CONDITIONAL TIME-VARYING INTEREST RATE RISK PREMIUM: EVIDENCE FROM THE TREASURY BILL FUTURES MARKET

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ABSTRACT

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Existing studies of the term structure of interest rates often use spot Treasury rates to represent default-free interest rates. However, part of the premium in Treasury rates is compensation for the risk that short-sellers may default. Since Treasury bill futures are default-free they provide cleaner data to estimate the interest rate risk premium. The mean excess return in default-free Treasury bill futures is zero. This suggests that the interest rate risk premium could be economically negligible. We find that although the mean unconditional premium is zero, futures returns contain economically and statistically significant *time varying conditional* interest rate risk premiums. The conditional premium depends significantly positively on its own conditional variance and its conditional covariance with the equity premium. The conditional premium is large in the volatile 1979-1982 period, but small afterwards.

1. Introduction

Do investors receive a premium for bearing interest rate risk? If they do, is the premium constant or does it vary systematically with other economic variables? The pure expectations hypothesis of the term structure says that the risk premium is zero, while the expectations hypothesis says that if there is a risk premium it is constant through time (Campbell 1986).

There is empirical evidence that part of the risk premium that researchers have found using spot Treasury rates is not a term premium. Instead it is a premium for counterparty default risk. To buy or sell Treasury forward contracts one has to buy and short-sell Treasury securities with different maturities. Traders in these synthetic forward contracts face the risk that their counter parties may default. Although Treasury securities are default free, short positions in Treasury securities, and hence long and short positions in synthetic forward contracts in Treasury securities, are not default free. In contrast, futures markets have a clearing association that serves as the guarantor of every contract and employs safeguards that virtually eliminate default risk. Kamara (1988) shows that spreads between implied forward Treasury bill rates and Treasury bill futures rates are positive and significantly positively related to measures of default risk, including the standard deviation of the change in spot rates. This implies that spreads between long- and short-term Treasury bill yields contain default premiums.

Fried (1994, p.70) concludes that using the term structure of spot yields alone "may be inappropriate in addressing some questions related to the term structure." Kamara (1997) presents evidence that time-variation in the default premia embedded in spot Treasury bill rates causes real GDP growth to be less correlated with the spot Treasury term structure than with a futures-implied term structure. Consequently, time variation in the spot Treasury term structure results from time variation in both nominal risk-free interest rates and forward default premiums.

Bekaert and Hodrick (2001) present three reasons why previous studies may have rejected the expectations hypothesis of the term structure: the assumptions of rational expectations and unlimited arbitrage may not hold; time-varying risk premiums may not be properly modeled; and peso problems or learning may lead to false rejections in finite samples. Our paper fits into Bekaert and Hodrick's second and third categories. We attempt to improve our understanding and measurement of the economic variables that affect a time varying interest rate risk premium, and we model peso problems in the term structure.

The 1979-82 period of time is often referred to as the "monetary regime" during which the Federal Reserve targeted money aggregates instead of interest rates. It is a period of very volatile interest rates. Absent a peso-problem the estimated coefficients of an econometric model of the term premium should be the same between the regime and non-regime observations. However, if as Lewis (1991) suggests, investors expected that the Federal Reserve might switch back to interest rate targeting, interest rates during the regime would reflect both current economic conditions and the possibility of the Federal Reserve switching back to interest rate targeting. As a result, the relationship between interest rates and contemporaneous economic data might change. Hamilton (1988), Lastrapes (1989), Bekaert, Hodrick and Marshall (2001), and Ang and Bekaert (2002) show that this could cause the regression coefficients of a model of the term premium to differ between the regime and non-regime time periods. We study this issue as well.

In this study we estimate unconditional and conditional term premiums using Treasury bills futures returns. Futures rates should provide cleaner data for studying the term premium. In contrast to the secondary Treasury market, futures markets have clearing associations that employ safeguards that virtually eliminate default on futures contracts. Telser and Higinbotham (1977)), Edwards (1983), and Brennan (1986) provide theoretical motivation for using futures rates instead of implied forwards rates to avoid the default risk inherent in implied forward rates. MacDonald and Hein (1989) and Kamara (1990) find that Treasury bill futures rates are significantly more accurate predictors of future spot rates than are implied forward rates. Poole (1978, p.63) argues that "it is probably not necessary to make any allowance for term premiums when using futures rates to gauge market expectations of future spot rates" We find that the mean Treasury bill futures excess return is economically negligible and statistically indistinguishable from zero. Hence, our unconditional tests do not reject the pure expectations hypothesis, which postulates that term premiums are zero. In contrast, the mean of the corresponding Treasury bill forward premium is economically and statistically significant, rejecting the pure expectations hypothesis.

Zero mean unconditional interest rate risk premiums need not imply that the interest rate risk premium is zero. The conditional interest rate risk premium can vary through time and be nonzero at each point in time, even when the mean unconditional premium is indistinguishable form zero. Hodrick (1987) reports such findings for currency premiums.

We investigate conditional one- and two-factor equilibrium models of the interest rate risk premium. The one-factor model predicts that the interest rate risk premium depends on its conditional variance. Affine models of the term structure (Cox, Ingersoll, and Ross,

1985) suggest that the term premium is positively related to interest rate volatility. Fama (1976), Shiller, Campbell, and Schoenholtz (1983), Engle, Lilien, and Robins (1987), and Klemkosky and Pilotte (1992), among others, document empirically that the interest rate risk premium in spot Treasury rates is related to interest rate variance.

Two lines of research motivate our two-factor model of the interest rate risk premium, which postulates that the term premium is also related to its conditional covariance with the equity premium. Mayers (1972), Stoll (1979), and Hirshleifer (1988) propose conditional two-factor equilibrium models based on CAPM models with non-tradable claims. Merton (1973) develops an intertemporal capital asset pricing model that relates an asset's risk premium to its conditional covariance with the equity market risk premium.

We design our empirical tests so that the futures risk premium and the term premium are identical. We find that both premiums contain a significant time varying conditional interest rate risk premium that depends positively on its conditional variance. In the two-factor model, the premiums also depend positively on its conditional covariance with the equity premium.

Longstaff (2000) presents evidence that at the very short end of the yield curve the pure expectations hypothesis holds. Longstaff used repo rates in his study to avoid the liquidity and secondary market default risk premiums that may be in interest rates on spot-traded Treasury bills. He found that the differences between average overnight repo rates and term repo rates for maturities of one week to three months were economically and statistically insignificant. We also find that our unconditional tests support the pure expectations hypothesis. However, unlike Longstaff, our conditional tests reject the expectations hypothesis theory of the term structure.

