Magnetic moment $(g - 2)_\mu$ and new physics

Intensity Frontier Review, August 2010
Introduction

New $(g - 2)$ experiment (and th. progress):

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (255\pm16\pm30) \times 10^{-11}$$

Outline

- Remarks on $a_{\mu}$, new physics
- New physics contributions to $a_{\mu}$ in general
  - Benchmark for any new physics scenario
  - Timely, complementary constraints
- Examples within SUSY, Little Higgs, . . .
  - Parameter measurements, model discrimination
Why new physics?

Big questions... point to TeV scale new physics

EWSB, Higgs, mass generation?

hierarchy $M_{Pl}/M_W$? Naturalness?

dark matter?

Grand Unification?

Tevatron, LHC: this decade = era of TeV-scale physics
→ discover signals for new physics

Quest: understand EWSB, understand new physics

Magnetic moment $(g - 2)_\mu$ and new physics

Introduction
Why is $a_\mu$ special?

$$\frac{a_\mu}{m_\mu} \bar{\mu}_L \sigma_{\mu\nu} \mu_R F^{\mu\nu}$$

Beautifully simple “textbook” quantity

CP- and Flavour-conserving, chirality-flipping, loop-induced

compare: EDMs, $b \to s\gamma$, $B \to \tau\nu$, $\mu \to e\gamma$

EWPO
New physics contributions to $a_\mu$

$g - 2 = \text{chirality-flipping interaction}$

$m_\mu = \text{chirality-flipping interaction as well}$

are the two related?
New physics contributions to $a_\mu$

$g - 2 = \text{chirality-flipping interaction}$

$m_\mu = \text{chirality-flipping interaction as well}$

are the two related?

New physics loop contributions to $a_\mu$, $m_\mu$ related by chiral symmetry

[Czarnecki, Marciano ‘01]

generally:

$$\delta a_\mu (\text{N.P.}) = \mathcal{O}(C) \left( \frac{m_\mu}{M} \right)^2,$$

$$C = \frac{\delta m_\mu (\text{N.P.})}{m_\mu}$$
Very different contributions to $a_\mu$

generally:
$$\delta a_\mu (\text{N.P.}) = \mathcal{O}(C) \left( \frac{m_\mu}{M} \right)^2, \quad C = \frac{\delta m_\mu (\text{N.P.})}{m_\mu}$$

classify new physics: *C very* model-dependent

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$\mathcal{O}(1)$
- radiative muon mass generation
  - [Czarnecki, Marciano '01]

$\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$
- supersymmetry (tan $\beta$), unparticles
  - [Cheung, Keung, Yuan '07]

$\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$
- extra dim. (ADD/RS) $(n_c)$
  - [Davioudasl, Hewett, Rizzo '00]
  - [Graesser, '00][Park et al '01][Kim et al '01]

$\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$
- $Z'$, $W'$, UED, Littlest Higgs (LHT)

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Magnetic moment $(g - 2)_\mu$ and new physics

New physics contributions to $a_\mu$ in general
Different types of new physics lead to very different $\delta a_\mu (\text{N.P.})$

- SUSY, RS, ADD, . . . : strong parameter constraints
- $Z'$, UED, LHT, . . . : ruled out if deviation confirmed

If new physics found at LHC:

- $a_\mu$ constitutes a benchmark for new physics models
- can sharply distinguish between different types of models
- timely, complementary constraints on models
  CP- and Flavour-conserving, chirality-flipping, loop-induced
Different types of new physics lead to very different $\delta a_\mu (\text{N.P.})$

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Now illustrate general points with examples
SUSY and $a_\mu$

\[ a_{\mu}^{\text{SUSY}} \approx 130 \times 10^{-11} \tan \beta \, \text{sign} (\mu) \left( \frac{100 \text{GeV}}{M_{\text{SUSY}}} \right)^2 \]

\[ \tan \beta = \frac{v_2}{v_1}, \mu = H_1-H_2 \text{ transition} \quad \text{— central for EWSB} \]

