

1. Does the divided difference method produce the Newton polynomial interpolant of the problem?

2. Does the order of the data points in the divided difference approach affect the interpolant obtained? What is your evidence?

3. Suppose you wanted to add a fourth data point to this data set.

    a. What changes in the above results would be necessary? Describe the process you would carry out in terms of modifying the divided difference tables and their results. *Do not actually carry this out.*

    b. Does it make any difference if the 4^{th} point extends the range of x or is within the above range? How do you know?

See reverse side