Though our tests reject the expectations hypotheses both during and after the monetary regime, the futures data suggest that the expectations hypotheses are a reasonable approximation of the behavior of the term premium outside the monetary regime of 1979-1982. The conditional term premium in futures Treasury bill rates outside the regime is economically small, and is substantially smaller than the conditional term premium in spot rates.

Our conditional tests also reject the affine models of the term structure of Vasicek (1977) and Cox, Ingersoll, and Ross (1985).

Our results also shed light on the wider debate about whether commodity and financial futures prices contain risk premiums. Studies of futures markets, for example, Dusak (1973), Carter, Rausser and Schmitz (1983) and Bessembinder (1992), reach conflicting conclusions regarding the existence of risk premiums in futures prices. We find that the conditional premiums in Treasury bill futures prices are substantial in 1979-1982, but are small afterwards.

Section 2 presents our empirical results and Section 3 our conclusions.

2. Empirical evidence

The Treasury bill futures contract calls for delivery of a 3-month (13-week) Treasury bill with one million dollars face value. We study the excess return on a long position in the Treasury bill futures contract over a quarterly horizon that begins exactly 13 weeks before the futures' delivery day and ends on the futures' delivery day. It is important to notice that we design our tests such that our investment period *always* ends on the futures' delivery date. Consequently, the uncertainty for our investors comes entirely from uncertainty regarding the interest rate on the *spot* 3-month Treasury bill on the delivery date. This is exactly the uncertainty faced by investors who buy six-month spot Treasury bills 3 months before the futures' delivery date, and sell them in the spot market on the futures' delivery date. Hence, in our tests, the futures risk premium is also the term premium.

This is an important point in our empirical design. To clarify, consider an economy with the following three securities today: Spot 3-month Treasury bills, spot 6-month Treasury bills, and a futures contract calling for delivery, 3 months from today, of one 3-month Treasury bill. Assume that all three securities have a face value of one dollar. Investors who wish to invest for 3 months can choose one of the following strategies:

1. Today, buy spot 3-month Treasury bills at a cost of $P_{3,0}$ per unit. After 3-months, redeem the bills for their face value. This is the riskless strategy. Let r_f denote the rate of return on this strategy.
2. Today, buy spot 6-month Treasury bills at a cost of $P_{6,0}$ per unit. After 3-months, sell them as 3-month bills for the spot price of $P_{3,1}$ per unit. The *excess* return on this strategy is $R_{6m3} = \frac{P_{3,1} - P_{6,0}}{P_{6,0}} - r_f$.
3. Today, buy one Treasury bill futures contract today with a price of F_0. After 3-months, pay F_0, accept delivery of the 3-month bill, and immediately sell it for $P_{3,1}$. Because the futures position does not require any initial cash flow, the *excess* dollar

return on Strategy 3 is ($P_{3,1} - F_0$). We define the excess rate of return on Strategy 3 as

$$R_{fut} = \frac{P_{3,1} - F_0}{F_0}.$$[1]

The excess return on strategy 2 is a forward rate measure of the term premium. The excess return on strategy 3 is the futures' risk premium. Strategies 2 and 3 contain the same risk – the uncertain price of $P_{3,1}$. Hence, when investors hold the Treasury bill futures contract to delivery, the futures' risk premium and the term premium are one and the same.[2]

2.1. Data

We assembled a sample of spot and futures rates on three-month Treasury bills from 1976 through 1998.[3] The spot rates are bid discount rates from the daily quote sheets of the Federal Reserve Bank of New York. The Treasury bills futures contracts, which began trading in 1976, expire on a quarterly basis (March, June, etc.). The futures rates are settlement discount rates on the nearest-maturity futures contracts observed exactly thirteen weeks before their delivery dates.[4] We collected the futures rates from various issues of the *International Monetary Market Yearbook* and from data kindly provided by the Chicago Mercantile Exchange.

Futures contracts are marked-to-market daily. As a result, the Treasury bill futures rate is less than its corresponding forward rate by what is termed "a convexity adjustment" (Hull 1999, p.108). The daily marking-to-market resets the value of the futures contract to zero each day. Consequently, in equilibrium, the futures rate must equal the risk neutral expectations of the underlying spot rate on the futures' delivery date

(Grinblatt and Jegadeesh 1996, provide a formal derivation). Our paper tests the expectations hypotheses and the possible risk premium in the Treasury bill futures market. Consequently, the absence of convexity in futures rates actually makes our tests “cleaner” than tests using forward interest rates. Put differently, the risk premium estimated using forward bill rates includes a convexity bias, whereas the risk premium estimated using futures bill rates does not. This difference could be important when interest rates are volatile because, like the term premium, the convexity bias is related to the variance of short-term interest rates (Campbell 1986).

2.2. Unconditional tests

Existing term structure studies document large term premiums in spot Treasury bill rates. However, Poole (1978), Kamara (1998, 1997), Fried (1994) and Longstaff (2000) argue that this is a premium for secondary market default and liquidity risks and not just a premium for interest rate risk. Poole (1978), Kamara (1998, 1997), Fried (1994) suggest that Treasury bill futures rates are more adequate measures of interest rate risk premiums.

Table 1 presents four unconditional tests of the expectations hypothesis. We measure the term premium in Treasury bill futures as $R_{fut} = (P_{3,1} - F_0)/F_0$.[5] For the entire sample the mean, annualized Treasury bill futures excess return is 0.0007 (i.e., 7 basis points) per year, with a t-statistic of 0.42, and a p-value of 0.68. Thus, the mean unconditional interest rate risk premium is economically small and statistically indistinguishable from zero. Stated alternatively, futures rates are unbiased forecasts of future spot rates. The conclusions for the 1979-82 monetary regime are similar. Thus, the results of the unconditional test using futures excess returns are consistent with the pure expectations hypothesis.

Table 1 also shows summary statistics for *excess* returns on Strategy 2 above, of buying spot 6-month Treasury bills and selling them after 3-months (denoted R_{6m3}). For the entire sample, the average excess return is 50 basis points with a p-value of 0.007, which suggests that the forward term premium is economically and statistically positive. Hence, the results of the unconditional test using spot excess returns reject the pure expectations hypothesis. During the 1979-82 regime the average excess return is much higher (94 basis points), but with a t-statistic of only 0.76 reflecting the substantially larger increase in the standard error.

While the standard errors of the futures and forward term premiums are about equal, the average term premiums are quite different. The difference in these two estimates of the term premium reinforces the point that something affects the average level of spot rates that does not affect futures rates. Based on existing literature, this difference may be a short-seller default risk premium that affects spot bill rates but not futures rates.

