Astr 509: Astrophysics III: Stellar Dynamics

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Lecture 14: Collisions I

Tidal Tails, Dynamical Friction

Collisions, Encounters, Tidal tails, etc.

- Andromeda is coming overhere at 100 km/s – expect fireworks 10^{10} years from now!

- **Two regimes for galaxy encounters:** fast, $v_\infty > v_f$ (elastic behavior, galaxies affect each other but do not merge, e.g. tidal tails) and slow $v_\infty < v_f$ (inelastic behavior – galaxies merge), where v_∞ is the relative velocity, and v_f is some critical velocity that depends on detailed structure of interacting galaxies.

- In the fastest encounters ($v_\infty >> v_f$), stars do not significantly change their positions – **impulse approximation**

- During the not-so-fast encounters, the orbital (kinetic) energy can be transferred to the internal energy (galaxies are not point masses – better described as viscous fluid that absorbs energy when deformed)

Fast Galaxy Encounters

- Impulse approximation: the potential energy doesn't change during the encounter, but the internal kinetic energy changes by, say, ΔK. This change of the kinetic (and total) energy takes the system out of virial equilibrium! What is the final equilibrium state? (before the encounter: $E = E_o$ and $K = K_o$, with $E_o = -K_o$)

- After the encounter, and before returning to the equilibrium: $K_1 = K_o + \Delta K$ $(= -E_o + \Delta K)$ and $E_1 = E_o + \Delta K$ (note that it is NOT true that $E_1 = -K_1$).

- After returning to the equilibrium: $E_2 = E_1$, and it must be true that $K_2 = -E_2$ because of virial theorem. Hence, $K_2 = -E_1 = -E_o - \Delta K = K_1 - 2\Delta K$! During the return to virial equilibrium, the system loses $2\Delta K$ of kinetic energy (which becomes potential energy because energy is conserved). Therefore, the (self-gravitating) system **expands**!

Slow Galaxy Encounters

- Need N-body numerical simulations for the full treatment

- In a special case when galaxies are very different in size, analytic treatment is possible to some extent

- Dynamical friction: a compact body of mass M (small galaxy) passes through a population of stars with mass m (large galaxy). The net effect is a steady deceleration parallel to the velocity vector (just like ordinary friction).

NGC 4676: When Mice Collide

Model from Toomre, A. & Toomre, J. 1972, Galactic Bridges and Tails, ApJ, 178, 623

