Lecture 14: Collisions I

Tidal Tails, Dynamical Friction
Collisions, Encounters, Tidal tails, etc.

- Andromeda is coming over here at 100 km/s – expect fireworks $10^{10}$ years from now!

- **Two regimes for galaxy encounters:** fast, $v_\infty > v_f$ (elastic behavior, galaxies affect each other but do not merge, e.g. tidal tails) and slow $v_\infty < v_f$ (inelastic behavior – galaxies merge), where $v_\infty$ is the relative velocity, and $v_f$ is some critical velocity that depends on detailed structure of interacting galaxies.

- In the fastest encounters ($v_\infty \gg v_f$), stars do not significantly change their positions – **impulse approximation**

- During the not-so-fast encounters, the orbital (kinetic) energy can be transferred to the internal energy (galaxies are not point masses – better described as viscous fluid that absorbs energy when deformed)
Fast Galaxy Encounters

- Impulse approximation: the potential energy doesn’t change during the encounter, but the internal kinetic energy changes by, say, $\Delta K$. This change of the kinetic (and total) energy takes the system out of virial equilibrium! What is the final equilibrium state? (before the encounter: $E = E_o$ and $K = K_o$, with $E_o = -K_o$)

- After the encounter, and before returning to the equilibrium: $K_1 = K_o + \Delta K \ ( = -E_o + \Delta K)$ and $E_1 = E_o + \Delta K$ (note that it is NOT true that $E_1 = -K_1$).

- After returning to the equilibrium: $E_2 = E_1$, and it must be true that $K_2 = -E_2$ because of virial theorem. Hence, $K_2 = -E_1 = -E_o - \Delta K = K_1 - 2\Delta K$! During the return to virial equilibrium, the system loses $2\Delta K$ of kinetic energy (which becomes potential energy because energy is conserved). Therefore, the (self-gravitating) system expands!
Slow Galaxy Encounters

• Need N-body numerical simulations for the full treatment

• In a special case when galaxies are very different in size, analytic treatment is possible to some extent

• **Dynamical friction:** a compact body of mass $M$ (small galaxy) passes through a population of stars with mass $m$ (large galaxy). The net effect is a steady deceleration parallel to the velocity vector (just like ordinary friction).
Fig. 22.—Model of NGC 4676. In this reconstruction, two equal disks of radius 0.7\(R_{\text{min}}\) experienced an \(e = 0.6\) elliptic encounter, having begun flat and circular at the time \(t = -16.4\) of the last apocenter. As viewed from either disk, the adopted node-to-peri angles \(\omega_A = \omega_B = -90^\circ\) were identical, but the inclinations differed considerably: \(i_A = 15^\circ\), \(i_B = 60^\circ\). The resulting composite object at \(t = 6.086\) (cf. fig. 18) is shown projected onto the orbit plane in the upper diagram. It is viewed nearly edge-on to the same—from \(\lambda_A = 180^\circ\), \(\beta_A = 85^\circ\) or \(\lambda_B = 0^\circ\), \(\beta_B = 160^\circ\)—in the lower diagram meant to simulate our actual view of that pair of galaxies. The filled and open symbols distinguish particles originally from disks A and B, respectively.
Another example: Antennae Galaxy
Galaxies NGC 4038 and NGC 4039 • Details

PRC97-34b • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA
Fig. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{\text{min}}$ suffered an $e \approx 0.5$ encounter with orbit angles $i = i_0 = 60^\circ$ and $\omega = \omega_0 = -30^\circ$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself $30^\circ$ from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.
Dynamical Friction

A compact body of mass $M$ passes through a sea of objects with mass $m$.

First solve for the effect of one body, and then add the effects of successive encounters.