The dashed line in Figure 1 shows that the futures excess return varies considerably over time, and is sometimes very large, especially in the Federal Reserve's alternative monetary regime that ran from late 1979 through late 1982. The lower panel of Table 1 shows summary statistics for the futures and forward term premiums during the reserve-targeting monetary regime. During the regime, the standard errors of both the futures and forward term premiums increased by a factor of about 2.5. In spite of this increased uncertainty, the mean futures term premium remains indistinguishable from zero. Thus, even in this period of high interest rate uncertainty, the average futures term premium is consistent with the pure expectations hypothesis. In contrast, the mean forward term premium almost doubled from 50 to 94 basis points. This increase may be partially due to greater counterparty default risk in the secondary Treasury bill market during the regime caused by the greater uncertainty of bill rates. Note that because of the increase in its standard error, the forward term premium is not different from zero during the regime.

We now explore the question of whether despite having zero mean unconditional premium, Treasury bill futures rates contain a time varying conditional interest rate risk premium.

2.3 Conditional univariate tests

Longstaff (2000) tested the expectations hypothesis of the yield curve by regressing the term premium, which he estimated using repurchase rates, on the long-term repo rate that was known at the beginning of the period. For a variety of repurchase contract maturities ranging from one-week to three-months, he found no statistically significant relationships between the term premium and the term repo rate. He viewed this as support for the expectations hypothesis. Our conditional univariate tests differ from Longstaff's in three ways: we estimate the term premium using futures and spot Treasury bill rates instead of repo rates; we use interest rate volatility instead of the level of the interest rate as our regressor; and our sample periods differ (Longstaff examines the period of 1991-1999).

To relate our study of the term premium to Longstaff's, we estimate his regression using both R_{fut}, and R_{6m3}, the futures and forward term premiums.

$$R_{i,t} = \alpha_0 + \alpha_1 \cdot y_{6,t} + \varepsilon_{i,t} \qquad i = fut,\ 6m3 \qquad (1)$$

Where y_6 is the yield on the 6-month bill (the corresponding long-term in our data).

Because of the possibility of a peso problem we allow the coefficients to change during the 1979-82 regime. We do this by adding a binary variable to the regression that has the value one during the regime and the value zero otherwise. We include the binary variable by itself to allow the intercept to change, and we add the product of the binary variable and the 6-month bill yield to allow the slope coefficient to change. Lewis (1991) reports that political pressure to reduce and stabilize interest rates may have caused investors to expect the Federal Reserve to revert back to an interest rate targeting monetary policy. If so, during the regime investors may have thought that interest rates were going to decrease from their high regime levels to the lower pre-regime levels. As a result, longer term interest rates during the regime may have been lower than they would have been had investors thought that the Federal Reserve would continue its reserve-targeting monetary policy. Thus, the coefficients that describe the term premium during the regime may not describe it outside of the regime.

Table 2 reports the results.[6] According to the pure expectations hypothesis, both the intercept and slope coefficient should be zero. If the expectations hypothesis describes the data, the intercept can be non-zero, but the slope coefficient should be zero. Using futures rates to estimate the term premium, during the entire sample the estimated intercept is –0.0007 with a p-value of 0.87, and the slope coefficient is 0.02 with a p-value of 0.75. These estimates are consistent with the pure expectations hypothesis. In contrast, during the regime period the intercept is significantly negative and the slope coefficient in the futures term premium regression is significantly positive. The null hypothesis that the sum of the intercept and the coefficient on the regime dummy are zero, and the null hypothesis that the sum of the coefficient on y_6 and the coefficient on the product of y_6 and the regime dummy are zero, are rejected, separately and jointly, at near-zero levels. Thus, during the regime the expectations hypothesis does not explain the futures term premium. The 6-month bill yield observed at the start of the quarter has predictive power for the futures excess return observed at the end of the quarter.

Using the forward term premium, the regression estimated over the entire sample yields an intercept of 0.0005 with a p-value of 0.90, and a slope coefficient of 0.058 with a p-value of 0.41. These results are consistent with the pure expectations hypothesis. During the regime the intercept becomes significantly negative and the slope coefficient significantly positive. These results are inconsistent with the expectations hypothesis.

These results suggest that the behavior of the term premiums changed during the regime: the 6-month Treasury bill rate at the beginning of the quarter predicts the subsequent excess returns during the regime, but not for the entire sample. The pure expectations hypothesis holds outside of the regime but neither it nor the expectations hypothesis hold in the regime.

Economic theory (Cox, Ingersoll and Ross 1985) and empirical studies (Fama 1976, Shiller, Campbell, and Schoenholtz 1983, Engle, Lilien, and Robins 1987, and Klemkosky and Pilotte 1992) suggest that the term premium is positively related to interest rate volatility. We test this relation for both the R_{fut}, and R_{6m3}.

$$R_{i,t} = \Box_0 + \Box_1 \cdot h_{i,t}^2 + \Box_{i,t} \qquad i = fut,\ 6m3 \tag{2}$$

The realized risk premium, $R_{i,t}$, is the expected risk premium, which is modeled as a linear function of its conditional variance, $h_{i,t}^2$, plus a random error, $\Box_{i,t}$.[7]

We estimate eq. (2) assuming that the time varying conditional volatility of Treasury bill rates follows the square root augmented GARCH(1,1) process proposed by Gray (1996) and Bekaert, Hodrick, and Marshall (1997). Here, $R_{f,t-1}$ is the lagged riskless (3-month) Treasury bill rate.

$$h_{i,t}^{2} = a_{1} \cdot \varepsilon_{i,t-1}^{2} + a_{2} \cdot h_{i,t-1}^{2} + a_{3} \cdot R_{f,t-1} \qquad i = fut,\ 6m3 \qquad (3)$$

GARCH models describe the time varying conditional variances by an autoregressive, moving average process of past squared shocks. In GARCH-M models, the conditional mean model, the error term and the conditional variance model are estimated jointly. The square root augmented GARCH(1,1) adds an interest rate level effect, as in the square root process of Cox, Ingersoll and Ross (1985), to the GARCH effects.[8] This allows the variance to increase and decrease along with the level of interest. Including the lagged level of the interest rate helps to accommodate the considerable shift in interest rate volatility during the 1979-1982 monetary regime. Still, because of the possibility of a peso problem we allow the coefficients in both (2) and (3) to change during the regime. We do this by including the regime binary variable and the product of the binary variable with each regressor. Hamilton (1988) found that if the heteroskedasticity in the variance of Treasury bill rates is modeled as a linear function of the lagged bill rate, the coefficients must be allowed to change during the regime.[9] Lastrapes (1989) presents evidence that the monetary regime affected the intercept in the conditional variance equation for nominal exchange rates. We go one step further and also allow the slope coefficients in the GARCH equation to change.