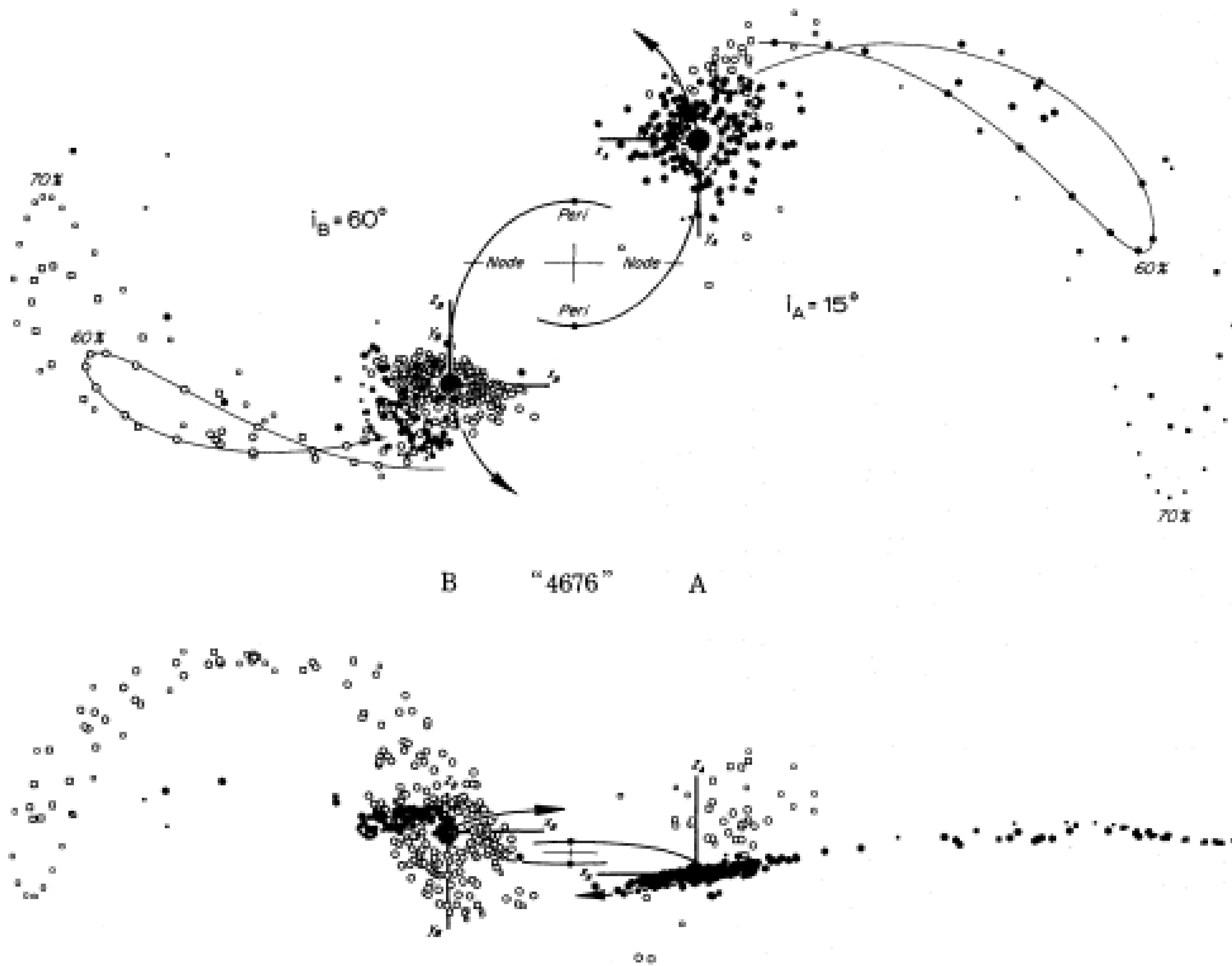
