Version 3.3
The Properties of Jovian Trojan Asteroids Listed in SDSS Moving Object Catalog 3

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ABSTRACT

We analyze 1187 observations of about 860 unique candidate Jovian Trojan asteroids listed in the 3rd release of Sloan Digital Sky Survey (SDSS) Moving Object Catalog. The sample is complete at the faint end to $r = 21.2$ mag (apparent brightness) and $H = 13.8$ (absolute brightness, approximately corresponding to 10 km diameter). A subset of 297 detections of previously known Trojans were used to design and optimize a selection method based on observed angular velocity that resulted in the remaining objects. Using a sample of objects with known orbits, we estimate that the candidate sample contamination is about 3%. The well-controlled selection effects, the sample size, depth and accurate five-band UV-IR photometry enabled several new findings and the placement of older results on a firmer statistical footing. We find that there are significantly more asteroids in the leading swarm (L4) than in the trailing swarm (L5): $N(L4)/N(L5) = 1.6 \pm 0.1$, independently of limiting object’s size. The overall counts normalization suggests that there are about as many Jovians Trojans as there are main-belt asteroids down to the same size limit, in agreement with earlier estimates. We find that Trojan asteroids have a remarkably narrow color distribution (root-mean-scatter of only $\sim 0.05$ mag) that is significantly different from the color distribution of the main-belt asteroids. The color of Trojan asteroids is correlated with their orbital inclination, in a similar way for both swarms, but appears uncorrelated with the object’s size. We extrapolate the results presented here and estimate that Large Synoptic Survey Telescope will determine orbits, accurate colors and measure light curves in six photometric bandpasses for about 100,000 Jovian Trojan asteroids.

Key words:

1 INTRODUCTION.

Jovian Trojan asteroids are found in two swarms around the L4 and L5 Lagrangian points of the Jupiter’s orbit (for a review see Marzari et al. 2001). The first Jovian Trojan was discovered a century ago by Max Wolf. Close to 2,000 Jovian Trojans were discovered by the end of 2003 (Bendjoya et al. 2004, hereafter B04). About half are numbered asteroids with reliable orbits (Marzari et al. 2001). Their total number is suspected to be similar to the number of the main belt asteroid (Shoemaker et al. 1989).

Trojans’ positions relative to Jupiter librate around L4 (leading swarm) and L5 (trailing swarm) with periods of the order hundred years. Their orbital eccentricity is typically smaller ($<0.2$) than those of main-belt asteroids, but the inclinations are comparable, with a few known Trojans having inclinations larger than 30 degree. The largest objects have diameters exceeding 100 km. They typically have featureless (D type) spectra and extremely low optical albedo (Tedesco 1989; Fernández, Sheppard & Jewitt 2003). These spectral properties are similar to those of cometary nuclei. However, there are also Trojans that have P or common C-type classification, mostly found in the trailing swarm (Fitzsimmons et al. 1994). The collisional grinding of Trojan asteroids is supported by their observed size distribution (Jewitt, Trujillo & Luu 2000, hereafter JTL).
Numerous studies of the origin of Jovian Trojans are based on two different hypothesis. According to one of them, the Jovian Trojans were formed simultaneously with Jupiter in the early phase of the solar nebula. The growing Jupiter could have captured and stabilized the planetesimals near its L4 and L5 points (Peale 1993). The other hypothesis assumes that the majority of Jovian Trojans were captured over a much longer period, and were formed either close to Jupiter, or were gravitationally scattered from the main belt or elsewhere in the solar system (Jewitt 1996). The spectral comet-like appearance of many Trojans is consistent with the scattering from the outer Solar System.

Depending on the importance of gas drag when Trojans formed, the L4 and L5 swarms could have different dynamics. The presence of significant gas drag helps stabilize orbits around the L5 point. On the other hand, these trailing objects have later evolution different from the leading swarm because planetary migration destabilizes L5 (Gomes 1998). Morbidelli et al. (2005) recently suggested a more complex picture: the present permanent Trojan populations are built up by objects that were trapped after the 1:2 mean motion resonance crossing of the Saturn and the Jupiter. Therefore, it is possible that size distributions, or detailed distributions of orbital parameters, could be different for the leading and trailing swarm. However, no such differences have yet been found (Marzari et al., 2001, and references therein).

It is noteworthy that there are severe observational biases in the sample of known Jovian Trojans due to their large distance. For example, although the numbers of main-belt asteroids and Trojans to a given size limit are similar, only about 1% of the known objects belong to the latter group. This is a consequence of the fact that a Trojan at a heliocentric distance of 5.2 AU is about 4 magnitudes fainter than a main-belt asteroid at a heliocentric distance of 2.5 AU (as observed in opposition, and not accounting for differences in albedo, which further diminishes the Trojan’s apparent magnitude).