If the radius vector between the two bodies is $r = x_m - x_M$, and $V = \dot{r}$, then

$$\left[ \frac{mM}{m + M} \right] \ddot{r} = -\frac{GMm}{r^2} \textbf{e}_r \quad (1)$$

Looks like a potential of a body with mass $(m + M)$ – reduced mass

Since

$$\Delta v_m - \Delta v_M = \Delta V \quad (2)$$
and (the center of mass is unaffected by the encounter)

\[ m \Delta v_m + M \Delta v_M = 0 \] (3)

we get

\[ \Delta v_M = - \left[ \frac{m}{m + M} \right] \Delta V. \] (4)

Once we find \( \Delta V \), we can compute \( \Delta v_M \).

How do we get \( \Delta V \)? We need to solve the equations of motions, but we already know the solution (this is a two-body problem, BT eq. 3-21)

\[ u(\Psi) = C \cos(\Psi - \Psi_o) + \frac{GM}{L^2} \] (5)

where \( u = 1/r \) and \( L \) is the angular momentum.

Here, \( L = bV_o \), and \( C \) and \( \Psi_o \) are determined by the initial conditions.
Dynamical Friction

We get

$$\tan(\Psi_o) = -\frac{bV_o^2}{G(M + m)},$$

(6)

which also determines the deflection angle

$$\theta_d = 2\Psi_o - \pi.$$  

(7)

The change of each velocity component are

$$|\Delta V_{\text{perp}}| = V_o \sin(\text{theta}_d) = \frac{2bV_o^3}{G(M + m)} \left[ 1 + \frac{b^2V_o^4}{G^2(M + m)^2} \right]^{-1}$$

(8)

$$|\Delta V_{\text{para}}| = V_o(1 - \cos(\text{theta}_d)) = 2V_o \left[ 1 + \frac{b^2V_o^4}{G^2(M + m)^2} \right]^{-1}$$

(9)

and, finally, for the parallel component of $v_M$

$$\Delta v_M = \left[ \frac{2mV_o}{m + M} \right] \left[ 1 + \frac{b^2V_o^4}{G^2(M + m)^2} \right]^{-1}$$

(10)
Dynamical Friction

When $M$ moves through many $m$, the perpendicular component of $\Delta v_M$ will sum to zero. How do we sum the parallel components?

The overall change of $\Delta v_M$ per unit time is equal to the change due to one $m$ star times the number of encounters per unit time, $dN/dt$.

$$dN/dt = f(v_m)dV/dt, \text{ where } dV = 2\pi b \, db \, V_0 \, dt \quad (11)$$

Hence,

$$\frac{dv_M}{dt} = f(v_m) V_0 d^3v_m \int_0^{b_{max}} \Delta v_M(b) 2\pi b \, db \quad (12)$$

Here $b_{max}$ is “the largest relevant” impact parameter – in practice it is determined by the behavior of $f(v_m)$.

The integral over $b$ can be (easily!) performed to get
\[
\frac{dv_M}{dt} = 2\pi \ln(1 + \Lambda^2) G^2 m(M + m) f(v_m) d^3 v_m \frac{(v_m - v_M)}{|v_m - v_M|^3}
\]  \hspace{1cm} (13)

Here

\[
\ln(\Lambda) = \frac{b_{\text{max}} V_0^2}{G(M + m)}
\]  \hspace{1cm} (14)

is the so-called Coulomb logarithm (n.b. typically $\Lambda >> 1$).

For an isotropic distribution of stellar velocities (note the integration limit!)

\[
\frac{dv_M}{dt} = -16\pi^2 \ln(\Lambda) G^2 m(M + m) \int_0^{v_M} f(v_m) v_m^2 dv_m v_M\frac{v_m^3}{v_M^3}
\]  \hspace{1cm} (15)

which is the famous Chandrasekhar dynamical friction formula.
Dynamical Friction

For small $v_M$ (compared to typical velocities of $m$ particles, i.e. their velocity dispersion),

$$\frac{dv_M}{dt} \propto -v_M \quad (16)$$

For large $v_M$,

$$\frac{dv_M}{dt} \propto -\frac{v_M}{v_M^3} \quad (17)$$

Applications: e.g. decay of globular cluster orbits – what a beautiful problem for the final exam!