Table 3 reports estimates of the GARCH-M model.[10] The table contains four sets of estimates: two for the futures premium, one for the entire sample and one for the change inside the regime; and two for the forward term premium, one for the sample and one for the change inside the regime. The key result is that during the entire sample and the regime, $\square_1$, the estimated coefficient of the variance of the futures term premium in the term premium regression is positive and significant with a p-value near zero. Based on this estimate, the conditional interest rate risk premium in the Treasury bill futures market covaries positively with its conditional time-varying variance.[11] This model rejects the expectations hypothesis of the term structure, which requires that if there is an interest rate risk premium, it is constant.

The coefficients of the futures term premium regression change considerably during the monetary regime. The intercept decreases and the slope coefficient increases. During the regime both the variance of the excess return, the regressor, and its coefficient increased. Together these caused a large increase in the conditional premium.

The estimated values of a_1, a_2 and a_3 support the assumption that the conditional variance follows an autoregressive moving average process of squared shocks.[12] The estimates of each of the GARCH terms are significant at conventional levels. Furthermore, the interest rate level effect, a_3, is significantly positive with a serial correlation and heteroskedasticity consistent t-statistic greater than 4. During periods of historically high interest rates, interest rate volatility is also high, as is the term premium. The coefficients of the GARCH process, though, also change significantly during the regime.

The solid line in Figure 1 plots the estimated time-varying, conditional, futures term premium. It increased from an average of about zero in the non-regime period, to as high as 11 per cent during the regime. One possible explanation for the large premiums in

1979-1982 may be the high leverage of futures contracts. For example, initial margins in 1982 were equivalent to a price change of about 0.40%. The plot shows that speculators often suffered substantial losses during the regime. It is also useful to note that the corresponding riskless returns on the underlying bill were unusually high and unusually volatile during 1979-1982, fluctuating between 7-17%.

The solid and dashed lines in Figure 1 show the different patterns of the projected risk premiums and realized excess returns in the 1979-1982 sub-period. While the expected premiums are positive and economically significant, the mean realized premium in 1979-1982 is economically and statistically negligible (about minus 3 basis points). The realized excess returns are those earned by investors (speculators) who bought the futures contracts and held them to delivery 3 months later. The realized excess return plot suggests that long speculators were often surprised by the realizations of interest rates in 1979-1982. Because the 1979-1982 regime is a short and unprecedented period this finding should not be viewed as evidence of market inefficiency. One possible explanation of the speculators' occasional large losses is that they may have expected the Federal Reserve to revert to an interest rate targeting policy. Anticipating such a policy might have led speculators to forecast a decrease in interest rates and gain on long futures contracts. The Federal Reserve may not have reverted to interest rate targeting as soon as the speculators anticipated it would. Once the Federal Reserve did revert to an interest rate targeting policy, speculators losses decreased.

The third and fourth columns of Table 3 report our estimated coefficients of the GARCH-M model of the forward term premium using R_{6m3} to measure the premium. Note that the coefficient of the conditional variance estimated using forward premiums during the non-regime period is 10.91, which is about 40 times the coefficient estimated using futures premiums, 0.266. The existing literature suggests that the coefficient of the variance of the forward term premium may be due to a combination of default and interest rate risk premia. Thus, the forward rate regression may give a biased estimate of the interest rate risk premium.

We find that both futures and spot rates contain significantly positive conditional premiums that covary positively with conditional interest rate variance. This is especially true during the monetary regime of 1979-1982. Outside the regime, however, conditional futures' premiums are substantially smaller than conditional spot premiums, [13] and appear to be less affected by fluctuations in conditional interest rate variance.

The estimates above also provide for a test of the term structure models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985). Vasicek (1977) postulates that that the term premium is constant, implying that $\square_1$ = 0 in eq. (2). The affine model of Cox, Ingersoll, and Ross (1985) postulates that the term premium is proportional to its variance, implying that $\square_0$ = 0 in eq. (2); and that the variance depends only on the level of the interest rate, implying that $a_1 = a_2 = 0$ in eq. (3).[14] Using the futures data we reject the Vasicek (1977) model, and the Cox, Ingersoll, and Ross (1985) model. The Cox et al. model is rejected because the variance of the term premium depends on its past values in addition to the level of the interest rate.[15] The inferences using forward premiums differ little from those based on futures premiums. Using R_{6m3}, we also reject both affine term structure models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985).

To summarize, we find that although the mean unconditional interest rate risk premium measured using futures rates is indistinguishable from zero; futures prices contain significant time varying conditional term premiums that relate significantly positively to their time varying conditional variance. This relation is similar to the one found in studies investigating spot Treasury quotes [e.g., Fama (1976), Shiller, Campbell, and Schoenholtz (1983), Engle, Lilien, and Robins (1987), and Klemkosky and Pilotte (1992)]. Consequently, our empirical evidence using futures rates strengthens our confidence in the spot market evidence that Treasury rates contain interest rate risk premium that covary positively with interest rate variance. Nevertheless, while our tests reject the expectation hypotheses, the futures data suggest that the expectations hypotheses are a reasonable approximation of the behavior of the term premium outside the monetary regime of 1979-1982.

2.4 Bivariate conditional tests

The univariate tests reported above assumed that the risk premium depends only on its underlying volatility. Yet, two important asset pricing theories imply that the risk premium in futures prices should also be related to the covariance of its return with some real variable, such as the equity market risk premium, rather than to just the variance of its return.[16] In this section we attempt to gain further insights into the determinants of the term premium by estimating a two-factor asset-pricing model.

A first strand of equilibrium asset pricing models include Mayers (1972), Stoll (1979) and Hirshleifer (1988). These CAPM-based models assume that some claims are non-tradable and that participation in futures markets is limited because of significant entry costs. These theories postulate that the equilibrium futures' risk premium depends on its covariance with the equity market risk premium, in addition to its residual (non-market) risk. The residual risk is the component of the risk premium's variance that remains after accounting for the covariance between the futures or forward premium and the equity market's risk premium. Thus, they postulate the following conditional two-factor model

$$R_{i,t} \;=\; \square_0 + \square_1 \cdot h^2_{i,t} + \square_2 \cdot h_{im,t} + \varepsilon_{i,t} \qquad\qquad i = fut,\ 6m3 \qquad (4)$$

Here, $h_{im,t}$ is the conditional covariance between the futures or forward premium and equity market's excess return. The two-factor model extends the one-factor model by adding a risk premium that depends on the conditional covariance with the equity premium.

