Astr 511: Galactic Astronomy

Winter Quarter 2015, University of Washington, Željko Ivezić

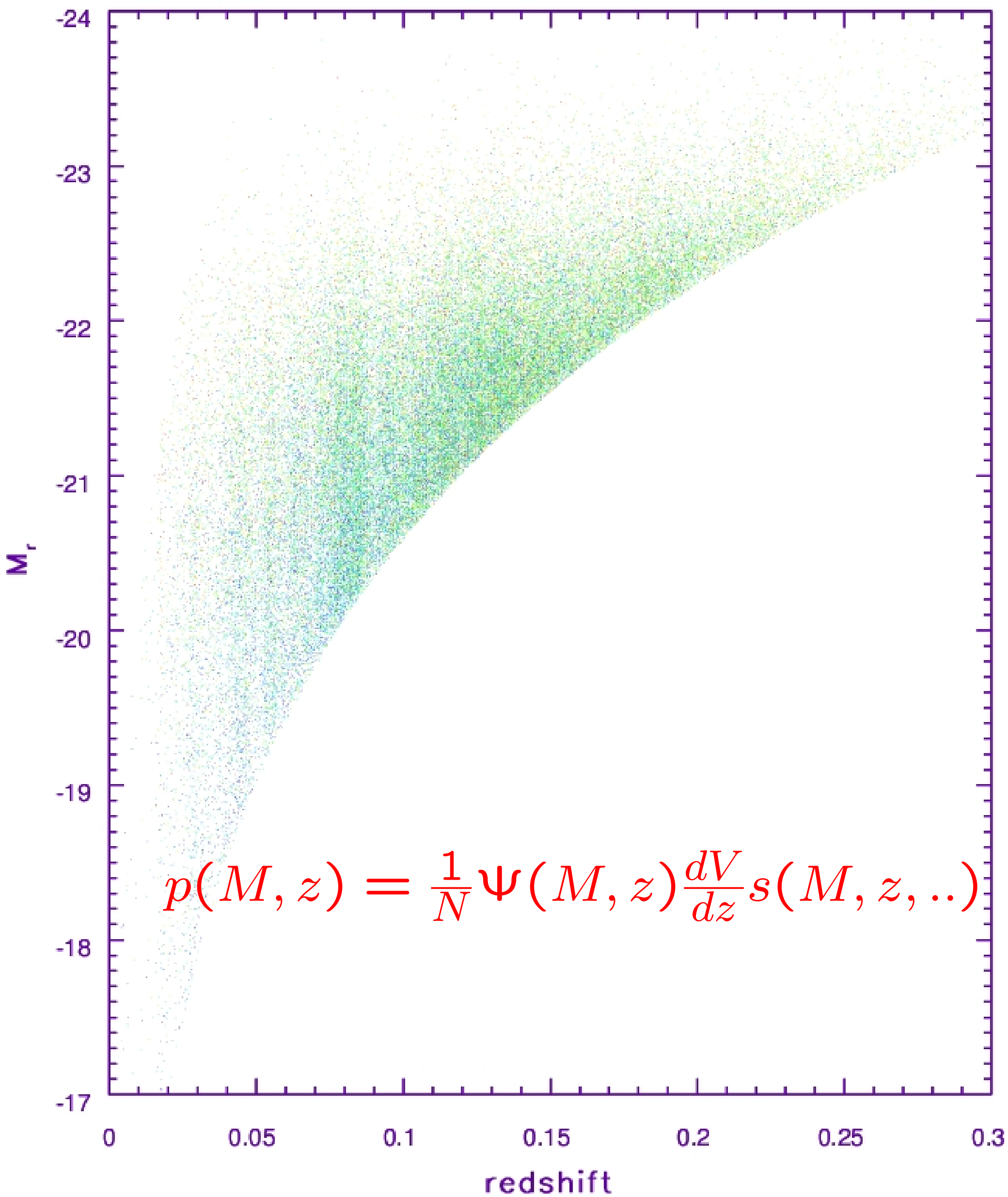
Lecture 4:

Luminosity and mass functions

Outline

- Basic concepts
- LF for SDSS galaxies
- Methods for estimating LF from data
- Stellar mass function in the Milky Way

Luminosity Function



$$p(M,z) = \frac{1}{N}\Psi(M,z)\frac{dV}{dz}s(M,z,..)$$

- Luminosity Function is the distribution in the luminosity–position plane; how many galaxies per unit interval in luminosity and unit volume (or redshift): $\Psi(M,z)$
- Imagine a tiny area with the widths ΔMr and Δz centered at some Mr and z in the plot to the left: count the number of galaxies, divide by $\Delta Mr \Delta z$, correct for the fraction of sky covered by your survey, and for the selection probability (a function of Mr, z, and possibly many other parameters): this gives you $\Psi(M,z)$.

Luminosity Function

- Luminosity Function is the distribution in the luminosity–position plane; how many galaxies per unit interval in luminosity and unit volume: $\Psi(M, z)$

- Often, this is a separable function: $\Psi(M, z) = \Phi(M)\, n(z)$, where $\Phi(M)$ is the absolute magnitude (i.e. luminosity) distribution, and $n(z)$ is the number volume density.

- Luminosity is a product of flux and distance squared (ignore cosmological effects for simplicity): $L = 4\pi D^2 F$

- The samples are usually *flux-limited* (meaning: all sources brighter than some flux limit are detected) – the minimum detectable luminosity depends on distance: $L > 4\pi D^2 F_{min}$, or for absolute magnitude $M < M_{max}(D)$ (c.f. the first plot)

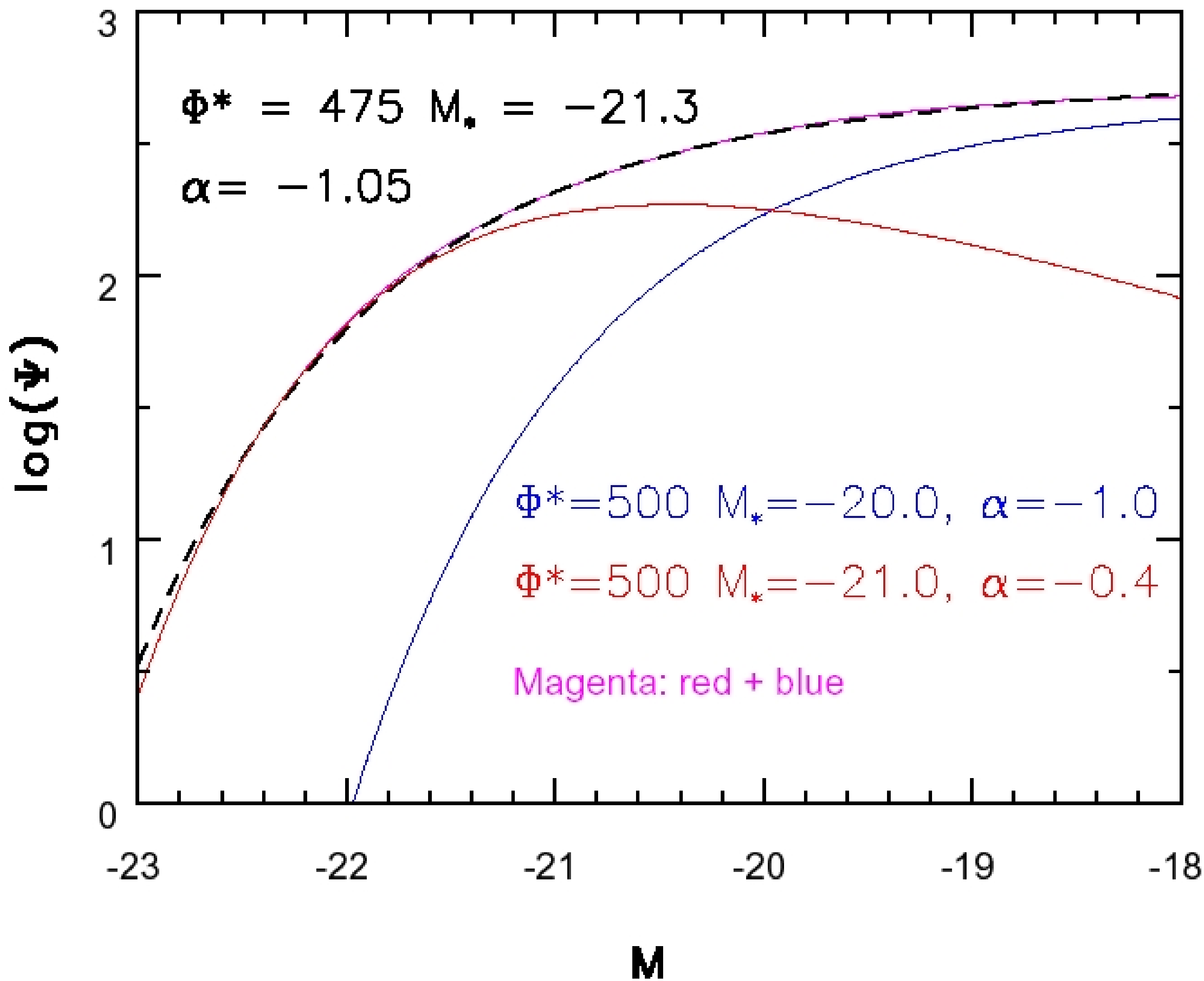
Schechter Function

Galaxy luminosity distribution resembles a power-law, with an exponential cutoff. This distribution is usually modeled by Schechter function:

$$\Phi(L) = \Phi^* \left(\frac{L}{L_*}\right)^{\alpha} \mathrm{e}^{-L/L_*} \tag{1}$$

