Astr 509: Astrophysics III: Stellar Dynamics

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Lecture 14: Kinetic Theory

The Gravothermal Catastrophy,

the Fokker-Planck Approximation

The Gravothermal Catastrophy

Positive vs. negative feedback

In a self-gravitating system, the emission of energy means increase of temperature (and decrease of size), which results in more intensive emission of energy, which in turn further increases the temperature (and decreases size), and so on, until the system collapses!

In the context of star formation, the fusion will eventually start at some temperature and its energy input will halt the contraction. Only a temporary delay!

The collisonless Boltzmann Equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left[v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right] = 0 \tag{1}$$

or

$$\frac{\partial f}{\partial t} + \mathbf{v} \nabla f = \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} \tag{2}$$

or

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0 \tag{3}$$

All these forms say the same thing:

$$\frac{Df}{Dt} = 0 \tag{4}$$

If collisions are taken into account, then

$$\frac{Df}{Dt} = \Gamma(f) \tag{5}$$

where Γ is the **collison term**. This form is called the **master equation**. How do we get Γ?

The Collison Term Γ

Define *the probability density that a star with the 6D coordinate* **w** *is scattered to the position* **w** + Δ**w***:* Ψ*(***w***,* Δ**w***)*.

The change of the distribution function for an infinitesimal volume with the 6D coordinate **w** is the number of stars scattered **into** this volume **minus** the number of stars scattered **out** of this volume:

$$\Gamma(f) = \int [\Psi(\mathbf{w} - \Delta\mathbf{w}), \Delta\mathbf{w}) f(\mathbf{w} - \Delta\mathbf{w}) - \Psi(\mathbf{w}, \Delta\mathbf{w}) f(\mathbf{w})] \, d^3\Delta\mathbf{w} \tag{6}$$

This is all very pretty, but where do we get Ψ(**w**, Δ**w**)?

The Fokker-Planck Approximation

Most of the gravitational scattering is due to weak counters with $|\Delta\mathbf{w}| << \mathbf{w}$. Therefore, can expand Ψf in a Taylor series, and truncate after the second term:

$$\Gamma(f) = \Psi(\mathbf{w} - \Delta\mathbf{w}), \Delta\mathbf{w})f(\mathbf{w} - \Delta\mathbf{w}) - \Psi(\mathbf{w}, \Delta\mathbf{w})f(\mathbf{w}) = \quad (7)$$

$$-\sum_{i=1}^{6} \frac{\partial}{\partial w_i}[f(\mathbf{w})D(\Delta w_i)] + \frac{1}{2}\sum_{i,j=1}^{6} \frac{\partial^2}{\partial w_i \partial w_j}[f(\mathbf{w})D(\Delta w_i \Delta w_j))] \quad (8)$$

where

$$D(\Delta w_i) \equiv \int \Delta w_i \Psi(\mathbf{w}, \Delta\mathbf{w}) d^3\Delta\mathbf{w} \quad (9)$$

are diffusion coefficients, and represent the expectation values for the change of w_i.

The Fokker-Planck Equation

When (8), (6) and (5) are combined, we get the Fokker-Planck equation:

$$\frac{Df}{Dt} = -\sum_{i=1}^{6} \frac{\partial}{\partial w_i}[f(\mathbf{w})D(\Delta w_i)] + \frac{1}{2}\sum_{i,j=1}^{6} \frac{\partial^2}{\partial w_i \partial w_j}[f(\mathbf{w})D(\Delta w_i \Delta w_j))] \tag{10}$$

But, where do we get D?

In most applications D are taken to be constant, and either computed using dynamical considerations, or simply postulated.

NB A similar approximation is used in radiation transfer analysis, including areas other than astronomy; e.g. models of cancer cell irradiation