A second strand of equilibrium asset pricing models that postulate the relation described by eq. (4) is a two-factor version of Merton's (1973) intertemporal capital asset pricing model. Merton's model says that the equilibrium risk premium of any asset includes a premium for systematic-market risk, as in the static CAPM. For futures' risk premiums, this component depends linearly on the covariance of the futures' risk premium with the equity market risk premium. In addition, Merton's (1973) model shows that the risk premium of any asset also depends linearly on its covariances with the state variables that determine the stochastic investment opportunity set. The two-factor version of Merton's model assumes that a single state variable fully describes the stochastic investment opportunity set. Merton suggests using a (Treasury) security whose return is perfectly negatively correlated with the future riskless (in terms of default) interest rate as a "single (instrumental) variable representing shifts in the investment opportunity set" (p. 879). Since the long futures position in our sample is held to delivery, R_{fut} is perfectly negatively correlated with the three-month Treasury bill rate. Hence, R_{fut} can proxy for that single state variable, and a two-factor Merton's model also implies the linear risk-return relation described in eq. (4).

Both strands of equilibrium models summarized above hypothesize that the coefficient of the systematic risk premium component in (4), λ_2, is positive.

Let R_m denote the quarterly excess return on the equity market portfolio. We measure it as the return on the Standard and Poor's 500 portfolio minus the riskless return on the corresponding Treasury bill. The dates of the equity excess returns exactly match the dates of the futures excess returns. Standard and Poor's 500 data are from the Center for Research in Security Prices (CRSP).

To estimate eq. (4) we must specify an empirical model for the conditional variance of the equity risk premium. We base the equity excess return specification on the empirical findings of Glosten, Jaganathan and Runkle (1993) and Scruggs (1998):

$$R_{m,t} = \lambda_0 + \lambda_1 \cdot \mathrm{h}^2_{m,t} + \lambda_2 \cdot \mathrm{R}_{\mathrm{f,t}} + \varepsilon_{m,t} \quad (5)$$

$$h^2_{m,t} = \mathrm{b}_0 + \mathrm{b}_1 \cdot \varepsilon^2_{m,t-1} + \mathrm{b}_2 \cdot \mathrm{h}^2_{\mathrm{m,t-1}} + \mathrm{b}_3 \cdot \mathrm{R}_{\mathrm{f,t-1}} \quad . \quad (6)$$

The equity excess return, R_m, depends on its time varying conditional variance, h^2_m , and on R_f, the riskfree (3-month Treasury bill) rate. Fama and Schwert (1977), Campbell (1987), Ferson (1989), Glosten, Jaganathan and Runkle (1993) and Scruggs (1998) report a strong negative empirical relation between the equity market monthly risk premium and the nominal Treasury bill yield. The GARCH(1,1)-M specification of h^2_m is augmented to include the lagged 3-month Treasury bill yield. Campbell (1987), Glosten, Jaganathan and Runkle (1993) and Scruggs (1998) also report a strong positive empirical relation between the volatility of the equity market excess returns and the nominal Treasury bill yield.[17]

Lastly, to make the estimation tractable we assume, similar to Scruggs (1998), that the conditional correlation between the two residuals is constant.

$$h_{im,t} = c_1 \cdot h_{i,t} \cdot h_{m,t} \qquad i = fut,\ 6m3 \qquad (7)$$

We continue to assume that the conditional futures' variance follows the square root augmented GARCH(1,1)-M process, eq. (3), as in the one-factor case.

We do not report results of tests that allow the coefficients to change during the regime. The reason is that with the large number of coefficients in (3)-(7) our sample size is not sufficiently large to obtain reliable estimates of (3)-(7) with additional variables for the regime (not even when we allow only the coefficients of the futures' equations (4) and (3) to change). Hence, the results in this section regarding the magnitude of the coefficients should be interpreted with great caution. It is important to emphasize, however, that allowing the univariate GARCH) coefficients in (2) and (3) to change during the regime did not affect our conclusions for the entire sample.[18]

Table 4 presents estimates of the conditional two-factor, bivariate GARCH-M model using both futures and forward term premiums.[19] The futures risk premium continues to depend significantly positively on its conditional variance, $\square_1$ is significantly positive. In addition, the estimate of $\square_2$ is significantly positive. Hence, we also find a positive empirical relation between the term premium and its conditional covariance with the equity premium. These results reject the expectations hypothesis of the term structure, which requires that if there is a risk premium that it is constant. While not reported, for brevity, the plot of the conditional futures' risk premium estimated using the two-factor model is similar to the plot of the conditional futures' risk premium estimated using the one-factor model, which is shown in Figure 1.

The results regarding the conditional equity risk premium are consistent with existing literature (Scruggs, 1998). The equity excess return relates significantly positively to its conditional variance, and significantly negatively to the Treasury bill yield, at less than a 0.00001 level in each case.[20] In addition, we also find a significant positive relation between the conditional variance of the equity premium and the nominal Treasury bill yield.

Table 4 also reports estimates using the forward term premium. These results are qualitatively similar to those estimated using the futures term premium.

3. Summary

The literature (Poole 1978, Kamara 1998, 1997, Fried 1994, and Longstaff 2000) suggests that the spot Treasury bill term premiums are biased estimates of the interest rate risk premium because they also include premiums for liquidity risk and for the risk that short sellers will default. In contrast, term premiums estimated using futures rates are more accurate measures of interest rate risk premiums because they do not include any default premium.

Moreover, because the daily marking-to-market of futures contracts resets their value to zero each day, futures rates do not include a convexity bias, and must be equal, in equilibrium, to the risk neutral expectations of the underlying spot rate on the futures' delivery date (Grinblatt and Jegadeesh 1996).

We have presented an investigation of the expectations hypothesis of the term structure and the possible risk premiums using futures Treasury bill rates. The pure expectations hypothesis says that futures rates equal expected future spot rates. Unconditional tests are consistent with the pure expectations hypothesis. The three-month futures rate is an unbiased forecast of the future spot rate. In contrast, the unconditional forward term premium in spot rates is economically and statistically positive.

Conditional tests of the futures' term premium, however, reject the expectations hypothesis. They reveal that futures rates contain a conditional risk premium that varies systematically through time. The conditional futures premium covaries positively with the conditional volatility of interest rates. This result holds over the 1979-1983 monetary regime sub-sample, as well as over our entire sample, 1976:I-1998:IV, albeit with substantially smaller coefficients. In addition, the conditional futures premium also covaries positively with the return on the stock market.

Though our tests reject the expectations hypotheses, the futures data suggest that the expectations hypotheses are a reasonable approximation of the behavior of the term premium outside the monetary regime of 1979-1982. The conditional term premium in futures Treasury bill rates outside the regime is economically small, and is substantially smaller than the conditional term premium in spot rates. Fluctuations in the conditional volatility of interest rates appear to have a much smaller effect on the conditional futures' premium than on the forward premium. This is consistent with the forward premium being an upwardly biased measure of the term premium.

Lastly, our conditional tests also reject the affine term structure models of Vasicek (1977), which predicts that the term premium is constant, and Cox, Ingersoll, and Ross

(1985), which predicts that the term premium is an affine function of interest rate volatility.