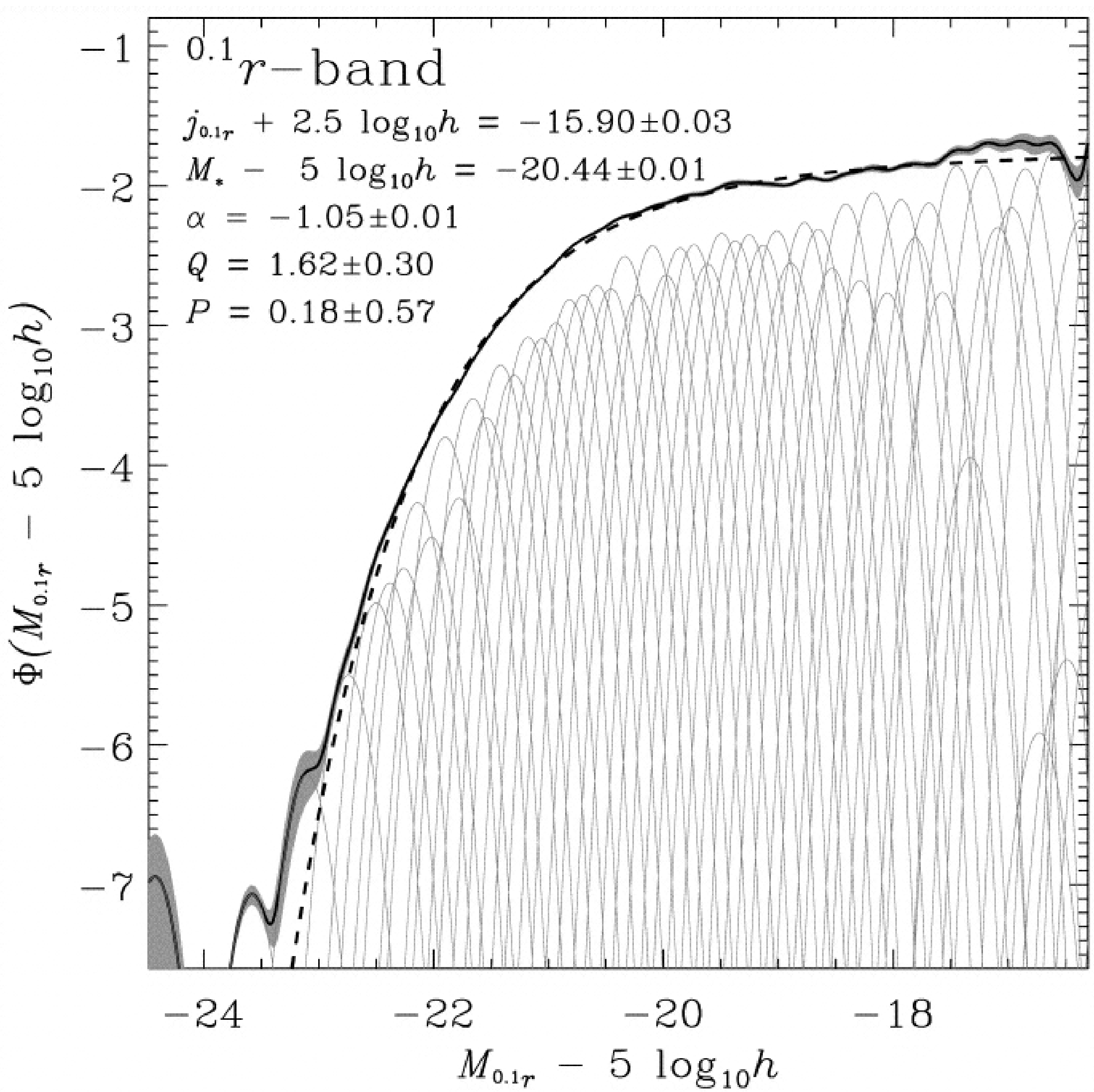
Or using absolute magnitudes:

$$\Phi(Mr) = 0.4\Phi^* \, \mathrm{e}^{-0.4(\alpha+1)(Mr-M^*)} \, \mathrm{e}^{-\mathrm{e}^{-0.4(Mr-M^*)}} \tag{2}$$



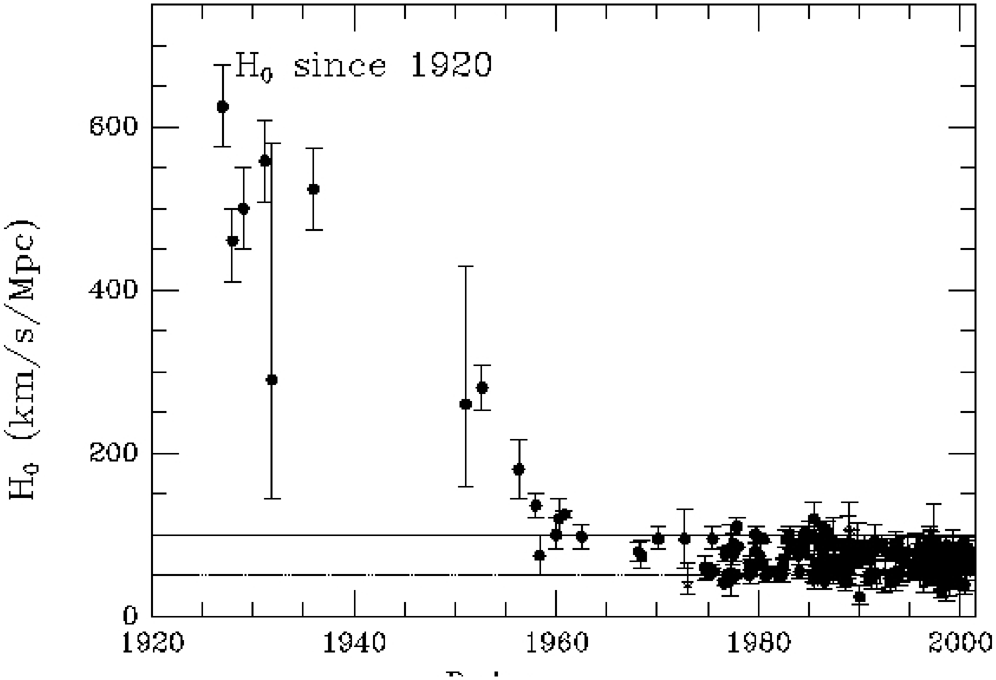
Φ* = 475 M* = −21.3
α= −1.05
Φ*=500 M*=−20.0, α=−1.0
Φ*=500 M*=−21.0, α=−0.4
Magenta: red + blue
log(Ψ)
M
0
1
2
3
-23
-22
-21
-20
-19
-18

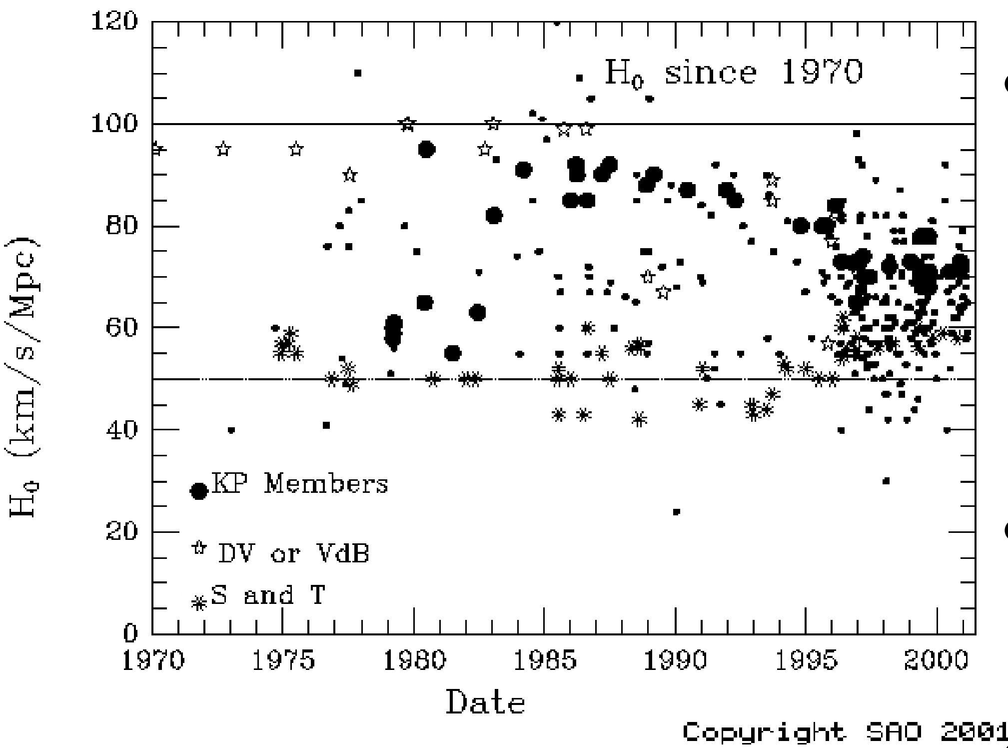
The LF in the SDSS r band



Note: this LF cannot be expressed as $\Phi(M, z) = f(M)\, g(z)$ – not separable!