Here we present an analysis of the properties of about 1000 known and candidate Jovian Trojan asteroids based on the data collected by Sloan Digital Sky Survey (SDSS, York et al. 2000). SDSS, although primarily designed for observations of extragalactic objects, is significantly contributing to studies of the solar system objects because asteroids in the imaging survey must be explicitly detected and measured to avoid contamination of the samples of extragalactic objects selected for spectroscopy. Preliminary analysis of SDSS commissioning data (Ivezić et al. 2001, hereafter I01) showed that SDSS will increase the number of asteroids with accurate five-color photometry by more than two orders of magnitude, and to a limit about five magnitudes fainter (seven magnitudes when the completeness limits are compared) than previous multi-color surveys (e.g. The Eight Color Asteroid Survey, Zellner, Tholen & Tedesco 1985). As we demonstrate below, the SDSS data extend the faint completeness limit for Trojan asteroids by about 1.5 magnitudes (to a limiting diameter of ∼10 km).

The large sample and accurate astrometric and five-band photometric SDSS data to a much fainter limit than reached by most previous surveys, together with suitable ways to quantify selection effects, allow us to address the following questions:

(i) What is the size distribution of Jovian Trojans asteroids with diameters larger than 10 km?
(ii) Do the leading and trailing swarms have the same size distribution (including both the distribution shape and the overall number above some size limit)?
(iii) What is their color distribution in the SDSS photometric system, and how does it compare to the color distribution of main-belt asteroids?
(iv) Is the color distribution correlated with inclination, as suggested by a preliminary analysis of SDSS data (Ivezić et al. 2002a, hereafter I02a)?
(v) Are the Trojans’ size and color correlated (as suggested by Bendjoya et al. 2004)?
(vi) Do the leading and trailing swarms have the same color distribution?
(vii) Is the size distribution correlated with inclination?

The SDSS asteroid data are described in Section 2, and in Section 3 we describe a novel method for selecting candidate Jovian Trojan asteroids from SDSS database. Analysis of the properties of selected objects, guided by the above questions, is presented in Section 4. We summarize our results in Section 5, and discuss their implications for the origin and evolution of Trojan asteroids.
eccentricity vs. semi-major axis distribution is shown. Note that there is no discernible correlation between the color and eccentricity for Jovian Trojan asteroids.

2 SDSS OBSERVATIONS OF MOVING OBJECTS

SDSS is a digital photometric and spectroscopic survey using a dedicated 2.5 m telescope at the Apache Point Observatory, which will cover 10,000 deg$^2$ of the Celestial Sphere in the North Galactic cap, and a smaller (∼225 deg$^2$) and deeper survey in the Southern Galactic hemisphere (Abazajian et al. 2003, and references therein). The survey sky coverage will result in photometric measurements for over 10$^8$ stars and a similar number of galaxies. The flux densities of detected objects are measured almost simultaneously (within ∼5 minutes) in five bands ($u, g, r, i,$ and $z$) with effective wavelengths of 3551 Å, 4686 Å, 6166 Å, 7480 Å, and 8932 Å (Fukugita et al. 1996; Gunn et al. 1998; Smith et al. 2002; Hogg et al. 2002). The photometric catalogs are 95% complete for point sources to limiting magnitudes of 22.0, 22.2, 22.2, 21.3, and 20.5 in the North Galactic cap. Astrometric positions are accurate to about 0.1 arcsec per coordinate (rms) for sources brighter than 20.5$^m$ (Pier et al. 2003), and the morphological information from the images allows robust star-galaxy separation (Lupton et al. 2001, 2002) to ∼21.5$^m$. The photometric measurements are accurate to 0.02 magnitudes (both absolute calibration, and root-mean-square scatter for sources not limited by photon statistics; Ivezić et al. 2004). The recent fifth public Data Release (DR5) includes imaging data for ∼8000 deg$^2$ of sky, and catalogs for 2.15 × 10$^8$ objects. For more details please see Abazajian et al. (2003) and references therein.

SDSS Moving Object Catalog (hereafter SDSS MOC) is a public, value-added catalog of SDSS asteroid observations (Ivezić et al. 2002b, hereafter I02b). It includes all unresolved objects brighter than $r = 21.5$ and with observed angular velocity in the 0.05–0.5 deg/day interval. In addition to providing SDSS astrometric and photometric measurements, all observations are matched to known objects listed in the ASTORB file (Bowell 2001), and to a database of proper orbital elements (Milani, 1999), as described in detail by Jurić et al. (2002, hereafter J02). J02 determined that the catalog completeness (number of moving objects detected by the software that are included in the catalog, divided by the total number of moving objects recorded in the images) is about 95%, and its contamination rate is about 6% (the number of entries that are not moving objects, but rather instrumental artifacts).

The third release of SDSS MOC used in this work contains measurements for over 204,000 asteroids. The quality of these data was discussed in detail by I01, including a determination of the size and color distributions for main-belt asteroids. An analysis of correlation between colors and asteroid dynamical families was presented by I02a. An interpretation of this correlation as the dependence of color on family age (due to space weathering effect) was proposed by Jedicke et al. (2004) and further discussed by Nesvorny et al. (2005). Multiple SDSS observations of objects with known orbital parameters can be accurately linked, and thus SDSS MOC also contains rich information about asteroid color variability, discussed in detail by Szabó et al. (2004).

The value of SDSS data becomes particularly evident when exploring the correlation between colors and orbital parameters for main-belt asteroids. Figure 1 uses a technique developed by I02a to visualize this correlation. A striking feature of this figure is the color homogeneity and distinctiveness displayed by asteroid families. This strong color segregation provides firm support for the reality of asteroid dynamical families. Jovian Trojans asteroids are found at $a$∼5.2 AU, and display a correlation between the color and orbital inclination (objects with high inclination tend to be redder). On the other hand, the color and orbital eccentricity (see Figure 2) do not appear correlated.