Table 1. Annualized Term Premium Measured Using Treasury Bill Futures and Implied Forward Rates, 1976:I-1998:IV

	R_{fut} - Futures term premium	R_{6m3} - Forward term premium
Mean	0.0007 (7 bps)	0.0050 (50 bps)
Standard Error	0.0162	0.0173
T-stat	0.4181	2.7559
P-value	0.6768	0.0071
Sub-period of September 1979-December 1982		
Mean	−0.0003 (−3 bps)	0.0094 (94 bps)
Standard Error	0.0414	0.0445
T-stat	−0.0231	0.7602
P-value	0.9819	0.4618

R_{fut} is the annualized futures' (quarterly) term premium, calculated as the percent change in the futures price from 3 months to delivery until the delivery day. R_{6m3} is the annualized (quarterly) forward term premium calculated as the excess rate of return from buying the 6-month spot bill and selling it 3 months later.

Table 2. Estimates of a time-varying interest rate risk premium using Longstaff's model, 1976:I-1998:IV

$$R_{i,t} = \square_0 + \square_1 \cdot y_{6,t} + \square_{i,t}, \qquad i = fut, 6m3$$

Coefficient	Estimated Coefficients using Futures Premium, R_{fut} (p-value)		Estimated Coefficients using Implied Forward Premium, R_{6m3} (p-value)	
	Entire Sample	Coefficient Change During Regime	Entire Sample	Coefficient Change During Regime
$\square_0$	-0.0007 (0.8734)	-0.1152 (0.0000)	0.0005 (0.9041)	-0.1149 (0.0005)
$\square_1$	0.0232 (0.7500)	0.8623 (0.0000)	0.0575 (0.4130)	0.8911 (0.0001)

R_{fut} is the annualized futures' (quarterly) term premium, calculated as the percent change in the futures price from 3 months to delivery until the delivery day. R_{6m3} is the annualized (quarterly) forward term premium calculated as the excess rate of return from buying the 6-month spot bill and selling it 3 months later. The term y_6 is the 6-month Treasury bill yield. Heteroskedasticity and autocorrelation consistent two-tailed p-values are in parentheses.

Table 3. Interest rate level augmented GARCH-M model with an indicator variable for the 1979-1982 monetary regime, 1976:I-1998:IV.

$$R_{i,t} = \square_0 + \square_1 \cdot h^2_{i,t} + \square_{i,t}, \qquad i = fut, 6m3$$

$$h^2_{i,t} = a_1 \cdot \square^2_{i,t-1} + a_2 \cdot h^2_{i,t-1} + a_3 \cdot R_{f,t-1}$$

Coefficient	Estimated Coefficients using Futures Rates, R_{fut} (p-value)		Estimated Coefficients using Implied Forward Rates, R_{6m3} (p-value)	
	Entire sample	Coefficient change during regime	Entire sample	Coefficient change during regime
$\square_0$	0.0005 (0.0000)	−0.0320 (0.0000)	0.0039 (0.0000)	−0.0323 (0.0000)
$\square_1$	0.2660 (0.0000)	15.6613 (0.0000)	10.9106 (0.0000)	3.0124 (0.0000)
a_1	0.4365 (0.0000)	−0.8091 (0.0000)	−0.1035 (0.0000)	−0.3832 (0.0000)
a_2	0.1492 (0.0000)	−0.0850 (0.0000)	−0.0099 (0.0000)	0.0389 (0.0000)
a_3	0.0004 (0.0000)	0.0166 (0.0000)	0.0007 (0.0000)	0.0243 (0.0000)

R_{fut} is the annualized futures' (quarterly) term premium, calculated as the percent change in the futures price from 3 months to delivery until the delivery day. R_{6m3} is the annualized (quarterly) forward term premium calculated as the excess rate of return from buying the 6-month spot bill and selling it 3 months later. R_f is the riskfree (3-month spot Treasury bill) rate. Heteroskedasticity and autocorrelation consistent two-tailed p-values are in parentheses.

Table 4. Bivariate interest rate level augmented GARCH-M model for the Treasury bill futures' interest rate risk premium, 1976:I-1998:IV.

$$R_{i,t} = \alpha_0 + \alpha_1 \cdot h^2_{i,t} + \alpha_2 \cdot h_{im,t} + \varepsilon_{i,t}, \qquad i = fut, 6m3$$

$$R_{m,t} = \beta_0 + \beta_1 \cdot h^2_{m,t} + \beta_2 \cdot R_{f,t} + \varepsilon_{m,t}$$

$$h^2_{i,t} = a_1 \cdot \varepsilon^2_{i,t-1} + a_2 \cdot h^2_{i,t-1} + a_3 \cdot R_{f,t-1}$$

$$h^2_{m,t} = b_0 + b_1 \cdot \varepsilon^2_{m,t-1} + b_2 \cdot h^2_{m,t-1} + b_3 \cdot R_{f,t-1}$$

$$h_{i,m} = c_1 \cdot h_{i,t} \cdot h_{m,t}$$

Estimated coefficient (p-value)	Futures risk premium	Forward risk premium
α_0	−0.0008 (0.0037)	−0.0015 (0.0000)
α_1	5.3111 (0.0000)	0.2512 (0.0000)
α_2	25.8770 (0.0000)	30.7798 (0.0000)
β_0	−5.9629 (0.0000)	−0.4752 (0.0000)
β_1	78.6341 (0.0000)	16.8061 (0.0000)
β_2	−2.7892 (0.0000)	−12.2230 (0.0000)
a_1	1.1999 (0.0000)	0.7726 (0.0000)
a_2	−0.0776 (0.0000)	−0.0204 (0.0000)
a_3	0.0002 (0.0000)	0.0004 (0.0000)
b_0	0.0768 (0.0000)	0.0284 (0.0000)

b_1	0.0072	0.0371
	(0.0000)	(0.0000)
b_2	0.0033	0.1495
	(0.3266)	(0.0000)
b_3	0.0203	0.5567
	(0.0000)	(0.0000)
c_1	0.0510	0.0984
	(0.0000)	(0.0000)

R_{fut} is the annualized futures' (quarterly) term premium, calculated as the percent change in the futures price from 3 months to delivery until the delivery day. R_{6m3} is the annualized (quarterly) forward term premium calculated as the excess rate of return from buying the 6-month spot bill and selling it 3 months later. R_m is the annualized (quarterly) excess return on the S&P 500 index. R_f is the riskfree (3-month spot Treasury bill) rate. Heteroskedasticity and autocorrelation consistent two-tailed p-values are in parentheses.

References

Anderson, Ronald W., and Jean-Pierre Danthine. (1983) "Hedger diversity in futures markets." *Economic Journal,* 93, 370-389.

