Astr 509: Astrophysics III: Stellar Dynamics

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Lecture 13: Disk Dynamics:

Spiral Arms and Bars as Instabilities

To remember:

- Spiral arms are not static structure (winding problem)
- Not all spirals are alike: more than one pattern
- Not clear if transient or quasy-steady phenomenon
- The appearance dominated by young luminous blue stars, but the overall density of *all* stars is elevated by 10-02% in spiral arms

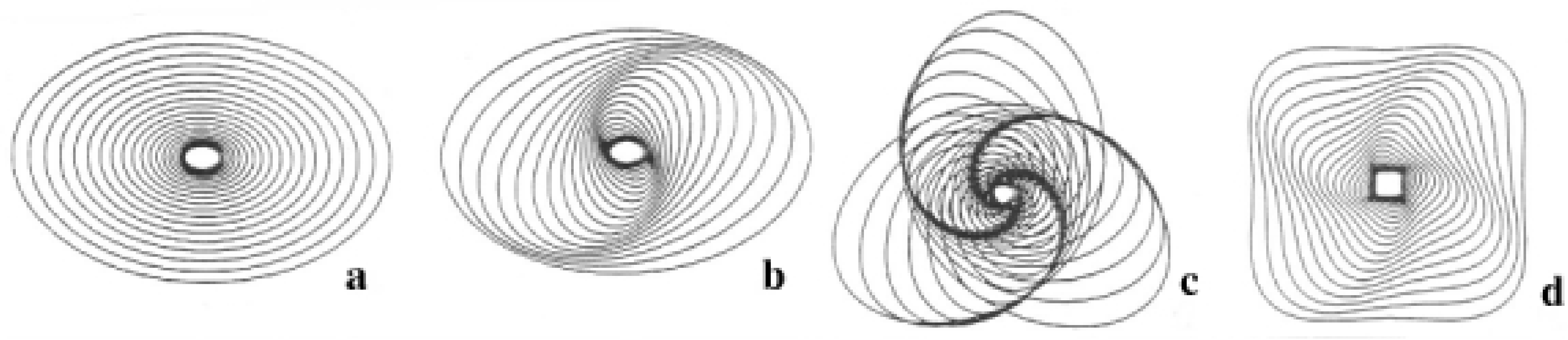
Theory of differentially rotating disks

- It is convenient to use a rotating coordinate system which revolves at some speed, say Ω_p

- For an axisymmetric disk with a flat rotation curve (a good first order approximation to the disk of a spiral galaxy), this rotation speed will match up with the rotation speed at some radius R – *corotation radius*. Particles inside this radius will appear to revolve in the direction of the frame rotation (prograde) while outside this corotation radius, they will be retrograde.

- Given Ω_p, at some radii open orbits become closed: for a star which completes two radial oscillations while performing one complete azimuthal trip in the rotating frame, we get elliptical orbits with the center at the center of the potential (so called 2/1 orbits). Similarly, a ratio of 3/2 gives a three-armed cloverleaf pattern, etc. If a number of 2/1 orbits is aligned concentrically and populated with stars, a bar is formed.

- When the majority of the stars are arranged in these spiral or bar patterns, the mass asymmetry will begin to affect the overall potential.



a) a bar can be produced by aligning a series of concentric elliptical orbits

b) if each ellipse is given an azimuthal offset proportional to $\sqrt{R}$, the effect is a two-armed spiral

c) a set of 3/2 orbits produces a three armed spiral

d) a set of 4/1 orbits produces a four armed pattern

Perturbations

If we perturb the axisymmetric potential of the rotating disk with a small non-axisymmetric component as above, it makes sense to define the rotation speed of that perturbation as the frame speed.

This potential is m-fold symmetric and may arise from a central bar pattern (m=2), an triaxial dark halo (m=2), some external perturbing agent such as a companion galaxy (m=1), or from large local mass concentrations within the disk of the galaxy (m=many).

At the corotation radius circularly orbiting particles feel a time-steady potential. In the case of an m=2 bar potential, a coresonant particle would perform small epicyclic orbits around a gyration point having constant phase and radial relationship with the bar.

