1 Course overview and curriculum content

This course focuses on developing and analyzing dynamics of biological systems and processes, from a fundamental and mechanistic standpoint, in mathematical terms and to better understand their behavior and functions. There is a combination of theories and applications drawn from many branches of biology and medicine. Students will gain the basic knowledge of a unified mathematical theory of biology, as well as experiences in applying differential equations, stochastic processes, and dynamical systems theory to biology.

Additional information for students in 523: As a course at the interdisciplinary boundary between physical and biological/life sciences, the Mathematical Biology courses in the Department of Applied Mathematics routinely expects to have enrollments from graduate students from a wide range of departments on campus: Biology, Bioengineering, Chemistry, Physics, Engineering departments, and Mathematics. So this course AMATH 523 is designed as a parallel to AMATH 423. The in-class instruction portion of the course will be the same as AMATH 423; however, a different learning experience for graduate students is reflected in the level and the challenges in the homework assignments. We also expected the students to have a more significant amount of reading.

AMATH 423/523 this year attempts to introduce a mathematical foundation for biological sciences on a par with mechanics to classical physics. It presents both the fundamentals as well as many applications, through modeling, based on both deterministic and stochastic mathematics. In particular, the course tries to show mathematical biology as a rigorous theory of the Nature just as powerful and legitimate as Newtonian natural philosophy. It begins with a new approach to continuous-time dynamics of populations of individuals, be it chemical species in a biochemical cell, cells in a tissue of cellular organisms in a microbiology setting, or animal and/or plant populations in an eco-system, based on individual's behavior: Dynamics in terms of trajectories and in terms of probability distributions. The topics to be covered are chosen from:

A comparitive study with theoretical mechanics
Continuous-time population dynamics
Foundation in individual's seemingly random behavior
Exponential distributed birth and death, and random walk
Discrete-time population dynamics
Continuous-time interacting population models
Discrete-time models for interacting populations
Enzyme kinetics and dynamics of chemical species
Simple oscillatory reactions
Hodgkin-Huxley theory and neural networks
Poisson process
One-dimensional random walks
Discrete-time Makov chain
Continuous-time Markov process

Birth-and-death process
Coupled oscillators
Epidemiology models and dynamic diseases
Pharmacokinetics and drug action

2 Learning Goals/Objectives

Although the subject matter of mathematical biology can be made difficult, we will attempt to present the course material in as simple a manner as possible. Model development, and how to connect biological problems to mathematical equations, will be emphasized. We will let students know clearly what they need to learn and what can be skipped.

This is a course on modeling of biological and biomedical systems and processes in terms of mathematics, the **graduate students** are expected to be proficient in mathematical problem solving of ordinary differential equations. The course does not require previous knowledge in biology beyond elemetary science background. The goal is to learn how to connect biological problems in words to mathematical formulations in equations.

Graduate students should leave the class knowing several concrete examples of mathematical models for biological systems and processes. In addition, they are expected to be able to generalize the various approaches presented in the class and to development models on their own for other biological problems.

For **graduate students**, homeworks are used to reinforce class lectures, as well as introduce additional mathematical analysis of problems discussed in classes. They serve two purpose: (1) To reinforce the mathematical skills beyond simple ordinary differential equations; and (2) to get their own "hand wet" by working on models developed in the class.

3 Required Texts, readings, films, websites, etc.

Required Reading:

1. I will provide you with several excerptions from the very good "Mathematical Biology: I. An Introduction" (Interdisciplinary Applied Mathematics) by James D. Murray, 3rd Ed., (2007) Springer. The book is available at University bookstore if you are truly interested in this subject; the $75(new)/$56(used) is actually worth spending.

2. "Stochastic Population Kinetics and Its Underlying Mathematicothermodynamics" by H. Qian (2019) In *The Dynamics of Biological Systems*, A. Bianchi et al. eds., Springer, New York, pp. 149–188. Download from course webpage.

Suggested Reading Texts on Stochastic Modeling:

"Random Walks in Biology" by H.C. Berg (1993) Princeton Univ. Press.

"Noise and Fluctuations: An Introduction" by D.K.C. MacDonold (2006) Dover.

"Stochastic Chemical Kinetics: Theory and (Mostly) Systems Biology Applications" by P. Érdi and G. Lente. (2014) Springer.

Reference Text:

"Elements of Mathematical Ecology" by M. Kot, (2001) Cambridge University Press.

"Chemical Biophysics: Quantitative Analysis of Cellular Systems" by D. A. Beard and H. Qian, (2008) Cambridge University Press.

4 Project and Presentation

There is a term project that consists of presentations and term papers. The students are encouraged to work as a group of no more than 3 people. Each student group will give two 10-12min in-class presentations of a paper that applies the modeling and mathematical techniques we have learned in the course (case study). These studies will be developed into final projects.

Two presentations: **Tu. February 25** (and continues if needed onto Th. February 27) in class (literature review and proposal) and **Tu. March 10** (and continues if needed onto Th. March 12) in class (final presentation).

Final term papers are due on the morning, **9AM**, of **Monday March 16**. The paper will be about 15 pages in length. It should be written with the style of a scientific research paper that includes: Introduction, Problem formulation and Models, Solution to Mathematical Models, Discussion, and References. Grading scheme of the term paper is based on the following five criteria, 20% each:

(1) Stylistics, i.e., spelling, grammar, referenecs, and handling of equations.
(2) Clarity of thoughts, writing, and implications.
(3) Originality and novelty and/or thoroughness of review.
(4) Accuracy of the mathematics and validity of the science.
(5) Mathematics, i.e., depth, explanations, exposition, and appropriateness.

5 Evaluation and grading

5-6 homework assignments with total 60% of the final grade. Class participation contributes to 10% of the final grade. A term project in the form of a term paper contributes to 30% of the balance.

6 Course Schedule

Twice a week, 80 mins per class. Tu. and Th. 10:00-11:20am, Loew Hall 106.

7 Important Dates

Frist day of class, Tuesday, January 7.
Tuesday Febeuary 18 and Thursday 20, guest lectures.

Tuesday Febeuary 25 (and possibly Th. Feb. 27) in class presentation on literature review and project plan.

Tuesday March 10 and Thursday March 12, finally presentation.

Term paper due, Monday, March 16.