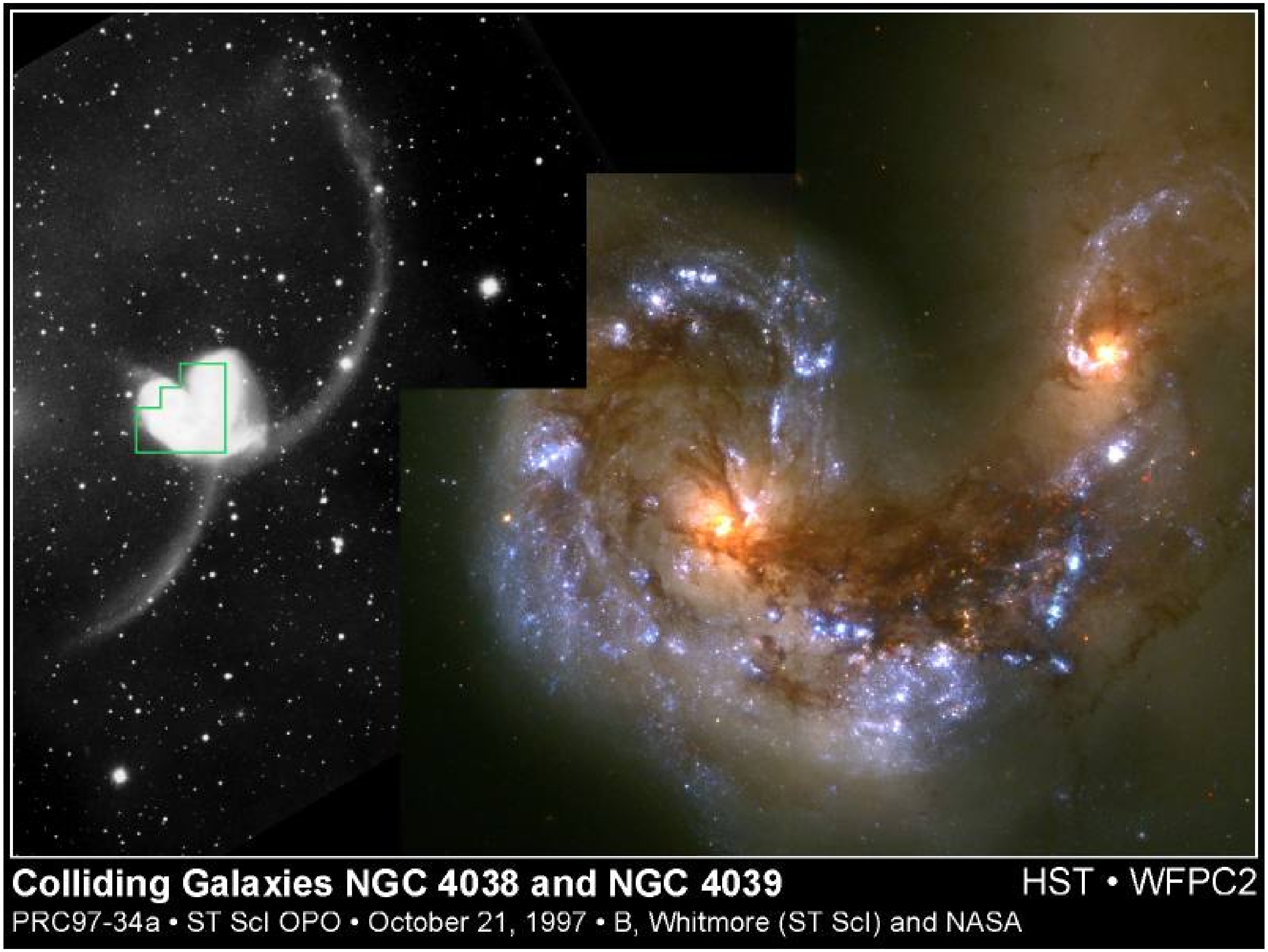