Ang, Andrew and Geert Bekaert, (2001) "Stock return predictability: Is it there?" Manuscript, Columbia University.

Ang, Andrew and Geert Bekaert. (2002) "Regime switches in interest rates." *Journal of Business & Economic Statistics,* 20, 163-182.

Bekaert, Geert, Robert J. Hodrick, and David A. Marshall. (1997) "On biases in tests of the expectations hypothesis of the term structure of interest rates." *Journal of Financial Economics,* 44, 309-348.

Bekaert, Geert, Robert J. Hodrick, and David A. Marshall. (2001) "Peso problem explanations for term structure anomalies." *Journal of Monetary Economics,* 48, 241-270.

Bekaert, Geert and Robert J. Hodrick. (2001) "Expectations Hypotheses Tests." *Journal of Finance,* 56, 1357-1394.

Berndt, E. K., B. H. Hall, R. E. Hall, and J. A. Hausman. (1974) "Estimation and Inference in nonlinear structural models." *Annals of Economics and Social Measurement,* 3/4, 653-665.

Bessembinder, Hendrick. (1992) "Systematic risk, hedging pressure, and risk premiums in futures markets." *Review of Financial Studies,* 5, 637-669.

Board of Governors of the Federal Reserve System. (1976 through 1991) *Federal Reserve Bulletin.* Washington, DC).

Bollerslev Tim, and Jeffrey M. Wooldridge. (1992) "Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances." *Economtric Reviews,* 11, 143-172.

Breeden, Douglas T. (1980) "Consumption risk in futures markets." *Journal of Finance*, 35, 503-520.

Brennan, Michael J. (1986) "A theory of price limits in futures markets." *Journal of Financial Economics,* 16, 213-233.

Campbell, John Y. (1986) "A Defense of traditional hypotheses about the term structure of interest rates." *Journal of Finance,* 41, 183-193.

Campbell, John Y. (1987) "Stock returns and the term structure." *Journal of Financial Economics,* 18, 373-399.

Carter, Colin A., Gordon C. Rausser, and Andrew Schmitz. (1983) "Efficient asset portfolio and the theory of normal backwardation." *Journal of Political Economy,* 91, 319-331.

Chicago Mercantile Exchange. (1976 through 1991) *International Monetary Market Yearbook.* Chicago, IL

(1981) "The relation between forward prices and futures prices." *Journal of Financial Economics,* 9, 321-46.

Cox, John C., Jonathan E. Ingersoll, Jr., and Stephen A. Ross. (1985) "A theory of the term structure of interest rates." *Econometrica,* 53, 385-407.

Duarte, Jefferson. (2003) "Evaluating alternative risk preferences in affine term structure models." *Review of Financial Studies,* forthcoming.

Dusak, Katherine. (1973) "Futures trading and investor returns: An investigation of commodity market risk premiums." *Journal of Political Economy,* 81, 1387-1406.

Edwards, Franklin R. (1983) "The clearing association in futures markets: guarantor and regulator." *Journal of Futures Markets,* 3, 369-392.

Engle, Robert F., and Kenneth F., Kroner. (1995) "Multivariate simultaneous generalized ARCH." *Econometric Theory,* 11, 122-150.

Engle, Robert F., David M. Lilien, and Russell P. Robins. (1987) "Estimating time-varying premia in the term structure: The ARCH-M model." *Econometrica,* 55, 391-407.

Fama, Eugene F. (1976) "Inflation uncertainty and expected returns on Treasury bills." *Journal of Political Economy,* 84, 427-448.

Fama, Eugene F., and William G. Schwert. (1977) "Asset returns and inflation." *Journal of Financial Economics*, 5, 115-146.

Ferson, Wayne E. (1989) "Changes in expected security returns, risk, and the level of interest rates." *Journal of Finance,* 44, 1191-1217.

Fried, Joel. (1994) "U.S. Treasury bill forward and futures prices." *Journal of Money Credit and Banking,* 26, 55-71.

Glosten, Lawrence R., Ravi Jaganathan, and David E., Runkle. (1993) "On the relation between the expected value and the variance of the nominal excess return on stocks." *Journal of Finance,* 48, 1779-1801.

Gray, Stephen F. (1996) "Modeling the conditional distribution of interest rates as a regime-switching process." *Journal of Financial Economics,* 42, 27-62.

Grinblatt, Mark, and Narasimhan Jegadeesh. (1996) "The relative price of Eurodollar futures and forward contracts." *Journal of Finance*, 51, 1499-1522.

Hamilton, James D. (1988) "Rational-expectations econometric analysis of changes in regimes: An investigation of the term structure of interest rates." *Journal of Economic Dynamics and Control,* 12, 385-423.

Hirshleifer, David. (1988) "Residual Risk, Trading costs, and commodity futures risk premia." *Review of Financial Studies,* 1, 173-193.

Hodrick, Robert J. (1987) *The empirical evidence on the efficiency of forward and futures foreign exchange markets.* New York, NY: Harwood Academic Publishers.

Hull, John C. (2000) *Options, futures and & other derivatives*, 4th ed. Upper Saddle River, NJ: Prentice-Hall.

Kamara, Avraham. (1988) "Market trading structure and asset pricing: Evidence from the Treasury bill markets." *Review of Financial Studies,* 1, 357-375.

Kamara, Avraham. (1990) "Forecasting accuracy and development of a financial market: The Treasury bill futures market." *Journal of Futures Markets,* 10, 397-405.

Kamara, Avraham. (1997) "The relation between default-free interest rates and expected economic growth is stronger than you think." *Journal of Finance,* 52, 1681-1694.

Keynes, John M. (1930) *A Treatise on money*, vol. 2, 135-44. London: MacMillan.

Klemkosky, Robert C., and Eugene A. Pilotte. (1992) "Time-varying term premia in U.S. Treasury bills and bonds." *Journal of Monetary Economics,* 30, 87-106.

Lastrapes, William D. (1989) "Exchange rate volatility and U.S. monetary policy: An ARCH Application." *Journal of Money, Credit, and Banking,* 21, 66-77.

Lauterbach, Beni. (1989) “Consumption volatility, production volatility, spot-rats volatility, and the returns on Treasury bills and bonds.” *Journal of Financial Economics,* 24, 155-179.

Lewis, Karen K. (1991) “Was there a “Peso problem” in the U.S. term structure of interest rates: 1979-82?” *International Economic Review,* 32, 159-173.

Longstaff, Francis A. (2000) “The term structure of very short-term rates: New evidence for the expectations hypothesis.” *Journal of Financial Economics,* 58, 397-415.

MacDonald, S. Scott, and Scott E. Hein. (1989) “Futures and forward rates as predictors of near-term Treasury bill rates.” *Journal of Futures Markets,* 9, 249-262.