The distribution of the positions of SDSS observing fields in a coordinate system centered on Jupiter and aligned with its orbit is shown in Figure 3. As evident, both L4 and L5 regions are well covered with the available SDSS data. There are 313 unique known objects (from ASTORB file) in SDSS MOC whose orbital parameters are consistent with Jovian Trojan asteroids (here defined as objects with semi-major axis in the range 5.0–5.4 AU). Since SDSS imaging depth is about two magnitudes deeper than the completeness limit of ASTORB file used to identify known Trojans, there are many more Trojan asteroids in SDSS MOC whose orbits are presently unconstrained. Nevertheless, they can be identified using a kinematic method described in the following Section.

3 SELECTION OF TROJAN ASTEROIDS FROM SDSS MOVING OBJECT CATALOG

The angular velocity of moving objects measured by SDSS can be used as a proxy for their distance determination and classification (see Figure 14 and Appendix A in I01). For example, Jovian Trojan asteroids are typically slower than main-belt asteroids because their distances from Earth are larger (the observed angular velocity is dominated by the Earth’s reflex motion). However, in addition to angular ve-
Jovian Trojan Asteroids in SDSS MOC

Figure 3. The distribution of the longitude of $\sim 440,000$ 9 $\times$ 13 arcmin$^2$ large SDSS observing fields in a coordinate system center on Jupiter and aligned with its orbit, as a function of observing epoch (green symbols). Fields obtained within 25 deg. from the opposition are marked by black symbols. The two dashed lines mark the relative longitudes of the L4 ($\lambda_{Jup} = 60$ deg, leading swarm) and L5 ($\lambda_{Jup} = -60$ deg, trailing swarm) Lagrangian points. Both swarms are well sampled in the third release of SDSS Moving Object Catalog.

velocity, the selection algorithm must also include the longitudinal angle from the opposition, $\phi$, because for large values of $|\phi|$ the main-belt asteroids can have angular velocity as small as Jovian Trojans. This behavior is illustrated in Figure 4.

We optimize criteria for selecting candidate Jovian Trojans with the aid of 482 observations of 313 Trojans from SDSS MOC that have known orbits extracted from ASTORB file (there are 43,424 unique objects with known orbits in the third release of SDSS MOC). These 482 observations are identified in orbital space using constraints $5.0 \, \text{AU} < a < 5.4 \, \text{AU}$ and $e < 0.2$, and hereafter referred to as the Known Trojans (KT). Of those, the majority (263) belong to the leading swarm.

We compare the angular velocity and $\phi$ distributions of these objects to those for the whole sample in Figure 4. We find that the following selection criteria result in a good compromise between the selection completeness and contamination:

\[ 0.112 - \left( \frac{\phi}{180} \right)^2 < v < 0.155 - \left( \frac{\phi}{128} \right)^2, \]

\[ -0.160 + \left( \frac{\phi}{134} \right)^2 < v\lambda < -0.125 + \left( \frac{\phi}{180} \right)^2, \]

for observations with $-25 < \phi < 25$. That is, only observations obtained relatively close to the opposition can be used to select a sample with a low contamination rate by main-belt asteroids. The adopted velocity limits are in good agreement with those proposed by JTL.

When applied to all objects from SDSS MOC, this selection results in a sample of 1187 candidate Trojans, including 272 observations of known objects (see Figure 5). Of the latter, 8 objects have semi-major axis too small to be a Trojan asteroid, which implies a contamination rate of 3%. SDSS MOC contains 297 observations of known Trojans obtained with $|\phi| < 25$, which implies that the kinematic selection method is 89% complete. The 264 detections of known Trojans in the kinematically selected sample correspond to 191 unique objects. Therefore, 1187 detections in the candidate sample correspond to about 858 unique objects.

The contamination rate could be higher than 3% because objects with known orbits tend to be brighter and thus have smaller measurement uncertainties for angular velocities than objects from the full candidate sample (for a de-
Figure 5. Analogous to Figure 4 except that all ~204,000 objects from SDSS Moving Object Catalog are shown (blue dots). The candidate Trojans are shown by black symbols, and the known Trojans are overplotted as red symbols.

Figure 6. A test of the selection robustness. The top two panels show all the objects from SDSS MOC (small blue dots), the known Trojans (red dots) and the candidate Trojans (black squares), as observed on the sky, in Jupiter’s coordinate system and in φ vs. λ_Jup diagram. Although λ_Jup was not used in selection, the known and candidate Trojans have similar λ_Jup distributions (the third panel from top, dotted and solid histograms, respectively), and different than for the whole sample, dominated by main-belt asteroids (dashed line). Note that these λ_Jup distributions are not corrected for the selection biases due to inhomogeneous coverage of λ_Jup − β_Jup plane (which are presumably similar for both known and candidate objects), and thus are not representative of the true distribution. The bottom panel compares the angular velocity distributions of Trojans and main-belt asteroids.

Detailed study of these errors and their correlation with other observables see I01). For this reason, we perform the following robustness test.

