AMATH 383 Homework Assignment #5
(Submitted in class: Feb. 11)

1. Problem 7.2

2. Problem 9.6

   Part b: Comparing Problem 10.1 with Problem 9.1, “Arms races”, Professor Tung seems to suggest that marriage is like a war. Please discuss the implications of these two mathematical models.

4. [Extra credit] Two species are interactive as a predator and a prey. While the predator consumes as much as it can find when food is scarce, it is not unreasonable that, during periods of abundance, the predator satiates and then feeds at a maximum per capita rate $B$, independent of the prey. Let $x(t)$ and $y(t)$ be the population sizes, at time $t$, of prey which feeds on an unlimited food source of its own, and the predator which feeds on the prey. Then if $x(t)$ is sufficiently large, $y(t)$ increases and vice versa. We also assume that $x$ is the sole food source for the predator. This verbal description yields the following mathematical model

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) - \frac{Bxy}{A + x},$$

$$\frac{dy}{dt} = sy \left(1 - \frac{y}{\nu x}\right),$$

where $r$, $K$, $s$, $\nu$, $A$, and $B$ are all positive constants.

(a) How many steady states are there for the system?

(b) Using parameter $K$ as the “unit” for population sizes $x$ and $y$, and use $B^{-1}$ as the “unit” for time, show that the above pair of equations can be simplified into

$$\frac{du}{d\tau} = ku(1 - u) - \frac{uv}{a + u},$$

$$\frac{dv}{d\tau} = \sigma v \left(1 - \frac{v}{\nu u}\right).$$

What are the $k$, $a$ and $\sigma$ in terms of the original parameters in Eq. (1)?

(c) Analyzing the type and stability of the non-trivial steady state of Eq. (2).