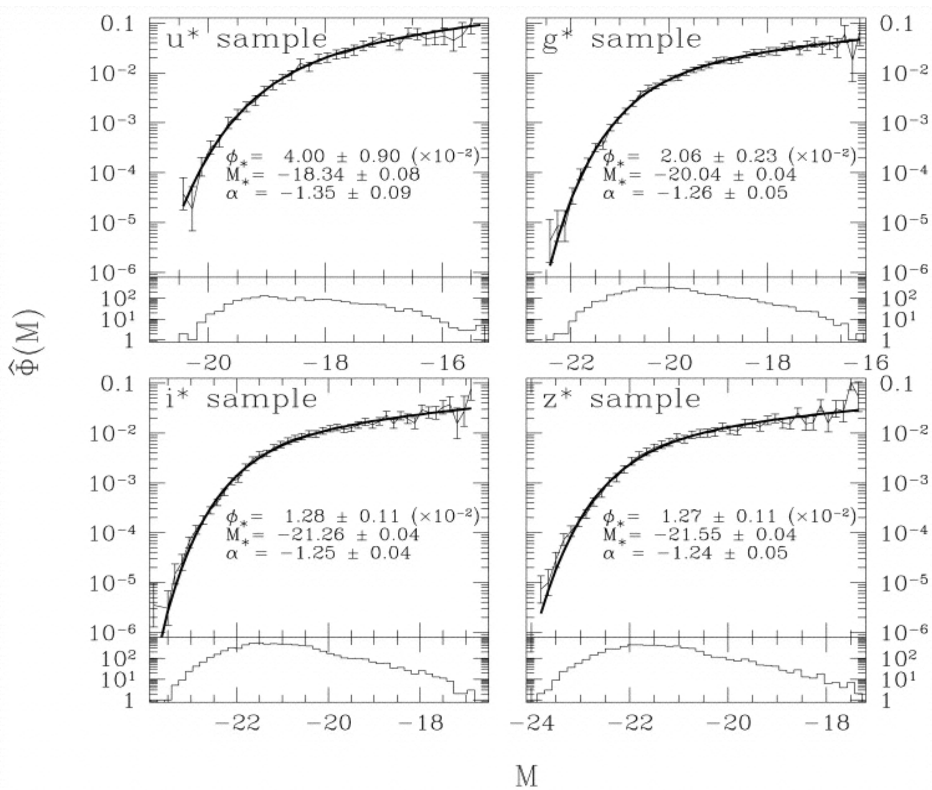
- The thick solid line is the SDSS r band luminosity function, and the gray band is its uncertainty.
- The dashed line is a Schechter-like fit that also includes the effects of changing luminosity and the number density with time (i.e. distance, or redshift). $Q > 0$ indicates that galaxies were more luminous in the past, and $P > 0$ that galaxies were more numerous in the past. For detailed discussion, see Blanton et al. 2003 (Astronomical Journal, 592, 819-838)

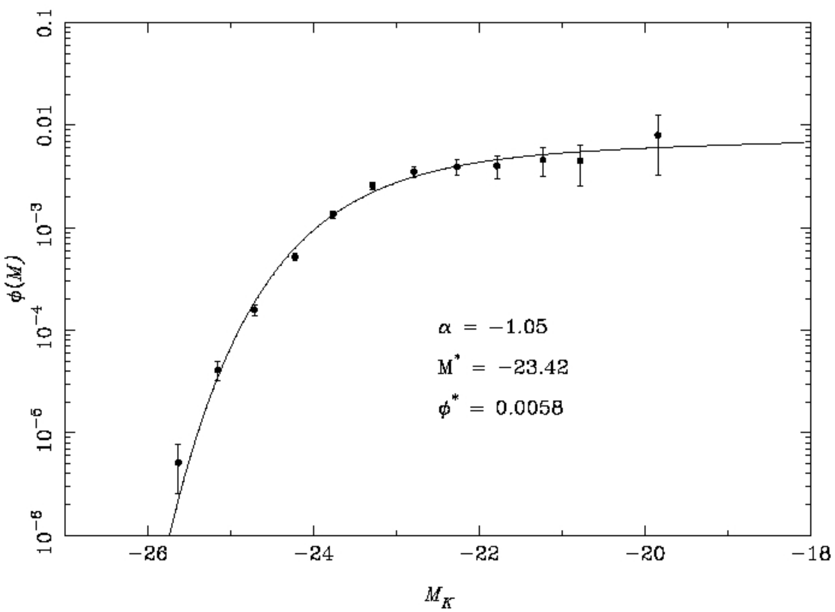




H_o as "a function of time"

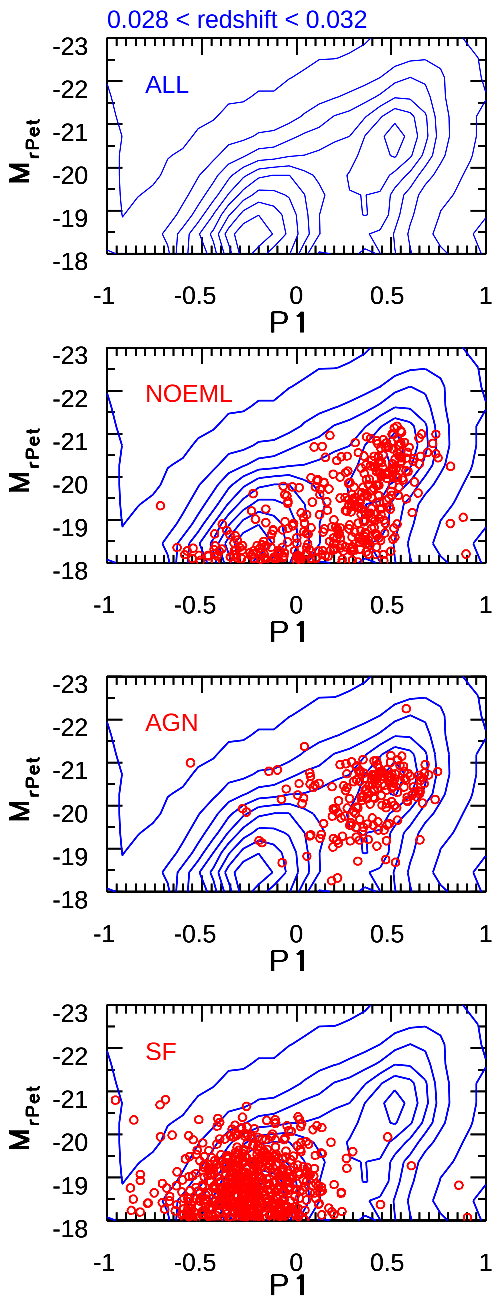
- the first three points: Lemaitre (1927), Robertson (1928), Hubble (1929), all based on Hubble's data
- the early low value (290 km/s/Mpc): Jan Oort
- the first major revision: discovery of Population II stars by Baade
- the very recent convergence to values near 65±10 km/sec/Mpc
- the best Cepheid-based value for the local H_o determination is 71±7 km/s/Mpc, the WMAP5 value based on cosmic microwave background measurements: 72±3 km/s/Mpc.
- WMAP9: 69.3±0.8 km/s/Mpc, and the Planck Mission: $H_o =$ 67.8 ± 0.8 km/s/Mpc ($h = 0.678$)
- Thusly, $-5\log_{10}(h) = 0.84$ mag!





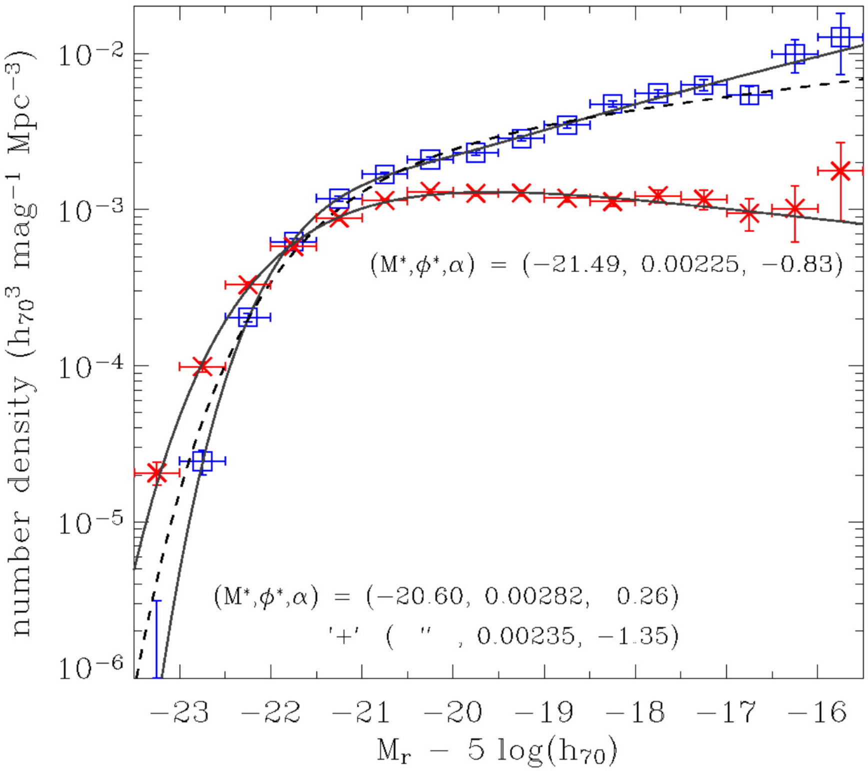
The dependence of LF on wavelength

- Top: SDSS ugiz bands
- Bottom: 2MASS K band
- The Schechter function is still a good fit, but best-fit parameters vary.
- Since the SEDs of galaxies are nearly one-dimensional families, once the LF for a sample selected by color or morphology is known, the LFs at other wavelengths can be simply obtained by shifting the M axis by the appropriate color difference.
- This doesn't work for the LFs in the top four panels because they are computed for the whole sample.