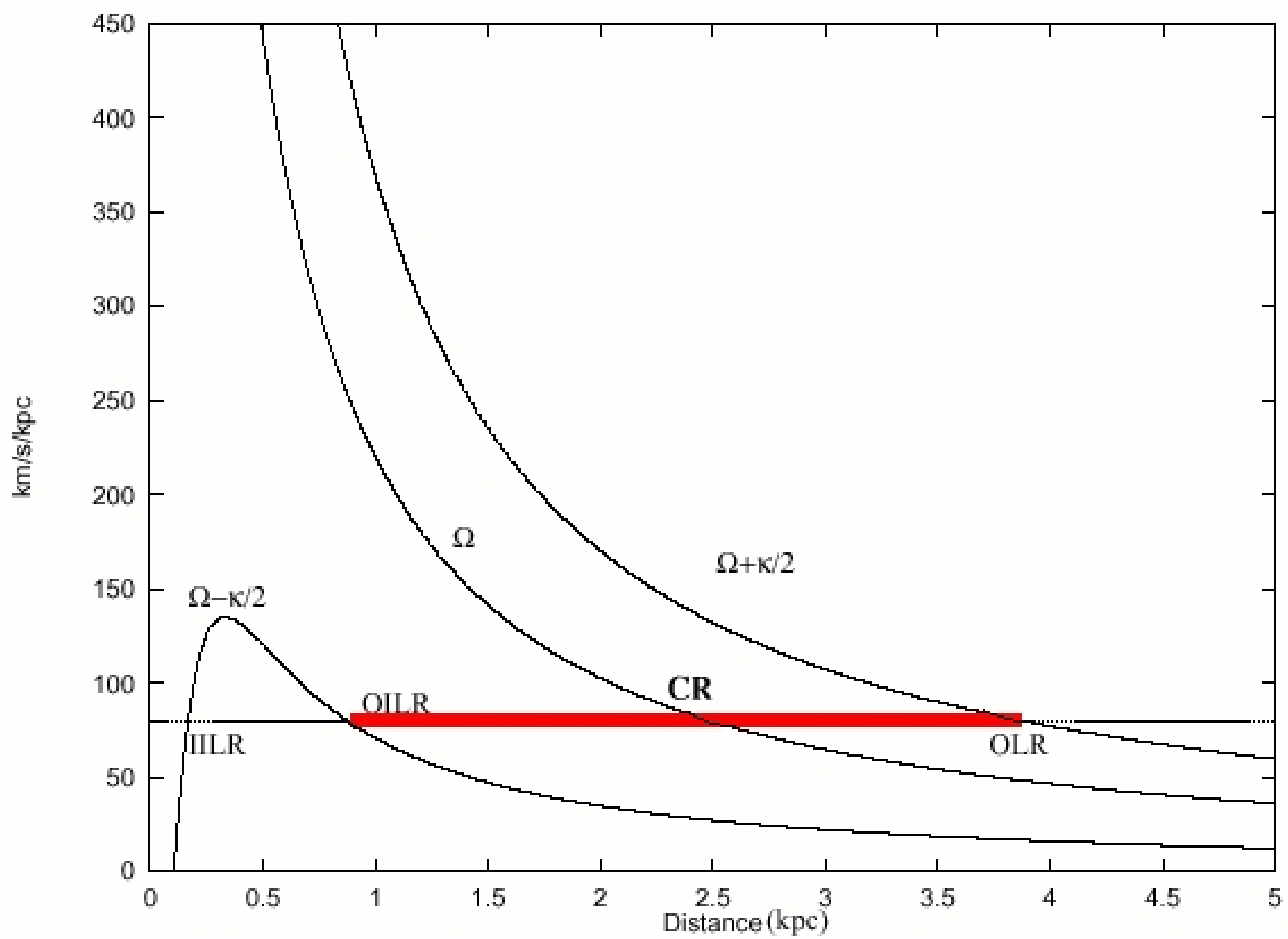
Particles at this radius feel an enhanced potential over their entire orbits. Since gravitational potentials are always attractive, this represents a barrier in the effective potential and is known as the corotation resonance (CR).

Two other resonances, discovered by and named after Lindblad, lie interior and exterior to the CR (LR are roughly equivalent to Kirkwood resonances seen in the asteroid belt due to Jupiter's perturbing force on the symmetric solar field).

The dispersion relations that follow from instability analysis relate the wavenumber k to the frequency ω ($\omega = m\Omega_p$). For CR, $\Omega = \Omega_p$, and for Lindblad resonances

$$\Omega = \Omega_p \pm \frac{\kappa}{m} \tag{1}$$

Lindblad resonances



- A a particle at the Inner Lindblad Resonance (ILR) it is at the top of its epicycle when the end of the bar swings by below it, and will be at the top of its next epicycle when the opposite end of the bar swings by. The particle is oscillating radially at an integral multiple of the driving frequency and at a constant phase, which represents a condition of forced oscillation – this is another barrier in the effective potential.

- The Outer Lindblad Resonance (OLR) is similar except that particles are moving relatively retrograde from the rotating bar. Both resonances present barriers to the radial potential profile in the disk.

- Waves are trapped in the annular regions between Lindblad resonances.

- Not all galaxies have inner Lindblad radii. The necessary condition for the formation of an ILR is a relatively rapid transition from a region of solid-body rotation to one of differential rotation (e.g. a flat rotation curve).

Stability of Differentially Rotating Disks

The dispersion relation for a gaseous disk in tight-winding limit

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 v_s^2, \tag{2}$$

where "tight-winding limit" implies that radial separation between two successive arms $\Delta R << R$.

An analogous relation for stellar disks is

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k|\, F[(m\Omega - \omega)/\kappa, k\sigma_R/\kappa] \tag{3}$$

where function F is defined in BT (eq.6-45).

E.g. for an axially symmetric perturbation in a gaseous disk ($m = 0$), and uniform rotation with $\kappa = 2\Omega$,

$$\omega^2 = 4\Omega^2 - 2\pi G\Sigma|k| + k^2 v_s^2, \tag{4}$$

Unstable if $\omega < 0$. If also $\Omega = 0$ (non-rotating potential), then disk is unstable if

$$|k| < k_J \equiv \frac{2\pi G \Sigma}{v_s^2} \tag{5}$$

Note how rotation helps to keep disk stable.

Toomre's **local** stability criterion for stellar disks:

$$Q \equiv \frac{\sigma_R \kappa}{3.36 G \Sigma} \tag{6}$$

MW disk is marginally stable.

Stability of Differentially Rotating Disks

What is tight-winding limit is not applicable?

There are no analytic methods - must perform numerical experiments.

Ostriker-Peebles criterion: more than 2/3 of kinetic energy must be in random motions for a disk to be stable to barlike modes. It implies that the MW disk is locally unstable.

Swing Amplifier and Feedback loops

A phrase coined by Alar Toomre (1981) to describe an effect discovered much earlier by Goldreich and Lynden-Bell (1965)

Swing works on leading waves and turns them into trailing waves giving strong amplification in the process. The mechanisms are shear and self-gravity.

In a rotating coordinate system:

$$\ddot{r} = \nabla\Phi_{eff} - 2\,(\Omega_b\,\mathbf{x}\,\dot{\mathbf{r}}) \tag{7}$$

Self-gravity tends to compress the spiral arm, and shear (Coriolis force) then acts differently on the two edges because of differential rotation.