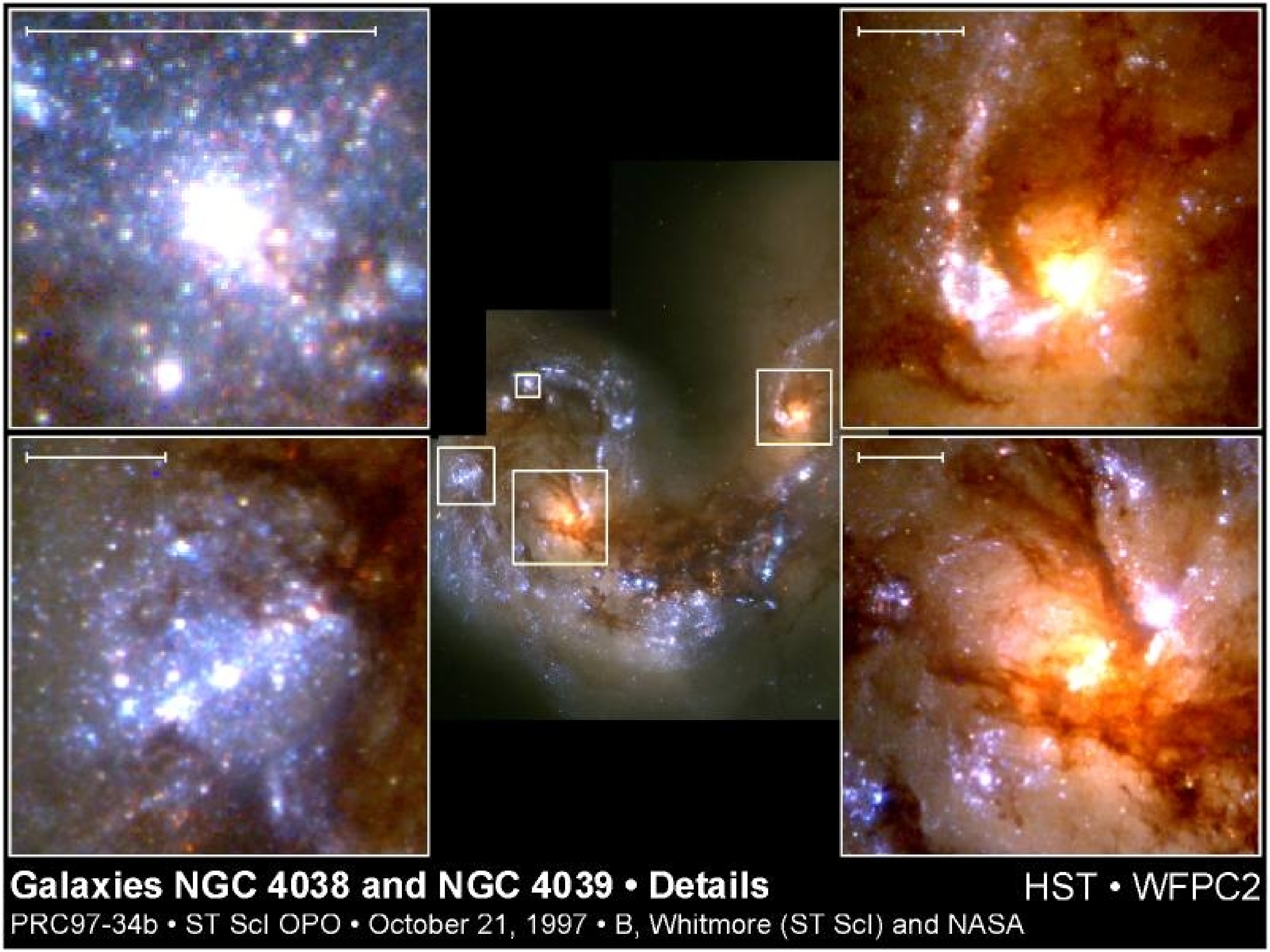