The above selection procedure does not include λ_Jup, the longitudinal angle between an object and Jupiter. If the selection is robust, the λ_Jup distributions for the known and candidate Trojans should be similar. As discernible from Figure 6, this is indeed the case and demonstrates that the contamination rate by non-Trojan asteroids in the candidate sample must be small. A similar conclusion is reached when comparing color distributions (see below). We refer to this sample hereafter as the Candidate Trojans (CT).

4 ANALYSIS OF THE PROPERTIES OF TROJAN ASTEROIDS

Using the sample of candidate Trojan asteroids selected as described above, here we analyze their distribution in the 3-dimensional size-color-inclination space, both for the full sample and separately for each swarm. The large size of the selected candidate sample allows accurate measurements of this distribution, and represents an especially significant improvement over the previous work when studying color distribution. The two largest homogeneous studies of spectral properties of Jovian Trojans are by Jewitt & Luu (1990) and Bendjoya et al. (2004). Jewitt & Luu obtained spectra...
of 32 Trojans and found that they are remarkably similar to cometary spectra. Bendjoya et al. obtained spectroscopic observations for 34 objects and, together with older observations, produced a sample of 73 objects. Therefore, accurate color information for over a thousand objects discussed here represents a substantial improvement.

4.1 The Numbers of Asteroids in L4 and L5 Swarms

It is usually assumed that the leading (L4) and trailing (L5) swarms contain similar number of asteroids down to the same size limit (e.g. JTL). Although the number of known objects in L4 and L5 differ (e.g. as listed in Bowell’s ASTORB file), this asymmetry is usually dismissed as due to complex selection biases in the sample of Trojans with known orbits (e.g. Marzari et al. 2001). On the other hand, Pál & Süli (priv. comm.) find using numerical simulations that the perturbations by Saturn produce different stability regions for L4 and L5. This effect is suspected to cause about a factor of 2 population size difference between the two swarms. It is reassuring that we obtained a statistically consistent result using the first method. We emphasize that there is no discernible difference in the shape of the spatial distribution of objects from the two swarms.

Interestingly, this number ratio is about the same as the leading-to-trailing ratio of Trojans with known orbits in Bowell’s ASTORB file. Although the selection effects are typically invoked to explain this asymmetry, it instead appears to be a real effect (we show below that the sample of known Trojans is indeed fairly complete to $r \sim 19.5$). On the other hand, the number ratio of asteroids in the two swarms could be dependent on object’s size, and the SDSS sample extends to smaller sizes than ASTORB file. We address this possibility in the next section.

4.2 Apparent and Absolute Magnitude Distributions

The differential apparent $r$ band magnitude distributions (for Trojans, Johnson’s $V \sim r + 0.25$) for known (KT) and candidate (CT) Trojans are shown in the top panel in Figure 9. The KT sample is complete to $r \sim 19.5$, and the CT sample is complete to $r \sim 21$. The formal cutoff for inclusion of moving objects in the SDSS MOC is $r < 21.5$. A slightly brighter completeness limit for Trojans can be understood as the removal of objects from a fairly narrow velocity space due to velocity errors (see fig. 6 from I01). Because the CT sample is complete to a $\sim 1.5$ mag deeper limit, it contains $\sim 4$ times more objects. It is noteworthy that the high completeness of KT sample indicated by the SDSS data (that is, the counts are practically identical for $r < 19.5$) argues that selection effects cannot be invoked to explain the L4–L5 asymmetry in the number counts of Trojans with known orbits listed in Bowell’s ASTORB file.

In order to investigate the dependence of various quantities (such as counts and colors) on object size, we transform apparent magnitudes to absolute magnitudes as follows. The dependence of apparent magnitude in the Johnson $V$ band on absolute magnitude, $H$, distance from Sun, $R$, distance

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3 Here we assume that the depth of SDSS imaging is constant, which is true to within several tenths of a magnitude.

4 The errors for these estimates are not larger than $\sim 0.5$ and indicate that the distribution of Trojans on the sky is not circularly symmetric around L4 and L5 points, as assumed by JTL.
from Earth, \( \Delta \), and viewing (phase) angle, \( \alpha \), can be expressed as

\[
V(R, \Delta, \alpha) = H + 5 \log(R\Delta) + F(\alpha).
\] (3)

Here \( V(R, \Delta, \alpha) = r + 0.44(g - r) \) is synthesized from SDSS measurements, \( F(\alpha) \) is the phase function, and \( H \) includes the dependence on diameter \( D \) (in km) and the V-band albedo, \( p_V \)

\[
H = 19.14 - 2.5 \log \left( \frac{p_V}{0.04} \right) - 5 \log(D).
\] (4)

Note that formally \( V(1,1,0) = H \). Given \( R \) and \( \phi \), \( \Delta \) and \( \alpha \) can be found from

\[
\Delta^2 + 2\Delta \cos(\phi) + 1 = R^2
\] (5)

and

\[
\alpha = \phi - \arccos \left( \frac{1 + \Delta \cos(\phi)}{R} \right).
\] (6)