The dependence of LF on galaxy type

- The top panel shows the distribution of SDSS galaxies in the absolute magnitude – color plane (in a narrow redshift range)
- In the bottom three panels, the same distribution is compared to the distributions for subsamples selected by their emission line properties
- Note that the most luminous galaxies ($M_r < -20$) are predominantly red ($P1 > 0.2$), while faint galaxies ($M_r > -19$) are blue ($P1 < 0.2$)



The dependence of LF on galaxy type

- The comparison of LFs for blue and red galaxies (from Baldry et al. 2004, ApJ, 600, 681-694)
- The red distribution has a more luminous characteristic magnitude and a shallower faint-end slope, compared to the blue distribution
- The transition between the two types corresponds to stellar mass of $\sim 3 \times 10^{10}$ $M_\odot$
- The differences between the two LFs are consistent with the red distribution being formed from major galaxy mergers.

The C^{-} method for estimating LF

- Lynden-Bell (1971, MNRAS 155, 95); a non-parametric method that works for separable LFs, $\Psi(L,z) = \Phi(L)n(z)$

- practically all non-parametric methods can be reduced to the C^{-} method (Petrosian 1992)

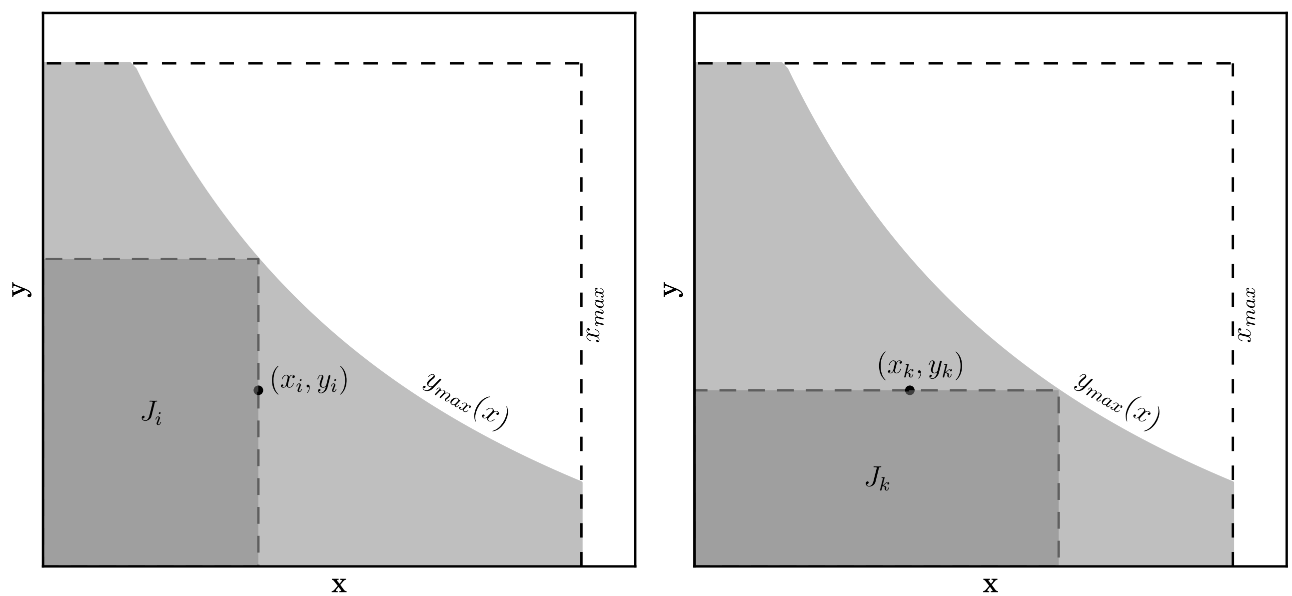
- parametric methods are usually based on maximizing likelihood (e.g. Marshall 1985)

- the simplest and most famous method, the V_{max} method (Schmidt 1968), requires binning in two axes simultaneously, while with the C^{-} method data is binned only one axis at a time (e.g. Fan et al. 2001)

- How do we know that separable LF is a good guess for our data?

C$^-$ method

- Given a set of measured pairs (x_i, y_i), with $i = 1 \ldots N$, and *known* relation $y_{max}(x)$, estimate the two-dimensional distribution, $n(x, y)$, from which the sample was drawn. Assume that measurement errors for both x and y are negligible compared to their observed ranges, that x is measured within a range defined by x_{min} and x_{max}, and that the selection function is 1 for $0 \le y \le y_{max}(x)$ and $x_{min} \le x \le x_{max}$, and 0 otherwise.

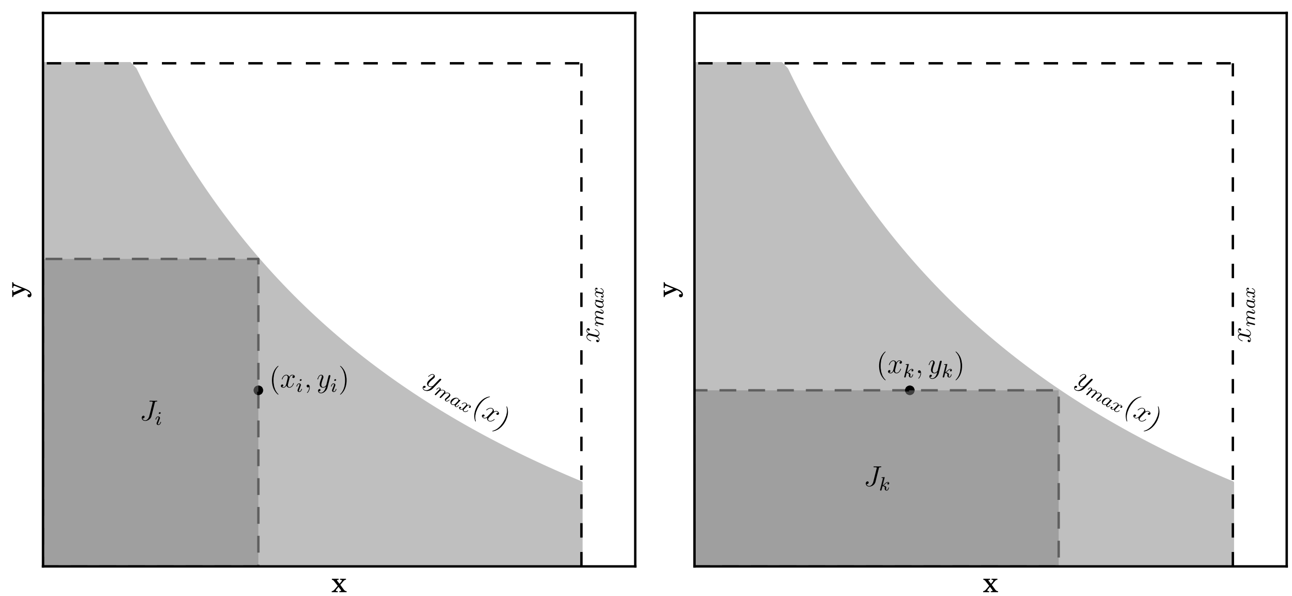


C^- method

- C^- method is applicable when the distributions along the two coordinates x and y are uncorrelated, that is, when we can assume that the bivariate distribution $n(x,y)$ is separable

$$n(x,y) = \Psi(x)\,\rho(y). \tag{3}$$

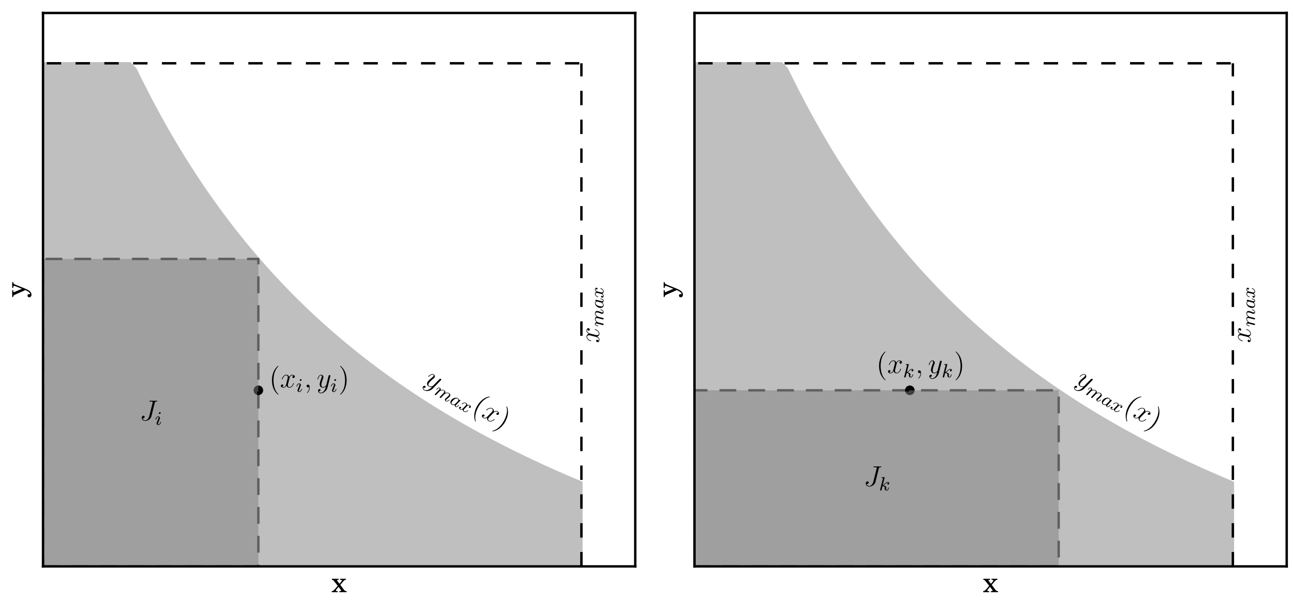
Therefore, before using the C^- method we need to demonstrate that this assumption is valid.