FIG. 22.—Model of NGC 4676. In this reconstruction, two equal disks of radius $0.7R_{\min}$ experienced an $e = 0.6$ elliptic encounter, having begun flat and circular at the time $t = -16.4$ of the last apocenter. As viewed from either disk, the adopted node-to-peri angles $\omega_A = \omega_B = -90°$ were identical, but the inclinations differed considerably: $i_A = 15°$, $i_B = 60°$. The resulting composite object at $t = 6.086$ (cf. fig. 18) is shown projected onto the orbit plane in the upper diagram. It is viewed nearly edge-on to the same—from $\lambda_A = 180°$, $\beta_A = 85°$ or $\lambda_B = 0°$, $\beta_B = 160°$—in the lower diagram meant to simulate our actual view of that pair of galaxies. The filled and open symbols distinguish particles originally from disks A and B, respectively.



Another example: Antennae Galaxy


Galaxies NGC 4038 and NGC 4039 • Details
HST • WFPC2
PRC97-34b • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

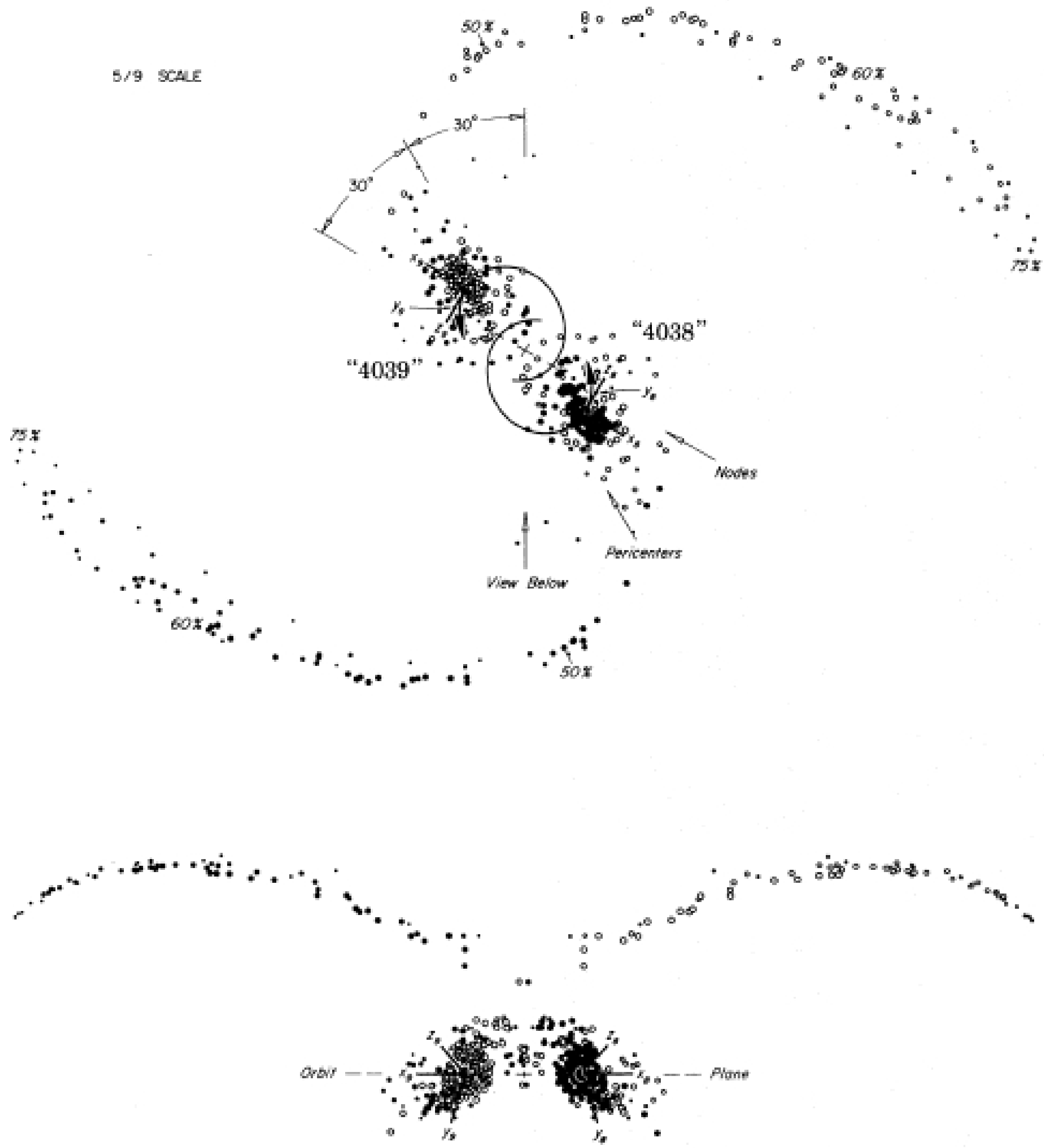


FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{\min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

Dynamical Friction

A compact body of mass M passes through a see of objects with mass m.

First solve for the effect of one body, and then add the effects of successive encounters.

If the radius vector between the two bodies is $\mathbf{r} = \mathbf{x}_m - \mathbf{x}_M$, and $\mathbf{V} = \dot{\mathbf{r}}$, then

$$\left[\frac{mM}{m+M}\right]\ddot{\mathbf{r}} = -\frac{GMm}{r^2}\mathbf{e}_r \tag{1}$$

Looks like a potential of a body with mass $(m+M)$ – *reduced mass*

Since

$$\Delta\mathbf{v}_m - \Delta\mathbf{v}_M = \Delta\mathbf{V} \tag{2}$$

and (the center of mass is unaffected by the encounter)

$$m\Delta\mathbf{v}_m + M\Delta\mathbf{v}_M = 0 \tag{3}$$

we get

$$\Delta\mathbf{v}_M = -\left[\frac{m}{m+M}\right]\Delta\mathbf{V}. \tag{4}$$

Once we find $\Delta\mathbf{V}$, we can compute $\Delta\mathbf{v}_M$.