When applying this procedure to observations discussed

Figure 7. The top panel shows the observed surface density map (number of detected objects per 4 deg\(^2\) large bin) of candidate Trojan asteroids in Jupiter’s coordinate system. The middle panel shows the number of SDSS fields observed in each bin (that is, the selection function), and the bottom panel shows the corrected surface density of Trojans (the ratio of the maps in the top and middle panels). The values are shown on a linear scale, increasing from blue to red (i.e. no objects are found in blue strips). The maximum value (coded red) in the top panel is 20 (Trojans per 4 deg\(^2\) large bin), 3.7 in the middle panel (SDSS observations per position, averaged over bin), and 5 (Trojans per deg\(^2\), averaged over 4 deg\(^2\) large bin). The purple (dark) regions contain no data.
Figure 8. The top panel shows the observed surface density of known Trojan asteroids from Bowell’s ASTORB file, analogously to Fig. 7. The distribution for each swarm is well described by a two-dimensional Gaussian. The second panel shows a model distribution that has the same shape as the Gaussian distribution implied by the top panel, but normalized to the observed counts of SDSS candidate Trojan (for each swarm separately), shown in the third panel with the same color scheme (red corresponds to 5 objects per deg^2). The best-fit L4:L5 number ratio is 1.6±0.1. The difference between the observed counts and this model distribution, normalized by the Poisson error bars, is shown in the bottom panel (the purple regions contain no data). The value of χ^2 per degree of freedom is 1.15.
Figure 9. The top panel shows the differential SDSS $r$ band distributions for known (squares) and candidate Jovian Trojan asteroids (circles). The SDSS candidate sample is $\sim$1.5 mag deeper than the sample of known objects. The bottom panel compares the differential absolute magnitude distributions in the Johnson’s V band. The dashed line is added to guide the eye and has the slope of 0.44. The SDSS data suggest that practically all Trojans brighter than V($1, 1, 0$) $\sim$12.3 ($r \sim$19.5), or approximately larger than 20 km, are already discovered and listed in ASTORB file.

Figure 10. The calibration of phase effects on observed magnitudes. The top panel shows the distance-corrected magnitudes as a function of phase for known Trojans observed at small latitudes ($|\beta| < 10$). Two different symbols corresponds to objects from L4 (star) and L5 (dot) swarms. The dotted line shows a best linear fit discussed in the text. The bottom panel shows a histogram of the scatter around this best fit.

here, $R$ and $F(\alpha)$ are not known. We adopt $R = 5.2$ AU and model the phase function as $F(\alpha) = k|\alpha|$. Therefore,

$$H \sim V(1, 1, 0) = V(1, 1, \alpha) - k|\alpha|.$$  

In order to determine coefficient $k$, we used known Trojan asteroids observed at low latitudes ($|\beta| < 10$). A least-square best-fit to the observed dependence of $V(1, 1, \alpha) - H$ on $|\alpha|$ gives $k = 0.066 \pm 0.018$. To the zero-th order, the transformation from apparent to absolute magnitudes for Trojans observed close to the opposition amounts to a shift of about 7 mag. In order to distinguish absolute magnitude for objects with known orbits from the estimates evaluated here, we will refer to $H$ and $V(1, 1, 0)$ for KT and CT samples, respectively.

It is noteworthy that the intercept of the best-fit line discussed above (see Figure 10) is consistent with 0. This shows that the $V$ band magnitudes synthesized from SDSS photometry and $H$ magnitudes for Trojans listed in ASTORB file are expressed on the same photometric system. This appears not be the case for a significant fraction of main-belt asteroids whose magnitudes (that are simply adopted from a variety of asteroid surveys) can have systematic errors as large as 0.5 mag (for more details see J02). The root-mean-square width of the residuals distribution shown in the bottom panel in Figure 10 is 0.3 mag, and represents an upper limit for the errors of our method for estimating $V(1, 1, 0)$ (e.g. photometric and other errors for $H$ listed in

8 I01 developed a method for estimating heliocentric distance of asteroids from their angular velocity measured by SDSS that is accurate to about 10% for main-belt asteroids. For Trojans, which have larger velocity errors, a smaller error is introduced by assuming a constant $R$. 

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Figure 11. The top panel compares the differential distributions of estimated absolute magnitudes in the Johnson V band for candidate Trojans separated into leading and trailing swarms. The counts have indistinguishable slopes, but the overall normalization is different. The bottom panel illustrates this difference by showing the ratio of cumulative counts for the two swarms. Note that within errors this ratio does not depend on absolute magnitude, or equivalently size, as marked on top (diameter in km).

The bottom panel in Figure 9 compares the differential absolute magnitude distributions in the Johnson's V band for known and candidate Trojans. The SDSS data suggest that practically all Trojans brighter than V(1, 1, 0) ∼ 12.3, or those with diameters approximately larger than 20 km, are already discovered.

We find that the differential absolute magnitude distribution is well described by

\[ \log(N) = C + \alpha H \]  

with \( \alpha = 0.44 \pm 0.05 \). This implies a differential size distribution index of \( q = 5\alpha + 1 = 3.2 \pm 0.25 \), valid for \( 9 < H < 13.5 \). This value is in good agreement with JLT, who obtained \( q = 3.0 \pm 0.3 \) using about 10 times smaller sample, and with Yoshida & Nakamura (2005), who obtained \( q = 2.9 \pm 0.1 \) using a sample of 51 objects.