C$^-$ method

- Define a *comparable* or *associated* set for each object i such that $J_i = \{j : x_j < x_i, y_j < y_{max}(x_i)\}$; this is the largest x-limited and y-limited data subset for object i, with N_i elements (see the left panel).

- Sort the set J_i by y_j; this gives us the rank R_j for each object (ranging from 1 to N_i)



C^- method

- Define a *comparable* or *associated* set for each object i such that $J_i = \{j : x_j < x_i, y_j < y_{max}(x_i)\}$; this is the largest x-limited and y-limited data subset for object i, with N_i elements.

- Sort the set J_i by y_j; this gives us the rank R_j for each object (ranging from 1 to N_i)

- Define the rank R_i for object i in *its* associated set: this is essentially the number of objects with $y < y_i$ in set J_i.

- If x and y are truly independent, R_i must be distributed *uniformly* between 0 and N_i.

C^- method

- If x and y are truly independent, R_i must be distributed *uniformly* between 0 and N_i.

- In this case, it is trivial to determine the expectation value and variance for R_i: $E(R_i) = E_i = N_i/2$ and $V(R_i) = V_i = N_i^2/12$. We can define the statistic

$$\tau = \frac{\sum_i (R_i - E_i)}{\sqrt{\sum_i V_i}} \tag{4}$$

  If $\tau < 1$, then x and y are uncorrelated at $\sim 1\sigma$ level!

C^- method

Assuming that $\tau < 1$, it is straightforward to show using relatively simple probability integral analysis (e.g., see Appendix in Fan et al. 2001), as well as the original Lynden-Bell's paper, how to determine cumulative distribution functions. The cumulative distributions are defined as

$$\Phi(x) = \int_{-\infty}^{x} \Psi(x')dx', \tag{5}$$

and

$$\Sigma(y) = \int_{-\infty}^{y} \rho(y')dy'. \tag{6}$$

Then,

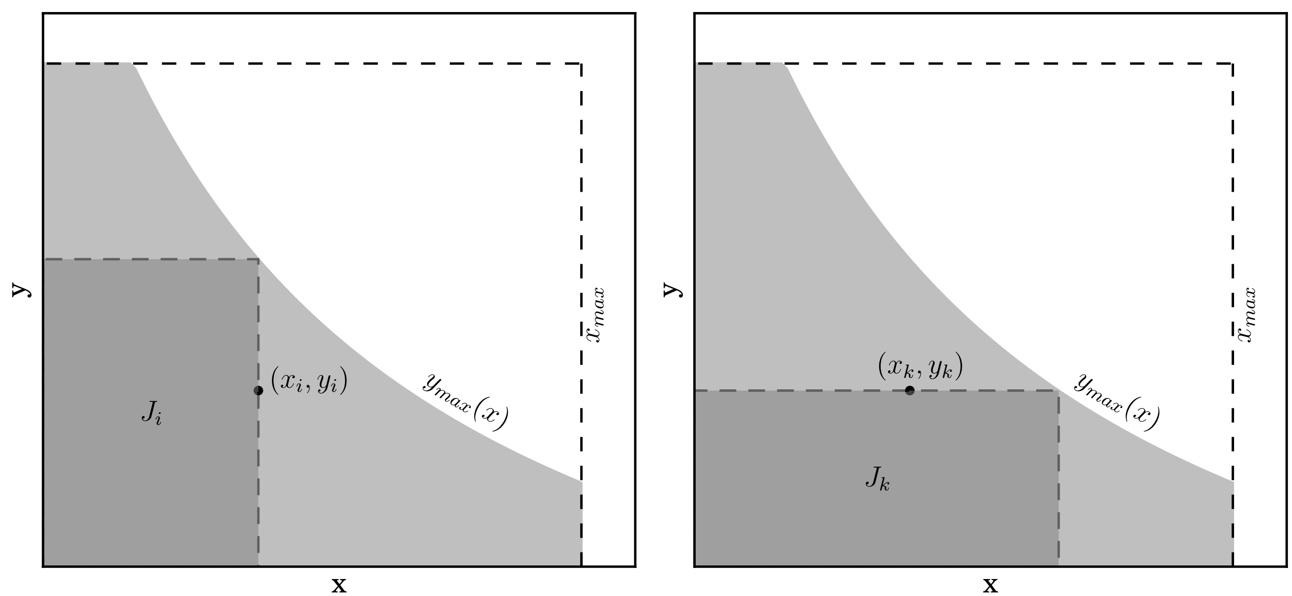
$$\Phi(x_i) = \Phi(x_1)\,\Pi_{k=2}^{i}(1 + 1/N_k) \tag{7}$$

where it is assumed that x_i are sorted ($x_1 \leq x_k \leq x_N$).

C^- method

Analogously, if M_k is the number of objects in a set defined by $J_k = \{j : y_j < y_k, y_{max}(x_j) > y_k\}$ (see the right panel of figure below), then

$$\Sigma(y_j) = \Sigma(y_1)\,\Pi_{k=2}^{j}(1 + 1/M_k). \tag{8}$$



astroML implementation of the C^- method

- Note that both $\Phi(x_j)$ and $\Sigma(y_j)$ are defined on non-uniform grids with N values, corresponding to the N measured values.

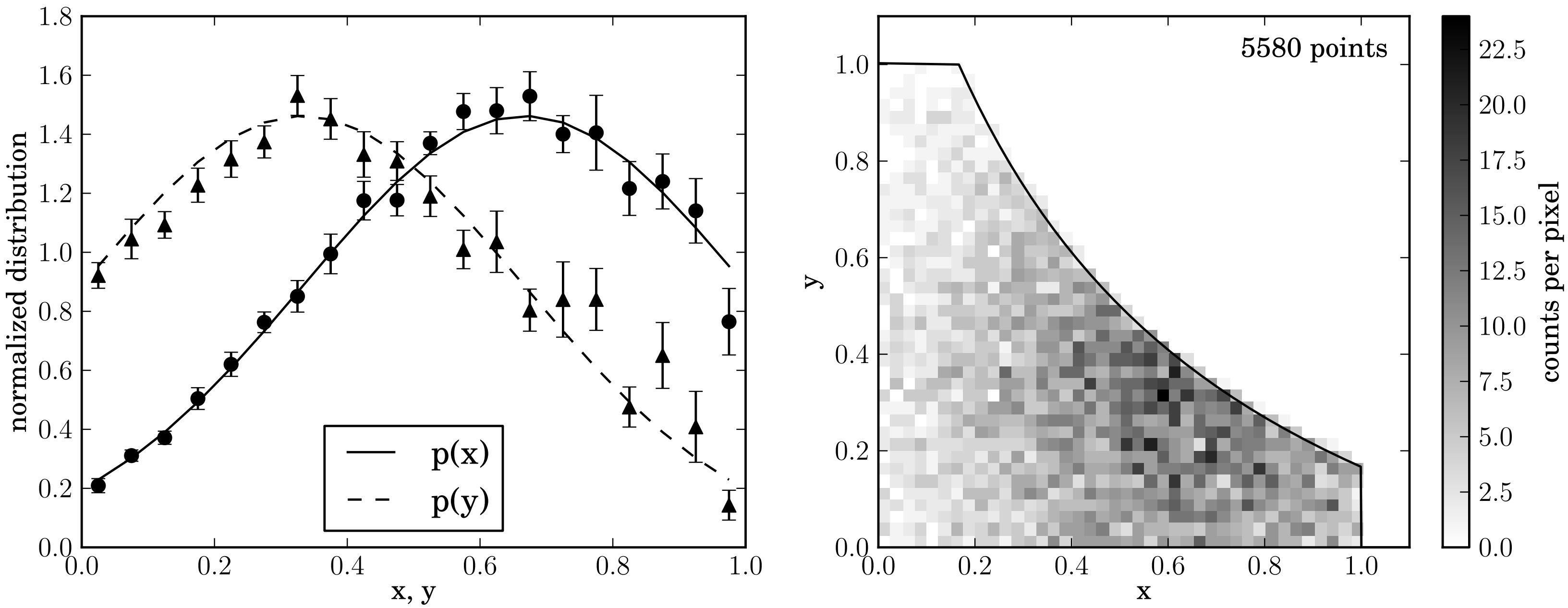
- Essentially, the C^- method assumes a piece-wise constant model for $\Phi(x)$ and $\Sigma(y)$ between data points (equivalently, differential distributions are modeled as Dirac's δ functions at the position of each data point).