If SUSY signals at LHC:
Need confirmation, precise SUSY parameter measurements
→ understand EWSB, ...
$a_\mu$ central complement for SUSY parameter analyses

- $a_\mu$ sharply distinguishes SUSY models
- breaks LHC degeneracies
- central, complementary in global analyses of SUSY parameters

LHC Inverse Problem (300fb$^{-1}$)

can’t be distinguished at LHC

[Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas ’10]

Constrained MSSM [Ellis, Olive, et al, update K. Olive]
$a_\mu$ central complement for SUSY parameter analyses

$\tan \beta = \frac{v_2}{v_1}$

central for understanding EWSB

LHC: $(\tan \beta)^{\text{LHC, masses}} = 10 \pm 4.5$ bad

[Sfitter: Lafaye, Plehn, Rauch, Zerwas '08, assume SPS1a]

$a_\mu$ improves $\tan \beta$ considerably

vision: test universality of $\tan \beta$, like for $\cos \theta_W = \frac{M_W}{M_Z}$ in the SM:

$$(t_\beta)^{a_\mu} = (t_\beta)^{\text{LHC, masses}} = (t_\beta)^{H} = (t_\beta)^{b}?$$
Bosonic SUSY

- partner states, same spin
- cancel quadratic div.s
- T-parity $\Rightarrow$ lightest partner stable

$\approx 10$ TeV
strong dyn.

$\approx 1$ TeV
states $W_H, l_H$ ...

$\approx 250$ GeV
SM, Higgs

no enhancement of $\frac{\alpha}{4\pi} \left( \frac{m_{\mu}}{M} \right)^2$

$a_{\mu}^{LHT} < 12 \times 10^{-11}$

Clear-cut prediction, sharp distinction from SUSY possible
Other examples

Randall-Sundrum models

Complementarity: LHC
- lowest KK-modes
- masses
- $a_\mu$ from KK-loops
- higher modes, details
- e.g. $C_{\text{Grav}} \propto M^2$, $C_H \sim 1$

What if the LHC does not find new physics — “Dark force”? [Pospelov, Ritz...]

- very light new vector boson
- very weak coupling
- motivated e.g. by dark matter, not by EWSB

$C \propto 10^{-8}$, $M < 1\text{GeV}$
- $a_\mu$ can be large
- could be “seen” by $a_\mu$-exp.

Magnetic moment $(g - 2)_\mu$ and new physics $a_\mu$, parameter measurements and model discrimination
Conclusions

- Big questions of TeV-scale (EWSB) motivate radically new ideas

- Understanding TeV-scale phenomena discovered at the LHC/Tevatron requires input from complementary experiments

\[ a^\text{N.P.}_\mu \text{ very model-dependent, typically } \mathcal{O}(\pm 10 \ldots 500) \times 10^{-11} \]
  - Benchmark for new physics scenario, unique

- New measurement of \( a_\mu \) will
  - sharply distinguish models, even with similar LHC signatures
  - exclude some models, pin down important details of others
  - break degeneracies
  - measure central parameters

\[ a_\mu \text{ will provide critical input and sharp constraints and will be timely complement of LHC in understanding TeV-scale physics} \]
1. Introduction
2. New physics contributions to $a_\mu$ in general
3. $a_\mu$, parameter measurements and model discrimination
4. Conclusions
5. Backup on LHC, fits, $\tan \beta$
6. Backup on complementarity to flavour-changing processes
7. Backup other models
8. Backup on SUSY

Magnetic moment $(g - 2)_\mu$ and new physics

Conclusions
However, reach for \((\chi, \tilde{\mu})\) worse, and more model-dependent

**Magnetic moment** \((g - 2)_\mu\) and new physics

Backup on LHC, fits, \(\tan \beta\)
Fits need $300 \text{fb}^{-1}$ (~10 years running)

Table 1: Result for the general MSSM parameter determination at the LHC in SPS1a. The left part neglects all theory errors, the right one assumes full theory errors. In all cases a set of 20 kinematic endpoints and the top-quark and lightest Higgs-mass measurements have been used. In the third and forth column we include the current measurement of $(g - 2)$. All masses are given in GeV.