We use the counts of the presumably complete bright \( (H < 12) \) subsample of known Trojans from ASTORB file to normalize the cumulative counts

\[ \log(N_{cum}) = 2.9 + 0.44 (H - 12) \]  

Assuming \( pv = 0.04 \) (Fernández, Sheppard & Jewitt 2003), \( D = 1 \) km corresponds to \( H = 19.14 \). The above result implies that there are about 1 million Jovian Trojans larger than 1 km, to within a factor of 2 (uncertainty comes from the error in \( \alpha \) and extrapolation over 5 mags; in addition, this normalization scales with the albedo approximately as \( \propto 0.04/pv \)). This estimate could be up to a factor of 2 too high if Trojans size distribution becomes shallower for objects smaller than \( \sim 5 \) km, as was found for main-belt asteroids (see IO1), and is suggested for Jovian Trojans by Yoshida & Nakamura (2005). These results are in good agreement with the normalization obtained by JTL and imply that there are about as many Jovians Trojans as there are main-belt asteroids down to the same size limit.

In previous section, we demonstrated that L4 has a significantly larger number of objects than L5. To examine whether, in addition to this difference in overall normalization, the slope of the size (i.e. \( H \)) distribution is different for the two swarms, we separately analyzed their counts. The slope of the size distributions for both the candidate Trojans (Figure 11) and for known Trojans (Figure 12) are the same within measurement uncertainties (with the slope error ∼0.05). Note that the L4-to-L5 count ratios shown in the bottom panel in Figures 11 and Figure 12 are different from the value of 1.6 discussed in Section 4.1 because \( \lambda - \beta \) selection effects (which are not a function of size) are not taken into account.

4.3 Color Distribution

One of the main advantages of the sample discussed here are accurate color measurements for a sample about two orders of magnitude larger than available before. Together with robust knowledge about the color distribution of main-belt asteroids in the SDSS photometric system (I01, I02a), we are
in a position to compare the colors of the two populations with an unprecedented level of detail.

We first correct color measurements for the phase effects using a linear color vs. phase angle approximation discussed in Section 4.2. We obtained the following best-fit relations for the colors corrected to zero phase angle

\[
(g - r)_c = (g - r) - 0.0051 |\alpha|, \quad (10)
\]

and

\[
(r - i)_c = (r - i) - 0.0056 |\alpha|, \quad (11)
\]

with the coefficient errors of about 0.001 mag/deg. No significant correlation with the phase angle was detected for the \(i - z\) color, and too few objects have accurate \(u - g\) color measurement to attempt a robust fit. As the mean value of \(|\alpha|\) is about 2 degree, these corrections are small compared to photometric accuracy.

The color distribution of Trojan asteroids is compared to the color distribution of main-belt asteroids in Figure 13. The mean colors and their standard deviation (not the error of the mean!) for candidate Trojans with color errors less than 0.05 mag are

- \(u - g = 1.45, 0.08\),
- \(g - r = 0.55, 0.08\),
- \(r - i = 0.22, 0.10\), and
- \(i - z = 0.13, 0.11\) (for reference, these colors correspond to Johnson’s \(B - V = 0.73, V - R = 0.45, R - I = 0.43\), using the photometric transformations from Ivezić et al. 2007; these values are in good agreement with previous work, e.g., Fornasier et al. 2004, Dotto et al. 2006). The two distributions are different, with the difference maximized in the \(i - z\) vs. \(r - i\) diagram. Using solar colors from I01, we compute the relative albedo for Trojan asteroids and compare it to the three dominant main-belt color types in Figure 14. As expected from previous work, Trojan asteroids are redder than main-belt asteroids at wavelengths longer than the visual band.

In addition to maximizing color differences between Trojan and main-belt asteroids, the \(i - z\) vs. \(r - i\) diagram is interesting because the distribution of candidate Trojans suggests bimodality. To quantify this effect in the subsequent analysis, we define a color index which is a linear combination of the \(r - i\) and \(i - z\) colors:

\[
t^* = 0.93 (r - i) + 0.34 (i - z) - 0.25, \quad (12)
\]

with the phase-angle correction

\[
t^*_c = t^* - 0.005 |\alpha|, \quad (13)
\]

The distribution of this color index for known and candidate Trojans is compared to that of the main-belt asteroids in Figure 15. The fact that the distributions for known and candidate Trojans are indistinguishable, while clearly different from that of the main-belt asteroids, is another demonstration of the robustness of kinematic selection method.

The distribution of this color index for known and candidate Trojans is compared to that of the main-belt asteroids in Figure 15. The fact that the distributions for known and candidate Trojans are indistinguishable, while clearly different from that of the main-belt asteroids, is another demonstration of the robustness of kinematic selection method.