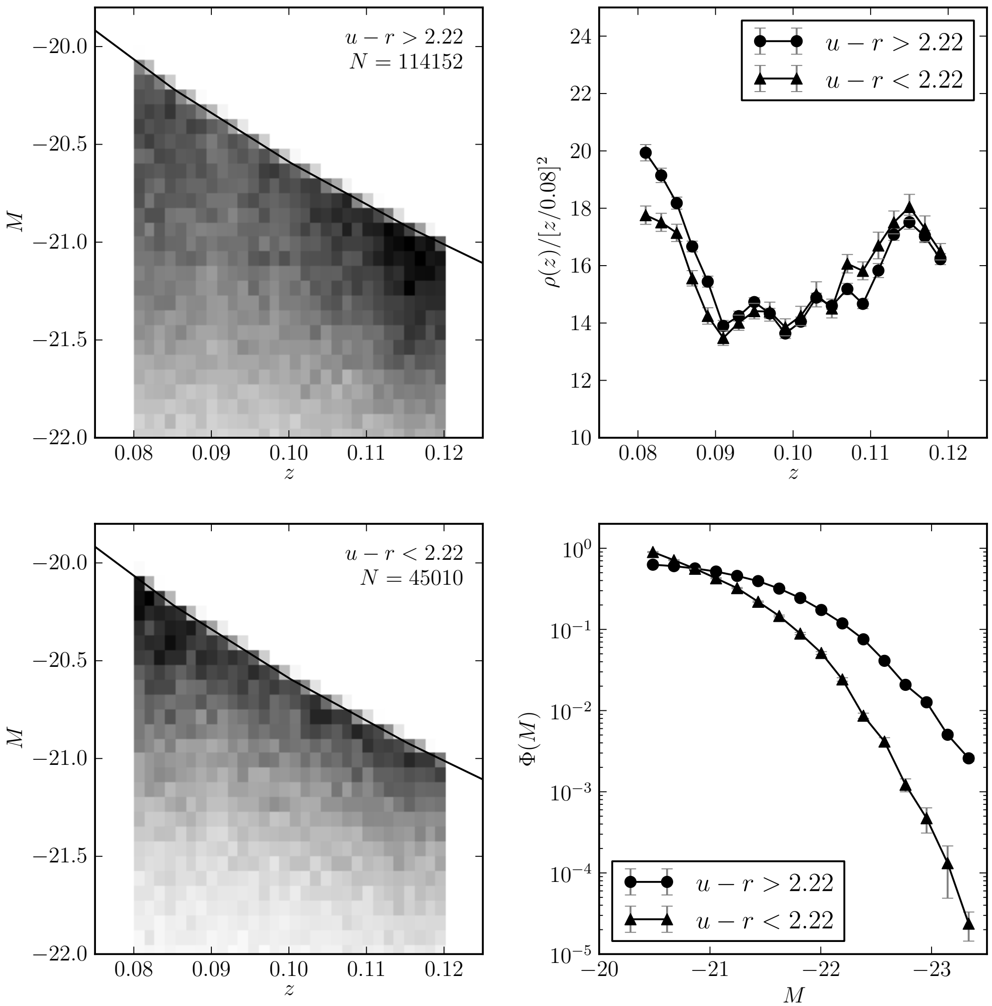
- As shown by Petrosian (1992), $\Phi(x)$ and $\Sigma(y)$ represent an optimal data summary.

- The differential distributions $\Psi(x)$ and $\rho(y)$ can be obtained by differentiating cumulative distributions in the relevant axis; an approximate normalization can be obtained by requiring that the total predicted number of objects is equal to their observed number.

- **astroML Book Figure 4.9:** The right panel shows a realization of truncated separable two-dimensional Gaussian distribution (with the truncation given by the solid line). The lines in the left panel show the true one-dimensional distributions of x and y, and the points are computed from the truncated data set using the C^- method (with error bars from 20 bootstrap resamples).

`http://astroml.github.com/book_figures/chapter4/fig_lyndenbell_toy.html`





- **astroML Book Figure 4.10:** The luminosity function for two $u-r$ color-selected subsamples of SDSS galaxies from the spectroscopic sample, with redshift in the range $0.08 < z < 0.12$ and flux limited to $r < 17.7$.
- The left panels show the distribution of sources as a function of redshift and absolute magnitude. The distribution $p(z, M) = \rho(z)\Phi(m)$ is obtained using Lynden-Bell's method, with errors determined by 20 bootstrap resamples, and shown in the right panels.

`http://astroml.github.com/book_figures/chapter4/fig_lyndenbell_gals.html`

Test of L-z Correlation. II

- In reality, the selection function is typically complex: $s(L, z|SED, ...)$ (no sharp faint limit!)

- First define a generalized comparable set (Fan et al. 2001; AJ 121, 54) $J_i = \{j : L_j > L_i\}$; this is a luminosity limited data subset for object i

- Then generalize N_i to

$$T_i = \sum_{j=1}^{N_i} \frac{s(L_i, z_j|SED_j)}{s(L_j, z_j|SED_j)}, \tag{9}$$

and redefine the rank accordingly

$$R_i = \sum_{j=1}^{N_i} \frac{s(L_i, z_j|SED_j)}{s(L_j, z_j|SED_j)}, \tag{10}$$

for $z_j < z_i$. It follows that $E(R_i) = T_i/2$ and $V(R_i) = T_i^2/12$.

LF normalization

- The C^- method does not know (or need) details about our sample; in particular, it cannot give us the overall LF normalization!

- We will use HW#2 problem to discuss normalization in more detail; we can talk about three levels of normalization in this case:

  1. The sample normalization: if we didn't have the selection effects, how many objects would our sample contain?

  2. Normalization to the full sky: we need to know the sky coverage for our sample (and have arguments why we can extrapolate to the whole sky).

  3. Extrapolation from the volume probed by the sample to some other position; here, we want to know LF at $Z = 0$.

LF normalization: the sample normalization

If we didn't have the selection effects, how many objects would our sample contain?

- To recap, the cumulative luminosity (absolute magnitude) function is $\Phi_c(M_j)$ and the cumulative distance distribution is $n_c(D_j)$ where $j = 1...N$.

- Both $\Phi_c(M_j)$ and $n_c(D_j)$ are direct outputs from the C$^-$ method; let us renormalize them as $\Phi_c(M_N) = 1$ and $n_c(D_N) = 1$, where it is assumed that M_j and D_j arrays are sorted so that M_N and D_N are their maxima (btw, C$^-$ would return $\Phi_c(M_N) = N$ and $n_c(D_N) = N$).

- The number of points, n, brighter than some arbitrary M^* and closer than D^* is then

$$n(M < M^* \text{ and } D < D^*) = C\,\Phi_c(M^*)\,n_c(D^*). \qquad (11)$$

where we (still) don't know C (n.b. C is dimensionless).

LF normalization: the sample normalization

- Now, if make sure that M^* and D^* are within our selection volume (the implied apparent mag must be above our cutoff) and thus unaffected by selection effects, then we get C from

$$N^o(M < M^o \text{ and } D < D^o) = C\,\Phi_c(M^o)\,n_c(D^o), \qquad (12)$$

which is almost the same expression as on the previous page, except that here $n(M < M^* \text{ and } D < D^*)$ is replaced by $N^o(M < M^o \text{ and } D < D^o)$: the actual number of objects in our sample that satisfy this condition.

- This is not mathematically optimal solution for C because N^o is a random variable, but with modern large samples this is nit-picking; the optimal procedure would integrate over the full sample, but nevertheless would still need to adopt an interpolation procedure for $\Phi_c(M)$ and $n_c(D)$...

- Given the real sample size, N, that is affected by selection effects, the "corrected" sample size is C!

LF normalization: the sample normalization

- The number of points per unit two-dimensional area, $dA = dM\,dD$, is then

$$\frac{d^2N}{dMdD} = C \left(\frac{d\Phi_c(M)}{dM} \right) \left(\frac{dn_c(D)}{dD} \right), \tag{13}$$

  where we now know C and can easily take (numerical) derivatives $d\Phi_c(M)/dM$ and $dn_c(D)/dD$ (where $\Phi_c(M)$ and $n_c(D)$ came from C^- and are normalized to 1).

- The quantities in parenthesis are differential distribution functions.

- When normalizing to the full sky (step #2), we need to know the fraction of sky, f_{sky}, covered by our sample; if justified, we need to multiply C by $1/f_{sky}$: $C_{sky} = C/f_{sky}$.

LF normalization: extrapolation

How do we go from $d^2N/(dMdD)$ to volume density?

$$\Phi(M,D) \equiv \frac{d^2N}{dMdV} = \frac{d^2N}{dMdD}\frac{dD}{dV}, \tag{14}$$

where $dV = 4\pi D^2 dD$. We have two cases of interest:

- **Case 1:** We seek the volume density vs. D, $\rho(D)$, and we don't care about M distribution:

$$\rho(D) = \int \Phi(M,D)dM = \frac{C_{sky}}{4\pi D^2}\left(\frac{dn_c(D)}{dD}\right), \tag{15}$$

  where we used the fact that $\int_{-\infty}^{\infty}\left(\frac{d\Phi_c(M)}{dM}\right)dM = \Phi_c(M_N) = 1$.

- Unit for $\rho(D)$ is the number of objects per (distance unit)^3 (remember that $n_c(D)$ was dimensionless and normalized to unity at $D = D_N$; the unit comes from taking derivative with respect to D, $dn_c(D)/dD$).

LF normalization: extrapolation

- Given $\rho(D)$, we can fit some function to it and **extrapolate** to get $\rho(D = D_0)$ (and thus the ratio $\rho(D)/\rho(D = D_0)$ for any D_0, including $\rho(D_0)/\rho(D_N)$).

- **Case 2:** We want to know the M distribution at some $D = D_0$, call it $\psi(M|D = D_0)$ (e.g. $D_0 = 0$ corresponding to solar neighborhood, as discussed in this HW). First, at $D = D_N$ (recall $n_c(D_N) = 1$)

$$\psi(M|D = D_N) = \int \Phi(M, D)dD = C_{sky}\left(\frac{d\Phi_c(M)}{dM}\right). \qquad (16)$$

- Then, extrapolating to D_0 (unit for ψ is the number of objects per mag; this is what we compare to the "true" LF)

$$\psi(M|D = D_0) = \psi(M|D = D_N)\,\frac{\rho(D_0)}{\rho(D_N)}. \qquad (17)$$

Evolving Luminosity Function

- The C^- method is simple and optimal, but it is valid only for uncorrelated variables (separable luminosity function). What do we do when the τ test suggests correlated variables?

