

AMATH 383 Homework Assignment #1

(Due in class: Oct. 10)

1. The first 4 numbers in Fibonacci sequence F_n with $n = 1, 2, \dots$ are 1, 2, 3, 5. Find the values for a, b, c, d in

$$G_n = an + bn^2 + cn^3 + dn^4,$$

such that the first four G_n fit the first four F_n . What is F_5 ? and what is G_5 ? Compare F_n and G_n for $0 \leq n \leq 100$ in a graph. What is the limit

$$\lim_{n \rightarrow \infty} \frac{G_n}{F_n}.$$

2. Exercise 2 of Chapter 1.
3. The populations of two interacting biological organisms X and Y , generation after generation, follow the pattern

$$\begin{cases} x_{n+1} = 6x_n - 4y_n, \\ y_{n+1} = 2x_n, \end{cases}$$

in which x_n and y_n are the population sizes of the n th generation of X and Y , respectively. If $x_0 = y_0 = 1$, find out the populations x_n and y_n for all $n \geq 0$.

Extra credit: There is a very easy way to solve this problem. However, a more systematic way to solve this problem is by assuming a general solution in the form of

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = c_1 \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \lambda_1^n + c_2 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \lambda_2^n$$

and determine the λ 's, a 's, b 's, and c 's. Consult Boyce and DiPrima's *Elementary Differential Equations* for solving a system of linear, constant coefficient, homogeneous first-order ODEs.