The distribution of this color index is bimodal. At first it appears that this bimodality is related to L4 vs. L5 separation, as illustrated in Figure 15. However, objects from L4 and L5 have different observed orbital inclination distribution due to observational selection effects (see Section 4.3.1). Instead, the differences in the L4 and L5 color
4.3.1 Correlation between Color and Orbital Inclination

As was already discernible in Figure 14, the color and orbital inclination for Jovian Trojan asteroids are correlated. This correlation is presented in a more quantitative way in the top panel in Figure 16 and in Table 1. As evident, objects with large orbital inclination tend to be redder. For example, the median $t^*$ color is -0.01 for objects with inclination less than 10 degree, while it is 0.04 for objects with inclination greater than 10 degree, and 0.06 for those with inclination greater than 20 degree. While these differences are not large, they are detected at a statistically significant level (the formal uncertainties are smaller than 0.01 mag). Equivalently, the median inclination for objects with $t^* < 0$ is 8.9 degree, while it is 13.4 for the redder objects. The marginal color distributions for subsamples selected by inclination are shown in the left panel in Figure 17.

The sample of candidate Trojans is much larger and fainter than the sample of known Trojans and can be used to test whether the color-inclination correlation extends to smaller sizes. Since the orbital inclination is unknown for the majority of candidate Trojans, we use as its proxy the latitude relative to Jupiter’s orbit, $\beta$. When the sample of known Trojans is separated by $\beta = 6$ deg, 89% of high-inclination and 66% of low-inclination objects are correctly classified. As evident from the middle panel in Figure 17, the differences in color histograms for subsamples of known Trojans separated by $\beta$ are still discernible, which justifies the use of $\beta$ as a proxy for inclination. The color histograms for candidate Trojans separated by $\beta$ are shown in the right panel in Figure 17. As they look similar to the analogous histograms for known Trojans, we conclude that the color-inclination correlation extends to smaller sizes.

Due to observational selection effects, the L5 subsample of known Trojans has a larger fraction of objects with large inclinations than the L4 subsample. This difference between L4 and L5, together with the color-inclination correlation, results in differences between their $t^*$ color distributions discernible in Figure 15. However, as shown in Figure 18 once the objects are separated by inclination, or by $\beta$, this difference between L4 and L5 objects disappears. We conclude that there is no evidence for different color-inclination correlations between the two swarms.

The similarity of the histograms shown in the middle and right panels in Figure 17 suggests that the color-inclination correlation cannot be a strong function of object’s size. Another “slice” through the observed color-inclination-size-swarm space is shown in Figure 19. We find no strong correlation between the Trojan size and color, except for a few large L4 objects with high inclination that have about ~0.05 mag redder $t^*$ color. Indeed, these few objects may be the reason for a claim by Bendjoya et al. (2004) that the spectral slope (i.e. color) is correlated with size in the size range 70–160 km.

5 DISCUSSION AND CONCLUSIONS

The kinematically-selected sample of candidate Jovian Trojan asteroids analyzed here is complete at the faint end to $r = 21.2$ mag, approximately corresponding to 10 km diameter, with a contamination rate of only ~3%. Similarity of the longitude (relative to Jupiter) and color distributions between known and candidate Trojans, and their difference from the distributions for main-belt asteroids which dominate the parent sample, strongly suggest that the kinematic selection is robust. The well-controlled selection effects, the sample size, depth and accurate five-band UV-IR photometry enabled several new findings and the placement of older results on a firmer statistical footing. The main results obtained here are:

(i) The differential size distribution of Jovian Trojan asteroids follows a power law, $n(D) \propto D^{-q}$, with the power-law index of $q = 3.20 \pm 0.25$, in agreement with previous work (e.g. JTL). This value of $q$ implies that the total mass is dominated by large objects. The overall normalization is tied to a complete sample of known Trojans and suggests that there moving objects from their observed apparent motions. While their method had satisfactory accuracy for studying main-belt asteroids, we found using a simple Monte Carlo simulation that it is not applicable here because the three times slower apparent motion of Trojans results in unacceptably large inclination errors.

Figure 14. A comparison of the relative albedo for Trojan asteroids (black dots) and the relative albedo for the three dominant main-belt color types (C type: blue circles, S type: red solid squares, V type: magenta open squares, for bands other than $z$ same as S). Due to large sample sizes, errors reflect systematic uncertainties in SDSS photometric calibration.
are about as many Jovians Trojans as there are main-belt asteroids down to the same size limit, also in agreement with earlier estimates.

(ii) The same power-law size distribution provides a good description for both the leading (L4) and trailing (L5) swarm. Their spatial distribution on the sky can be described with two elliptical Gaussian distributions (σ = 14°, β = 9°) that have different normalization: there are 1.6±0.1 more objects in the leading than in the trailing swarm. The cumulative number of Jovian Trojan asteroids (per deg²) as a function of absolute magnitude H and a position in Jupiter’s coordinate system (λ, β, in degree) can be estimated from

\[ n(H, \lambda, \beta) = N_{\text{cum}}(H) \frac{f(\lambda)}{2\pi \sigma \lambda \sigma \beta} e^{-\frac{\beta^2}{2\sigma \beta}} \]

where \( N_{\text{cum}}(H) \) is given for \( H < 13.5 \) by eq. 9 and

\[ f(\lambda) = 0.62 e^{-\frac{(\lambda - 60°)^2}{2\sigma \lambda}} + 0.38 e^{-\frac{(\lambda + 60°)^2}{2\sigma \lambda}} \]  

(iii) The two orders of magnitude increase in the number of objects with accurate color measurements allowed us to demonstrate that Trojan asteroids have a remarkably narrow color distribution (root-mean-scatter of only ~0.05 mag) that is significantly different from the color distribution of the main-belt asteroids.