- Recent work by Kelly, Fan and Vestergaard (2008, ApJ 682, 874) describes a powerful and completely general Bayesian approach (see their Appendix A for a nice introduction to Bayesian methodology). While too complex for homework, this is a fantastic method – if you ever come again across the problem of estimating a general multi-dimensional distribution that is sampled with non-negligible and possibly complex selection function, remember it!

Stellar Mass Function

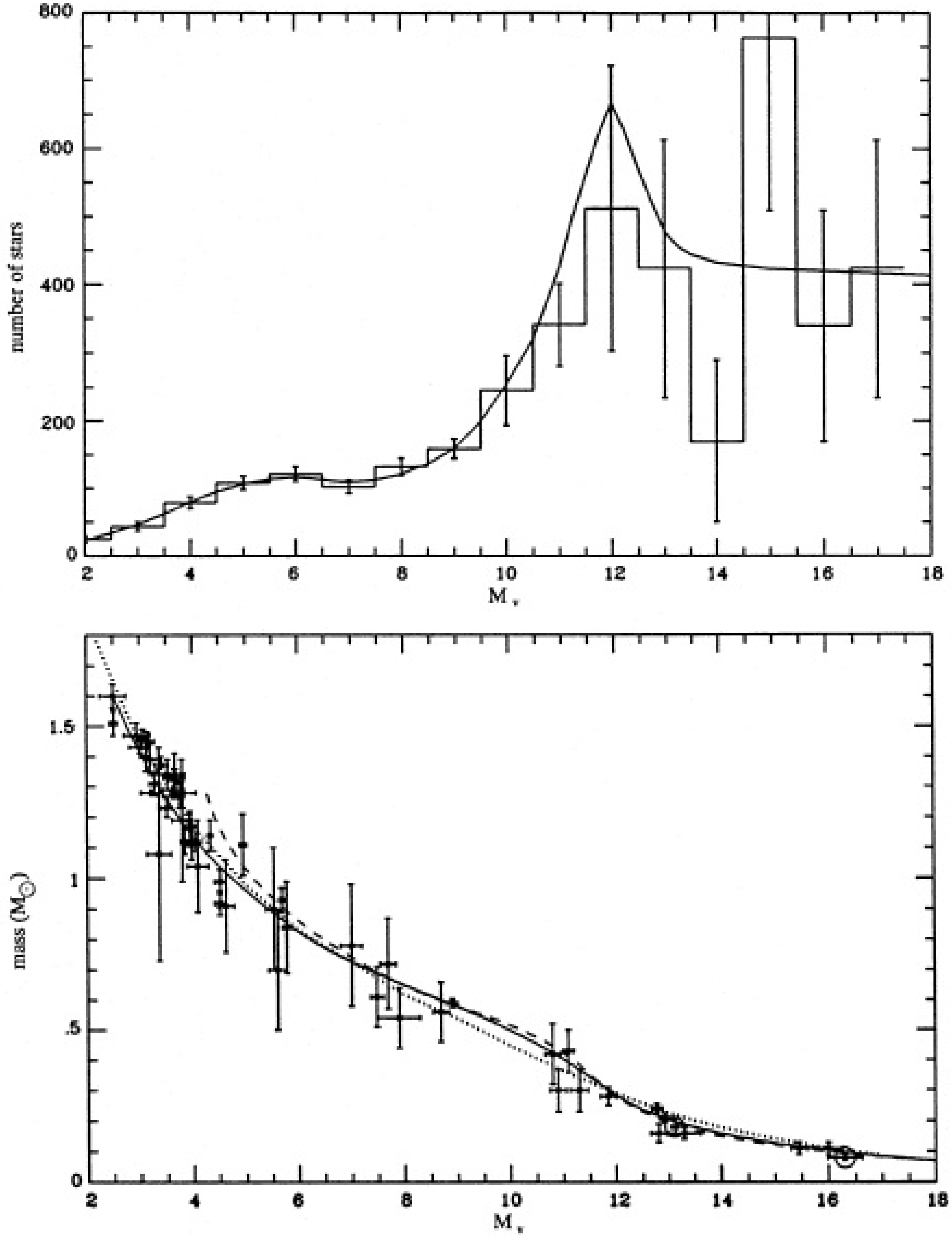
- Analogously to luminosity function, the mass function is the distribution of mass of stars, galaxies, etc.

- The term *Stellar mass function* can refer to the distribution of galaxies with respect to their stellar mass (mass of all their stars), or to the distribution of mass of stars in the Milky Way!

- The distribution of mass of stars in the Milky Way is often parametrized by a power law, $dN/dM \propto M^{-\alpha}$, with $\alpha = 2.35$ (called Salpeter function in this context; FYI: power law is called the Pareto distribution in statistics...)

- Kroupa, Tout & Gilmore (1993, MNRAS 262, 545) proposed a three-part power law

Mass Function of Disk stars



- Determination of a three-part power law mass function by Kroupa, Tout & Gilmore (1993)
- Top: the measured number of stars per M_V bin
- Bottom: the mass-luminosity relation adopted in deriving the mass function

Mass Function of Disk stars

- Determination of a three-part power law mass function by Kroupa, Tout & Gilmore (1993)
- Present-day mass function (PDMF): dot-dashed line
- Initial mass function (IMF): solid line
- Note that the PDMF and IMF are equal below about 1 solar mass.

Low-mass stars in the Galactic disc 575

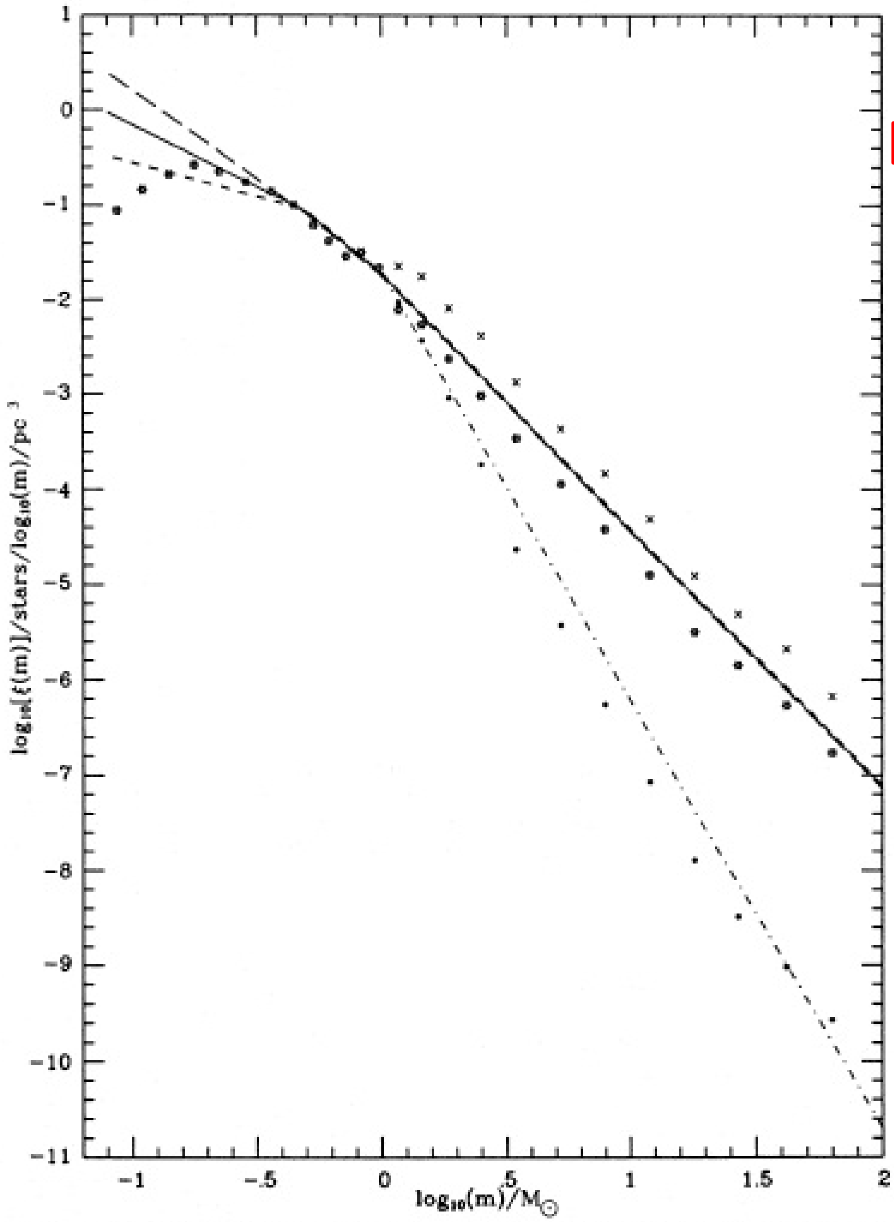


Figure 22. The stellar initial mass function (IMF) and present-day mass function (PDMF). The solid line represents the IMF given by equation (13), and the long- and short-dashed lines are for the cases $\alpha_1 = 1.85$ and 0.70, respectively. The PDMF ($\alpha_3 = 4.5$, Section 2) is indicated by the dot–dashed line. At masses below about 1 $M_\odot$ the PDMF equals the IMF. As a comparison, we show the PDMF derived by Scalo (1986) by solid dots. He corrects for stellar evolution; for a Galactic disc age of $T_0 = 9$ Gyr the IMF is indicated by stars, and for $T_0 = 12$ Gyr by crosses.

