Homework # 4

1. Prob. 1 on P. 215 of the "Reading material on biochemical reaction kinetics".

2. Prob. 4 on P. 216 of the same "Reading material". Note in the old edition of the text, there is a typo in the second equation: It should be $\frac{dy}{dt}$ not $\frac{dx}{dt}$. You can find the Descartes' rule of signs from online wikipedia.

Studying simple mathematical models like this has been an active area of biological research. For examples see

`http://prola.aps.org/abstract/PRL/v84/i23/p5447_1`
`http://www.pnas.org/cgi/content/abstract/97/13/7148`

3. Consider the system of nonlinear chemical reaction

$$A \xrightarrow{k_1} X, \;\; B + X \xrightarrow{k_2} Y + D, \;\; 2X + Y \xrightarrow{k_3} 3X, \;\; X \xrightarrow{k_4} C,$$

where ks are the rate constants, and the reactant concentrations of chemical species A and B are kept at constant values of a and b, respectively, for all time.

(a) Write the governing differential equation system, according to the *law of mass action*, for the concentrations of X and Y; nondimensionalize the equation so that they becomes

$$\frac{du}{d\tau} = 1 - (\beta + 1)u + \alpha u^2 v, \quad \frac{dv}{d\tau} = \beta u - \alpha u^2 v,$$

in which u and v are the corresponding variables for the concentrations of X and Y, $\tau = k_4 t$, $\alpha = (k_1 a)^2 k_3 / k_4^3$, and $\beta = k_2 b / k_4$.

(b) Determine the positive steady state and show that there is a bifurcation value $\beta_c = 1 + \alpha$ at which the steady state becomes unstable in a Hopf bifurcation way.

(c) Show that in the vincinty of $\beta = \beta_c$, the limit cycle has a period of $2\pi/\sqrt{\alpha}$.