Astr 102: Introduction to Astronomy
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Lecture 16:
Cosmic Microwave Background
and other evidence for the Big Bang
Observational Cosmology: observations that allow us to test our models for evolution of the whole Universe!

- Dark matter: rotation curves
- Dark matter: gravitational lensing
- Gamma-ray bursts
- Standard candles: supernovae (type Ia)
- Nucleosynthesis
- Cosmic microwave background
- Cosmological models
Observational Cosmology

Key observations that support the Big Bang Theory

1. **Expansion: the Hubble law** if the Universe is expanding, it must have been much smaller (and therefore denser and hotter) in the past

2. **The light element abundance (nucleosynthesis)** implies a very specific relationship between density and temperature in early Universe

3. **Cosmic Microwave Background** a relic of extremely hot Universe
Expansion of the Universe

- Discovered as a linear law \( v = H D \) by Hubble in 1929.
- With distant SNe, today we can measure the deviations from linearity in the Hubble law due to cosmological effects.
- The curves in the top panel show models of the Universe with different amounts of dark matter and dark energy.
- The data imply an accelerating Universe at low to moderate redshifts but a decelerating Universe at higher redshift (the purple line is the so-called “concordance” model).
- A “prediction” of the standard expansion model is that the Universe was much (much much much much!) denser and hotter in the past: as the Universe expands, it cools and becomes less dense.
The Formation of Light Elements

- Expansion of the Universe and extrapolation to past using laws of physics imply high temperatures and densities early in the Universe.
- At high temperatures only neutrons (13%) and protons (87%) exist.
- As the temperature was decreasing with time, the nucleosynthesis began and all the neutrons were incorporated into He nuclei, while the leftover protons remained as hydrogen nuclei.
- After this first wave of nucleosynthesis was completed, the universe consisted of roughly 25% He and 75% H (by weight).
- Do abundance measurements support this prediction?
The Light Element Abundance as a Cosmological Constraint

- The deuterium ($H^2$), He$^3$, He$^4$ and Li$^7$ abundances depend on the single parameter: the current density of ordinary matter made out of protons and neutrons (baryonic matter), often expressed as the fraction of the total matter/energy density, $\Omega_b$ (more about this next time).
- A single value of the baryon density $\Omega_b$ fits 4 abundances simultaneously.
- This value ($\sim 0.04$) is much smaller than $\Omega_{\text{matter}} \sim 0.24$ measured by other means (CMB, SNe, dynamical methods). Hence, most of matter is in non-baryonic form. Candidates are massive neutrinos, WIMPS (weakly interacting massive particles), axions, etc.
Cosmic Microwave Background (CMB)

- The CMB was discovered by Penzias & Wilson in 1965 (although there was an older measurement of the “sky” temperature by McKellar using interstellar molecules in 1940, whose significance was not recognized).
- The CMB is the best black-body spectrum ever measured, with $T = 2.73$ K. It is also remarkably uniform across the sky (to one part in $\sim 10^{-5}$), after dipole induced by the solar motion is corrected for.
- The existence of CMB was predicted by Gamow in 1946.
- Fluctuations in the CMB at the level of $\sim 10^{-5}$ were first detected by the COBE satellite.
- The WMAP satellite has recently measured these fluctuations at a much higher angular resolution.
Cosmic Microwave Background (CMB)

- The CMB fluctuations, recently observed by WMAP at a high angular resolution, show a characteristic size of $\sim 1^\circ$
- How do we mathematically describe this behavior? How do we compare models to these observations?
Spherical Harmonics

- **Don’t be afraid of math!**
- Recall: a one-dimensional periodic function can be expanded into a Fourier series; the (squared) amplitudes vs. frequency plot shows the contribution of each mode: the **power spectrum**
- Similarly, a two-dimensional function defined on a sphere can be expanded in **spherical harmonics**; the power spectrum shows the contribution of each characteristic size
- Spherical harmonics are extensively used in quantum mechanics: the **Schrödinger equation** in spherical coordinates
Hydrogen Schrodinger Equation