Mayers, David. (1972) “Non-marketabe assets and capital market equilibrium under uncertainty.” In *Studies in the Theory of CapitalMmarkets*, edited by Michael Jensen, pp. 223-248. New York, NY: Prager.

Merton, Robert C. (1973) “An intertemporal capital asset pricing model.” *Econometrica,* 41, 867-887.

Press, William H., Brian P. Flannery, Saul A. Teukolsky, and William T. Vetterling. (1986) *Numerical Recipes* New York, NY: Cambridge University Press.

Poole William. (1978) “Using T-bill futures to gauge interest-rate expectations.” *Economic Review*, Federal Reserve Bank of San Francisco, September, 63-75.

Richard, Scott, F., and M. Sundaresan. (1981) “A continuous time equilibrium model of forward prices and futures prices in a multigood economy.” *Journal of Financial Economics,* 9, 347-371.

Scruggs, John T. (1998)," Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: A two-factor approach." *Journal of Finance,* 53, 575-603.

Scruggs, John T., and Paskalis Glabadanidis. (2003) "Risk premia and the dynamic covariance between stock and bond returns." *Journal of Financial and Quantitative Analysis*, forthcoming.

Shiller, Robert J., John Y. Campbell, and Kermit L. Schoenholtz. (1983) "Forward rates and future policy: Interpreting the term structure of interest rates." *Brookings Papers on Economic Activity*, 1, 173-217.

Stoll, Hans R. (1979) "Commodity Futures And Spot Price Determination And Hedging In Capital Market Equilibrium." *Journal of Financial and Quantitative Analysis,* 14, 873-894.

Telser, Lester G., and Harlow N. Higinbotham. (1977) "Organized futures markets: Costs and benefits." *Journal of Political Economy,* 85, 969-1000.

Vasicek, Oldrich. (1977) "An equilibrium model of the term structure." *Journal of Financial Economics,* 5, 177-188.

Endnotes

[1] Because the futures position does not require any initial cash flow, its rate of return is not well defined. The common practice is to use the percent change in futures prices as the risk premium (Dusak 1973; Breeden 1980, Carter, Rausser and Schmitz 1983; Bessembinder 1992).
[2] The two strategies are not exactly equivalent because of trading frictions, default risk, and the daily marking-to-market of futures contracts (Cox, Ingersoll and Ross 1981, Richard and Sundaresan 1981, Kamara 1988). For simplicity, the example above ignores those differences. We will, however, address the effects of marking-to-market below.
[3] Our sample ends in 1998 because trading in Treasury bill futures contracts fell dramatically in the late 1990s. Nineteen ninety-eight seems the last year when the contract was still sufficiently liquid to avoid stale prices. For example, in our data, the trading volume and open interest for the March 1995 contract are 7,194 and 15,520 contracts, respectively; but, trading volume and open interest for the September 1998 contract are 722 and 3267 contracts. Our last observation (on December 1998 for the March 1999 contract) has a trading volume of only 74 contracts and an open interest of 1,500 contracts. We repeated all our tests without it. Our conclusions are unaffected.
[4] Starting with the June 1983 contract, the delivery date of the futures contract moves within the delivery month. (The purpose is to maximize the number of bills available for delivery and reduce the likelihood and severity of possible squeezes.) As a result, one-half of our observations after 1982 overlap by one week (1/13 of an interval). We address this issue by correcting the standard errors in all the regressions below for serial-correlation of, at least, first-order. At the end of 1997 the futures contract became a cash-settled contract.
[5] Our calculations of returns account for the fact that Treasury trades settle one day after the trade day. We also adjust futures returns.
[6] Bekaert, Hodrick, and Marshall (1997) show that these regressions suffer from a small sample bias, in favor of the expectation hypothesis, due to the persistence of short-term interest rates.
[7] The hedging pressure literature that began with Keynes (1930) also advances that the futures' risk premium depends on the variance of the interest rate. Hedging pressure models (Anderson and Danthine 1983), however, distinguish between hedgers (agents who trade in both the underlying asset and the futures market) and speculators (agents who face an infinite cost of entry into the underlying cash market). As a result, the futures' risk premium depends on the net hedging position. We do not test these models because there are no net hedging data for the Treasury bill futures market for large periods in our sample. Moreover, the existing data suggest that hedgers in the Treasury bill futures market are almost always net short.
[8] The constant term is omitted because the square root process introduces a de facto intercept (Gray 1996, p. 49.)
[9] Ang and Bekaert (2002) use a Markov measure of regimes and find that interest rates in the U.S., U.K. and Germany display two regimes corresponding to business expansions and contractions.
[10] We calculated quasi-maximum likelihood estimates and standard errors in two stages using the RATS software. First, we calculated maximum likelihood estimates using the Berndt, Hall, Hall and Hausman (1974) algorithm. This yields consistent estimates, but the standard errors need to be adjusted (Bollerslev and Wooldridge 1992). We then calculated quasi-maximum likelihood estimates and serial correlation and heteroskedasticity-consistent standard errors using the Broyden, Fletcher, Goldfarb and Shanno algorithm (Press et. al. 1986). We iterated the two procedures until we achieved convergence of both the estimates and the log-likelihood function.
[11] Our finding is similar to that of Bekaert, Hodrick and Marshall (2001) who report that their regime-switching model fits the term structure better when they allow for a time-varying term premium.
[12] Tests of higher order GARCH models do not reject the GARCH(1,1) model.
[13] Conditional futures' premiums are always smaller than conditional spot premiums outside the regime.
[14] Recent affine term structure models (Duarte 2003) allow both $\square_0$ and $\square_1$ to be significant.
[15] Note, however, that we are testing hypotheses of continuous-time models using discrete-time quarterly observations.
[16] Lauterbach (1989) finds that spot Treasury term premiums depend on conditional volatilities of consumption and production in 1964-1979, but do not depend on them in the post-1979 period.

[17] We also tested a model with an asymmetric variance term following Glosten, Jaganathan and Runkle (1993) but could not reject the null hypothesis of symmetry. Scruggs (1998) also finds that when the conditional equity variance depends on the Treasury bill yield, the estimate of the asymmetric term is indistinguishable from zero.

[18] Our small sample and Scruggs and Glabadanidis (2003) are additional reasons why one should interpret the results in this section with great caution. Examining monthly stock and bond returns during 1953-1997, they find that the ability of conditional two-factor models (Scruggs 1998) to explain intertemporal variations in stocks and bond returns is sample-sensitive.

[19] For a discussion of Multivariate GARCH models, see Engle and Kroner (1995).

[20] Ang and Bekaert (2001) also find a robust negative conditional relation between equity returns and interest rates, with univariate coefficients around –3.