AMATH 383 Homework Assignment #5
(Due in class: Nov. 7)

1. If random time T (≥ 0) has a probability density function (pdf) $f_T(t)$, then
$$F_T(t) = \int_0^t f_T(s)\mathrm{d}s$$
is called the cumulative distribution function (cdf) of T. It represents the probability for $T \leq t$: $F_T(t) = \Pr\{T \leq t\}$. Show that if T has an upper bound M: $T \leq M$, then
$$\langle T \rangle = \int_0^\infty \Big[1 - F(t)\Big]\mathrm{d}t.$$

2. If T_1 is a non-negative random time with probability density function $f_1(t)$, then
$$\int_0^\infty f_1(t)\mathrm{d}t = 1.$$
And its mean value (also called expected value) and variance can be obtained from
$$\langle T_1 \rangle = \int_0^\infty t f_1(t)\mathrm{d}t,$$
$$\text{Var}[T_1] = \left(\int_0^\infty t^2 f_1(t)\mathrm{d}t\right) - \langle T_1 \rangle^2.$$
Now consider a second random time T_2 with probability density function $f_2(t)$. If T_1 and T_2 are statistically independent, then the probability density function for the random time $T_3 = T_1 + T_2$ is
$$f_3(t) = \int_0^t f_1(s) f_2(t-s)\mathrm{d}s.$$
(a) When T_1 is an exponentially distributed random time with mean time τ_1, and T_2 is an exponentially distributed random time with mean time τ_2, what is the distribution for T_3?

(b) Same T_1 and T_2 as in (a), what is the mean and the variance of T_3?

3. Let T_1 be a non-negative random time with probability density function $f_1(t)$ and cumultive distribution function $F_1(t)$, and T_2 be a non-negative random time with probability density function $f_2(t)$ and cumultive distribution function $F_2(t)$. Assuming T_1 and T_2 are statistically independent, and consider the random variable
$$T^* = \max\{T_1, T_2\}.$$
(a) What is the probability distribution of T^*?

(b) Can you give a verbal explanation of the two terms you obtained for $f_{T^*}(t)$ in (a)?

(c) If T_1 and T_2 are both exponentially distributed random time with mean values τ_1 and τ_2 respectively, what is the mean time for T^*?

4. Exercise 5 of Chapter 9.