Time: MW 4:00-5:20; Place: LOW 101

CourseWebsite : http : //faculty.washington.edu/hqian/amath572/

Textbook: Stochastic Methods: A Handbook for the Natural and Social Sciences (4th Edition)
by Crispin Gardiner, Springer (2009)

Reference Book 1: A First Course in Stochastic Processes (2nd Edition)
by S. Karlin and H.M. Taylor, Academic Press (1975)

Reference Book 2: Probability Theory: The Logic of Science
by E. T. Jaynes, Cambridge University Press (2003)

Reference Book 3: Theory And Applications Of Stochastic Processes: An Analytical Approach
by Z. Schuss, Springer (2010)

Reference Book 4: Numerical Solution of Stochastic Differential Equations
by P.E. Kloeden and E. Platen, Springer (1995)

Syllabus

1. **Review on Probability and Random Variables:**

   The first week, I shall give an overview of why stochastic dynamics is important. Then in the second and third weeks, we shall review the materials on the theory of probability, and also have a brief histroical account. **Required reading**: Ch. 1, Ch. 2; optional reading: a paper by Montroll (on the course web). I shall give lectures on related material, but not follow the book exactly. This is a review: In principle you should know this material *a priori*.

   New science, even social sciences, needs new mathematics. The stochastic mathematical view of the world is fundamentally different from that of Newtonian mathematics. It is not an alternative; It is a more complete, more sophisticated, and more realistic perspective. The deterministic theory is only a special extreme case.

   Some people, such as Bernoulli (1700-1782), Laplace (1749-1827), and E. T. Jaynes (1922-1998), actually articulate an even stronger view that the therory of probability is the language of science.

   Statistical physics as an example: kinetic of gases and fluid dynamics; But in physics the stochastic effect is only considered as a perturbation to the deterministic theory. It is a part of the Nature due to atomic theory of the matters, thermal fluctuations, or imcomplete information. It was Darwin who put the stochasticity, called "variation" into a much more positive spot-light. In Darwin's view, the stochasticity is a basis of living process. It is the constructive orgin of life. According to a deterministic world view, things can only converge; but according to Darwinian view of the world, there are diversities.

   Probability review. *(1) Distribution and dynamics, (2) Joint and conditional probabilities, (3) Mean values and probability density, (4) Characteristic function and correlation functions, (5) Binomial, Poisson and Gaussiani distributions, (6) Central limit theorem.*

2. **Markov Processes (Chapter 3)**

   What is a stochastic process? Markov process and the Chapman-Kolmogorov equation, discrete space and time;

   Continuous-time Markov processes and its representation;

   Diffusion processes — Fokker-Planck equation;

Deterministic processes — Liouville's equattion;

Kolmogorov forward and backward equations.

4. **Stationary Processes and Correlation Function (Section 3.7)**

   Stationary and homogeneous Markov processes;

   Autocorrelation function for Markov processes;

   Reversibility and entropy production;

   Time reversibility, symmetry, detailed balance, and potential;

   Relative entropy and entropy production (see reading material).

5. **Fokker-Planck Equation (FPE) (Chapter 5)**

   FPE in 1D, FPE in 2D;

   Reversibility and circulation;

   FPE in several dimentions;

   First passage time problem.

6. **Master Equations and Jump Processes (Chapter 11)**

   Random walk and birth-death processes;

   Approximation of Master equations by FPEs;

   Mean first passage times;

   Birth-death systems with many variables and the chemical master equation;

   Poisson representation.

7. **Brownian Motion, Diffusion Processes, and Stochastic Differential Equations (Chapter 4)**

   Einstein's Brownian motion theory;

   Diffusion equation;

   Langevin equation;

   Stochastic integration and stochastic differential equations.

8. **Approximation Methods for Diffusion Processes**

   Small noise perturbation (Chapter 7);

   Elimination of fast variables (Chapter 8);

   Beyond the white noise limit (Chapter 9);

   Kramers' proble and barrier crossing (Chapter 14).

8. **Lévy Process and Financial Applications (Chapter 10)**

   Central limit theorem and the origin of Lévy process;

   The Paretian processes;

   Stochastic description of financial markets and the theory of Black-Scholes;

   Fractional Brownian motion and Gaussian processes.