The electron in the hydrogen atom sees a spherically symmetric potential, so it is logical to use spherical polar coordinates to develop the Schrodinger equation. The potential energy is simply that of a point charge:

\[ U(r) = \frac{-e^2}{4\pi \varepsilon_0 r} \]

The expanded form of the Schrodinger equation is shown below. Solving it involves separating the variables into the form

\[ \Psi(r, \theta, \phi) = R(r) P(\theta) F(\phi) \]

The starting point is the form of the Schrodinger equation:

\[ \frac{-\hbar^2}{2\mu} \frac{1}{r^2 \sin \theta} \left[ \sin \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial^2 \Psi}{\partial \phi^2} \right] \]

\[ -U(r) \Psi(r, \theta, \phi) = E \Psi(r, \theta, \phi) \]

\[ \frac{d^2 \Phi}{d\phi^2} + m_\ell^2 \Phi = 0 \quad \text{with solution} \quad \Phi_{m_\ell}(\phi) = \frac{1}{\sqrt{2\pi}} e^{im_\ell \phi} \]

\[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \left[ \ell(\ell + 1) - \frac{m_\ell^2}{\sin^2 \theta} \right] \Theta = 0 \]

\[ \ell = 0, 1, 2, 3, \ldots \quad m_\ell = 0, \pm 1, \pm 2, \ldots , \pm \ell \]

\[ \Theta_{\ell,m_\ell}(\theta, \phi) m_\ell(\phi) = Y_{\ell,m_\ell}(\theta, \phi) \]

<table>
<thead>
<tr>
<th>\ell</th>
<th>m_\ell</th>
<th>Y_{\ell,m_\ell}(\theta, \phi) = \Theta_{\ell,m_\ell}(\theta) \Phi_{m_\ell}(\phi)</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( (1 / 4\pi)^{1/2} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( (3 / 4\pi)^{1/2} \cos \theta )</td>
</tr>
<tr>
<td>1</td>
<td>\pm 1</td>
<td>( \mp (3 / 8\pi)^{1/2} \sin \theta e^{\pm i\phi} )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( (5 / 16\pi)^{1/2} (3 \cos^2 \theta - 1) )</td>
</tr>
<tr>
<td>2</td>
<td>\pm 1</td>
<td>( \mp (15 / 8\pi)^{1/2} \sin \theta \cos \theta e^{\pm i\phi} )</td>
</tr>
<tr>
<td>2</td>
<td>\pm 2</td>
<td>( (15 / 32\pi)^{1/2} \sin^2 \theta e^{\pm 2i\phi} )</td>
</tr>
</tbody>
</table>

\[ \Phi_{m_\ell}(\phi) = \frac{1}{\sqrt{2\pi}} e^{im_\ell \phi} \]

\[ \Theta_{\ell,m_\ell}(\theta) = \left[ \frac{2\ell + 1}{2} \frac{(\ell - m_\ell)!}{(\ell + m_\ell)!} \right]^{1/2} P_{\ell,m_\ell}^m(\theta) \]

\[ P_{\ell,m_\ell}^m(\theta) = \text{associated Legendre polynomial} \]
$l=2$

$m=0$

$m=1$

$m=2$
Cosmic Microwave Background (CMB)

- The CMB fluctuations, recently observed by WMAP at a high angular resolution, show a characteristic size of $\sim 1^\circ$.
- A statistical description of the anisotropies is given by the power spectrum. The power spectrum encodes constraints on cosmological parameters.
WMAP Power Spectrum

Basic flat WMAP parameters: \( W_L = 0.71, W_m = 0.29 \) (\( W_c = 0.24, W_b = 0.047 \)), \( n = 0.93, h = 0.71 \).

WMAP + other: \( W_L = 0.71, W_m = 0.27 \) (\( W_c = 0.23, W_b = 0.044 \)), \( n = 0.93, h = 0.71 \).
The origin of CMB

- **A historical note:** the accuracy of CMB measurements is improving fast – the accuracy delivered by WMAP is truly spectacular!