(iv) We find that the color of Trojan asteroids is correlated with their orbital inclination, in a similar way for both swarms, but appears uncorrelated with the object’s size.

(v) We did not detect a size-inclination correlation.

These results have direct implications for the theories of Trojans origin. The detected difference in the normalization between leading and trailing swarms suggests that there was at least some period during which their formation and/or evolution was different. Similarly, the color-inclination correlation suggests that there must have been a process in the past which is responsible for the increased fraction of red objects at high orbital inclinations. Gas dynamics and planetary migration are good candidates for such a process, as recently discussed by Tsiganis et al. (2005). A possible explanation for this correlation is that when asteroids on the temporary eccentric orbits encounter the Sun, their minimal distance from the Sun is related to the inclination we observe today. In this picture the space weathering effects and volatization would vary with the inclination. A detailed analysis of these possibilities is beyond the scope of this paper and we leave it for future work.

While the increase in sample size enabled by SDSS is considerable, very soon new large-scale sky surveys, such as Pan-STARRS (Kaiser et al. 2002) and LSST (Tyson 2002), may obtain even more impressive samples, both in size, diversity of measurements and their accuracy. For example, LSST will scan the whole observable sky every three nights in two bands to a 5σ depth equivalent to \( V = 25 \) (about 2.5 mag deeper than SDSS). Using the size distribution determined here, we estimate that LSST, which may have its first light in 2014, will collect a sample of about 100,000 Jovian Trojan asteroids and provide both orbits, accurate color measurements and light curves for the majority of them. A significant fraction (20–30%) of this sample will be obtained by Pan-STARRS4, which is supposed to have its first

### Table 1.

The statistics of various color indices show prominent inclination dependence. The subsets are selected by the inclination range, \( \text{inc} \), \( N \) is the number of objects in each bin, and \( \text{Err} \) is the standard error of the mean.

<table>
<thead>
<tr>
<th>inc</th>
<th>N</th>
<th>( g-t )</th>
<th>Err</th>
<th>( t-i )</th>
<th>Err</th>
<th>( i-z )</th>
<th>Err</th>
<th>( B-V )</th>
<th>Err</th>
<th>( V-R )</th>
<th>Err</th>
<th>( R-I )</th>
<th>Err</th>
<th>( t )</th>
<th>Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>153</td>
<td>0.56</td>
<td>0.01</td>
<td>0.21</td>
<td>0.01</td>
<td>0.11</td>
<td>0.02</td>
<td>0.73</td>
<td>0.02</td>
<td>0.45</td>
<td>0.01</td>
<td>0.42</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>10–20</td>
<td>227</td>
<td>0.58</td>
<td>0.01</td>
<td>0.24</td>
<td>0.01</td>
<td>0.13</td>
<td>0.01</td>
<td>0.75</td>
<td>0.01</td>
<td>0.47</td>
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<td>0.47</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>20–30</td>
<td>71</td>
<td>0.60</td>
<td>0.02</td>
<td>0.26</td>
<td>0.01</td>
<td>0.16</td>
<td>0.01</td>
<td>0.77</td>
<td>0.02</td>
<td>0.48</td>
<td>0.01</td>
<td>0.48</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### Figure 15.

The left panel compares the distribution of the synthetic color index \( t^* \) for known (dashed line) and candidate (solid line) Trojan asteroids to that of the main-belt asteroids (dotted line). The middle and right panels compares the \( t^* \) distribution separately for L4 (solid line) and L5 (dashed line) swarms. The differences between the two swarms are due to a color-inclination correlation and different sampling of orbital inclinations due to observational selection effects (see Section 4.3.1).
light around 2009. These samples will undoubtedly reinvigorate both observational and theoretical studies of Jovian Trojan asteroids.

ACKNOWLEDGMENTS

We thank Elisabetta Dotto for a discussion that helped improve the presentation. This work has been supported by the Hungarian OTKA Grants T042509, the “Magyary Zoltán” Higher Educational Public Foundation and the Szeged Observatory Foundation. We acknowledge generous support by Princeton University.

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web Site is http://www.sdss.org/

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Figure 19. Color-magnitude diagrams for subsamples of candidate Trojans separated into L4 (top) and L5 (bottom) objects, and further into low-inclination (left) and high-inclination (right) objects. Small dots represent individual objects and large circles are the median values of $t^*$ color in 1 mag wide bins of absolute magnitude. The $1\sigma$ envelope around the median values is computed from the interquartile range. Note the cluster of $V(1,1,0) < 11$ objects in top right panel that have slightly redder objects than the rest of the sample.

University, the United States Naval Observatory, and the University of Washington.

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Figure 16. The top panel shows the distribution of known Trojans in the inclination vs. color diagram using linearly spaced contours. Individual objects are also shown and separated into L4 (blue dots) and L5 (red crosses) swarms. Note that L5 objects tend to have larger inclination due to observational selection effects. The middle panel is analogous, except that orbital inclination is replaced by its proxy $\beta$ (latitude relative to Jupiter’s orbit). The bottom panel is analogous to the middle panel, except that it shows candidate Trojans.

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