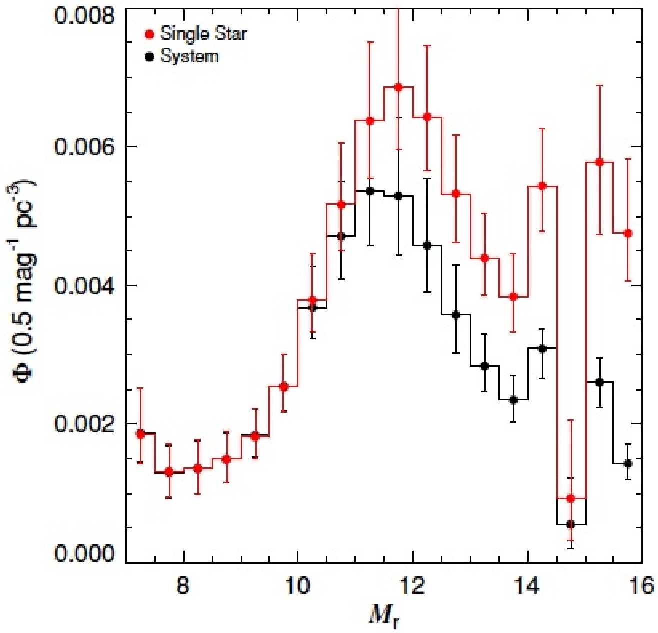


Figure 21. Single-star (red filled circles) and system (black filled circles) LFs. Note that the major differences between our system and single-star LFs occur at low luminosities, since low-mass stars can be companions to stars of any higher mass, including masses above those sampled here.

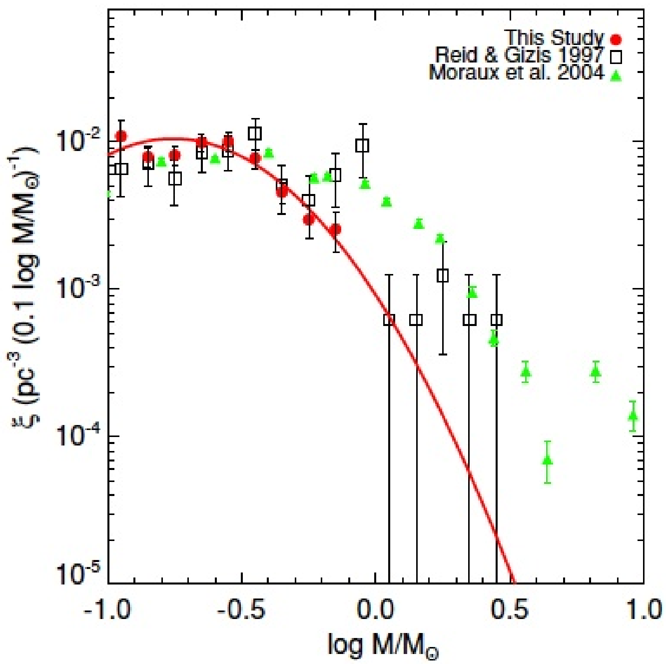
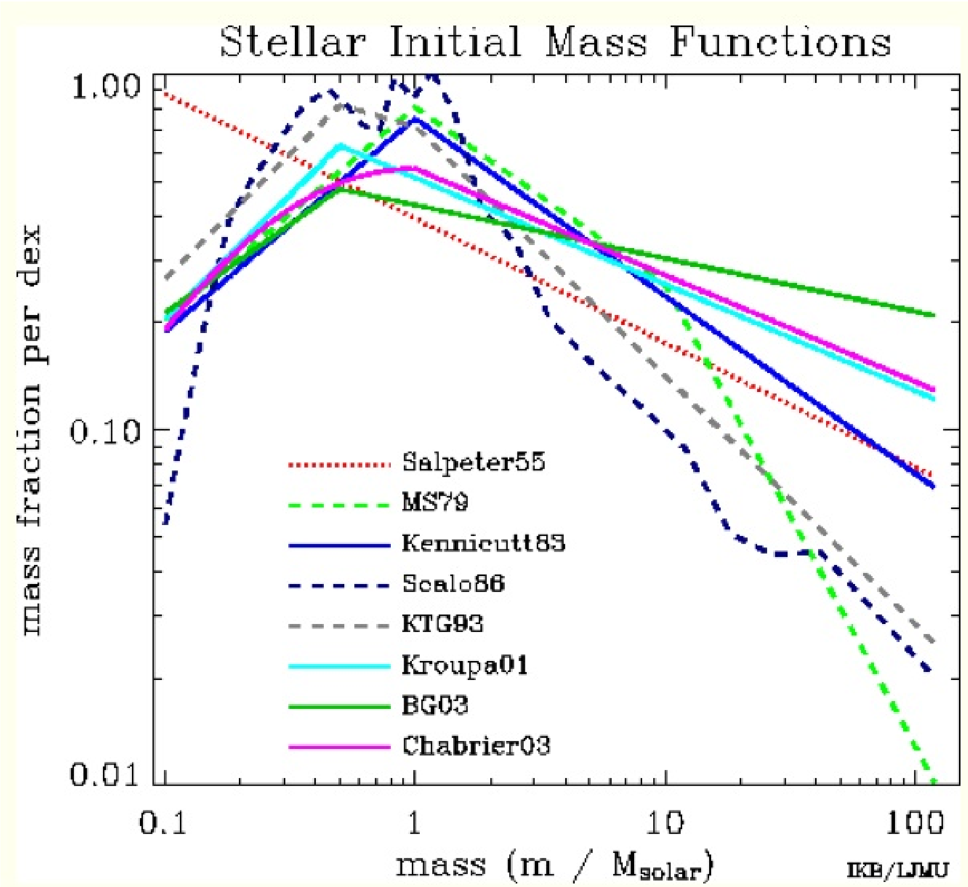


Figure 27. Shown are the single-star MF and best lognormal fit from this study (red filled circles and solid line), the Reid & Gizis (1997, open squares), MF (open squares), and the Pleiades MF Moraux et al. (2004, green triangles). The best fit extrapolated from our study systematically under-predicts the density at masses outside the bounds of our data.

Luminosity and Mass Function of Low-mass stars

- Bochanski et al. (2010, AJ 139, 2679)
- Based on SDSS data for 15 million low-mass stars!
- The mass range: 0.1–0.8 solar masses (corresponding to $7 < M_r < 16$)
- The turn-over and a local maximum well detected!
- Data well described by a log-normal distribution (over the probed mass range).



Initial Mass Function

- The stellar initial mass function (IMF) is used for computing stellar masses and colors of galaxies in cosmology.
- There is substantial variation between different estimates (left).
- Kroupa (2001) claimed a variable IMF (MNRAS 322, 231).

From Ivan Baldry:

`http://www.astro.ljmu.ac.uk/~ikb/research/imf-use-in-cosmology.html`

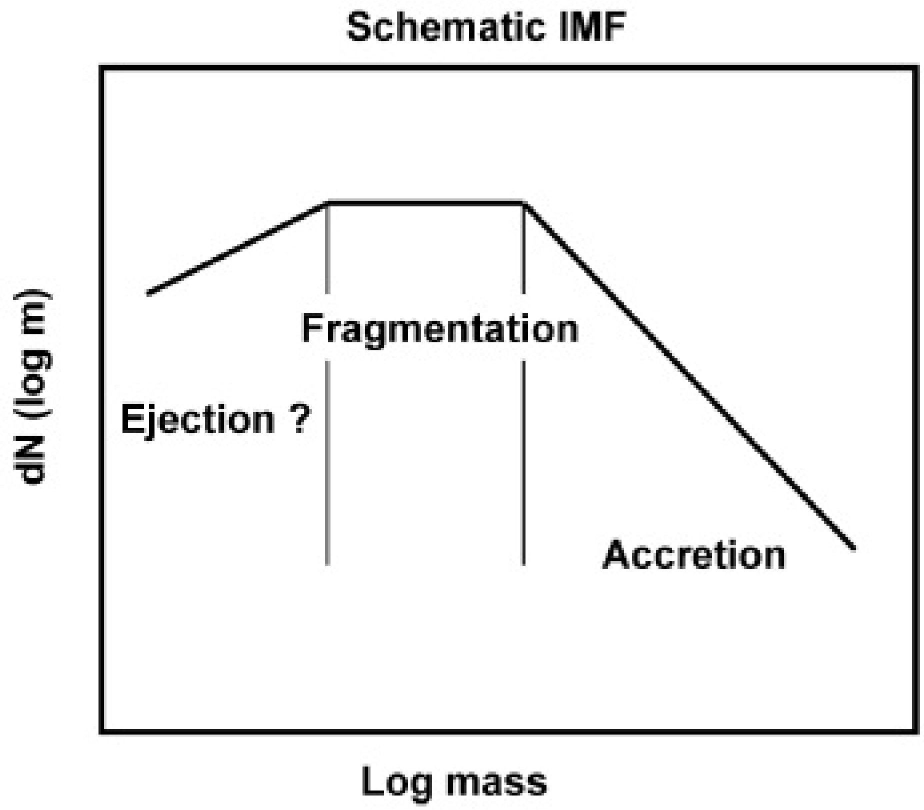


Fig. 11.— A schematic IMF showing the regions that are expected to be due to the individual processes. The peak of the IMF and the characteristic stellar mass are believed to be due to gravitational fragmentation, while lower mass stars are best understood as being due to fragmentation plus ejection or truncated accretion while higher-mass stars are understood as being due to accretion.

From W. Chen

Initial Mass Function

- An approximate understanding of the origin of different slopes.
- A hard problem to solve! (e.g. turbulence, magnetic fields...)