How do we get $\Delta\mathbf{V}$? We need to solve the equations of motions, but we already know the solution (this is a two-body problem, BT eq. 3-21)

$$u(\Psi) = C\cos(\Psi - \Psi_o) + \frac{GM}{L^2} \tag{5}$$

where $u = 1/r$ and L is the angular momentum.

Here, $L = b\mathbf{V_o}$, and C and Ψ_o are determined by the initial conditions.

We get

$$\tan(\Psi_o) = -\frac{bV_o^2}{G(M+m)}, \tag{6}$$

which also determines the deflection angle

$$\theta_d = 2\Psi_o - \pi. \tag{7}$$

The change of each velocity component are

$$|\Delta V_{perp}| = V_o \sin(theta_d) = \frac{2bV_o^3}{G(M+m)}\left[1 + \frac{b^2V_o^4}{G^2(M+m)^2}\right]^{-1} \tag{8}$$

$$|\Delta V_{para}| = V_o(1 - \cos(theta_d)) = 2V_o\left[1 + \frac{b^2V_o^4}{G^2(M+m)^2}\right]^{-1} \tag{9}$$

and, finally, for the **parallel** component of $\mathbf{v}_M$

$$\Delta\mathbf{v}_M = \left[\frac{2mV_o}{m+M}\right]\left[1 + \frac{b^2V_o^4}{G^2(M+m)^2}\right]^{-1} \tag{10}$$

Dynamical Friction

When M moves through **many** m, the perpendicular component of $\Delta \mathbf{v}_M$ will sum to zero. How do we sum the parallel components?

The overall change of $\Delta \mathbf{v}_M$ per unit time is equal to the change due to one m star times the number of encounters per unit time, dN/dt.

$$dN/dt = f(v_m) dV/dt, \text{ where } dV = 2\pi b \, db \, V_o \, dt \tag{11}$$

Hence,

$$\frac{d\mathbf{v}_M}{dt} = f(\mathbf{v}_m) \mathbf{V_o} \, d^3\mathbf{v}_m \int_0^{b_{max}} \Delta \mathbf{v}_M(b) 2\pi b \, db \tag{12}$$

Here b_{max} is "the largest relevant" impact parameter – in practice it is determined by the behavior of $f(\mathbf{v}_m)$.

The intergral over b can be (easily!) performed to get

$$\frac{d\mathbf{v}_M}{dt} = 2\pi \ln(1+\Lambda^2) G^2 m(M+m) f(\mathbf{v}_m) d^3\mathbf{v}_m \frac{(\mathbf{v}_m - \mathbf{v}_M)}{|\mathbf{v}_m - \mathbf{v}_M|^3} \quad (13)$$

Here

$$\ln(\Lambda) = \frac{b_{max} V_0^2}{G(M+m)} \quad (14)$$

is the so-called Coulomb logarithm (n.b. typically $\Lambda >> 1$).

For an isotropic distribution of stellar velocities (note the integration limit!)

$$\frac{d\mathbf{v}_M}{dt} = -16\pi^2 \ln(\Lambda) G^2 m(M+m) \frac{\int_0^{v_M} f(v_m) v_m^2 dv_m}{v_M^3} \mathbf{v}_M \quad (15)$$

which is the famous **Chandrasekhar dynamical friction formula**.

Dynamical Friction

For small v_M (compared to typical velocities of m particles, i.e. their velocity dispersion),

$$\frac{d\mathbf{v}_M}{dt} \propto -\mathbf{v}_M \tag{16}$$

For large v_M,

$$\frac{d\mathbf{v}_M}{dt} \propto -\frac{\mathbf{v}_M}{v_M^3} \tag{17}$$

Applications: e.g. decay of globular cluster orbits – what a beautiful problem for the final exam!