- **Why do we have CMB?** It is a remnant of hot radiation field from the beginning of the Universe – at some time in the past radiation and matter were in equilibrium!

- **Radiation-matter equilibrium:** there are enough energetic (UV) photons to keep matter (mostly hydrogen) ionized; the scattering of photons by electrons “equilized” the matter temperature throughout the Universe. How do we know this?

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**Figure 18.** Dramatic change took place in CMB power spectrum measurements around the turn of the 21st century. Although some rise from the COBE level was arguably known even by 1997, a clear peak around \( \ell \approx 200 \) only became established in 2000, whereas by 2003 definitive measurements of the spectrum at \( \ell \lesssim 800 \), limited mainly by cosmic variance, had been made by the WMAP.
The origin of CMB

• As the universe expands, the energy density of radiation $\propto R^{-4}$ (because the total energy, which is proportional to $T^4$, is also proportional to the product of volume and characteristic wavelength) and that of matter $\propto R^{-3}$. Therefore, at some time in the past radiation and matter must have been in equilibrium.

• From the photon-to-nucleon number ratio ($\sim 10^9$), one can estimate that the radiation temperature at that time was $\sim 3000$. Since $T \propto 1/R \propto (1+z)$, the corresponding redshift is $z \sim 1000$, or $\sim 380,000$ years after the Big Bang (next time)
Universe is Ionized!

Photons can scatter off unbound electrons

Universe is Neutral!
Photons are too low energy

No unbound “free” electrons, so nothing to scatter photons
Photons start at optical wavelengths at $z \sim 1300$

They then redshift with the expanding universe to radio wavelengths
We should see these very cool photons in every direction!
The CMB is incredibly smooth!

- No galaxies or stars existed yet! (Completely ionized right before the CMB photons were liberated).

- Density of the Universe was uniform to better than 0.01%.

- Whole Universe was like the surface of a red giant star (T~3000K)!
But, the CMB is not quite perfectly smooth...

... because the Universe wasn’t perfectly smooth!

\[ z = 28.62 \]
But, the CMB is not quite perfectly smooth...

- Slight "overdensities", pulled matter towards and away from us.

- Slight motions at $z \sim 1300$ caused some regions to be slightly Doppler redshifted or blueshifted!
Detailed modeling of the whole power spectrum also constrains other cosmological parameters, such as the Hubble constant (six in total).

What do we learn from $P(k)$?

- Models (computations based on physics) are compared to data:
  - The position of the first peak is very sensitive to $\Omega_{tot}$, which is a measure of the total energy content of the Universe (more details next time), and not very sensitive to other parameters.
  - Roughly, $l_1 = 220/\Omega_{tot}$. Thus, the measurement of the angular size of the “spots” in the CMB fluctuation map is essentially a direct measurement of $\Omega_{tot}$!
  - Recently, the WMAP satellite measured the peak position to be $l_1 \sim 216 \pm 4$ (implying an angular size of $180/l_1 \sim 0.83$ degree, which is just slightly larger than full Moon), and thus $\Omega_{tot} = 1.02 \pm 0.02$. 

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**Figure 4**  Sensitivity of the acoustic temperature spectrum to four fundamental cosmological parameters. (a) The curvature as quantified by $\Omega_{tot}$. (b) The dark energy as quantified by the cosmological constant $\Omega_k (w_k = -1)$. (c) The physical baryon density $\Omega_b h^2$. (d) The physical matter density $\Omega_m h^2$. All are varied around a fiducial model of $\Omega_{tot} = 1$, $\Omega_k = 0.65$, $\Omega_b h^2 = 0.02$, $\Omega_m h^2 = 0.147$, $n = 1$, $z_i = 0$, $E_i = 0$. 

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The Cosmological Concordance Model

- A large number of fundamentally different observations are explained with the same model: the expansion of the Universe after the Big Bang.
- There is no other theory except the Big Bang that can explain all these observations simultaneously.
- How do we combine very different observations, such as expansion of the Universe, the abundances of light elements, dark matter distribution, cosmic microwave radiation, and others, into a coherent model for evolution of the Universe?