<table>
<thead>
<tr>
<th>tan $\beta$</th>
<th>$\Delta \chi^2_{\text{ILC}}$</th>
<th>$\Delta \chi^2_{\text{ILC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>102.7±7.1</td>
<td>189.5±6.2</td>
</tr>
<tr>
<td>$M_2$</td>
<td>185.5±7.0</td>
<td>96.6±6.4</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-362.7±7.8</td>
<td>-364.7±6.8</td>
</tr>
<tr>
<td>$\Delta \chi^2_{\text{ILC}}$</td>
<td>73</td>
<td>22000</td>
</tr>
<tr>
<td>ILC</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tau_1$</td>
</tr>
<tr>
<td>$\Omega h^2$</td>
<td>0.17±0.07</td>
<td>$(4±2) \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Backup on LHC, fits, $\tan \beta$
Sfitter SUSY fits

$(\mu|M_1|M_2)$

$(83|368|136)$

$(352|99|192)$

DS1–12 can’t be distinguished at LHC $(300 fb^{-1})$ if SPS1a realized [Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas ’10]

$(-184|101|354)$

<table>
<thead>
<tr>
<th></th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>DS4</th>
<th>DS7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan\beta$</td>
<td>$12.3\pm5.6$</td>
<td>$12.4\pm5.0$</td>
<td>$14.9\pm9.8$</td>
<td>$8.9\pm5.9$</td>
<td>$13.8\pm7.5$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$102.7\pm7.1$</td>
<td>$189.5\pm6.2$</td>
<td>$107.2\pm9.2$</td>
<td>$383.2\pm9.1$</td>
<td>$105.0\pm6.9$</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$185.5\pm7.0$</td>
<td>$96.\pm6.4$</td>
<td>$356.9\pm8.7$</td>
<td>$114.2\pm10.7$</td>
<td>$194.7\pm7.3$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$-362.7\pm7.8$</td>
<td>$-364.7\pm6.8$</td>
<td>$-186.0\pm8.5$</td>
<td>$-167.0\pm9.6$</td>
<td>$353.0\pm7.7$</td>
</tr>
<tr>
<td>$\Delta\chi^2_{ILC}$</td>
<td>73</td>
<td>22000</td>
<td>1700</td>
<td>25000</td>
<td>0.4</td>
</tr>
<tr>
<td>ILC $\tilde{\tau}_1$</td>
<td>$\tilde{\chi}_1^0$</td>
<td>$\chi_1^-$</td>
<td>$\chi_3^0$</td>
<td>$\chi_1^+$</td>
<td>$\tilde{\tau}_1$</td>
</tr>
<tr>
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<td>$0.17\pm0.07$</td>
<td>$(4\pm2)\cdot10^{-4}$</td>
<td>$0.14\pm0.08$</td>
<td>$(8\pm4)\cdot10^{-4}$</td>
<td>$0.16\pm0.07$</td>
</tr>
</tbody>
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Magnetic moment $(g - 2)\mu$ and new physics

Backup on LHC, fits, $\tan\beta$
Importance: $\tan \beta$, $\mu$ are central parameters in EWSB

\[
\mu^2 + M_{H_u}^2 = B_\mu \cot \beta + \frac{1}{2} M_Z^2 \cos 2\beta
\]

\[
\mu^2 + M_{H_d}^2 = B_\mu \tan \beta - \frac{1}{2} M_Z^2 \cos 2\beta
\]
Table 1 shows the result of our SPS1a analysis. For comparison and to make the effect of the additional $(g-2)$ data easily visible, we include the result without $(g-2)$ data from Tables VIII and IX of Ref. [31]. We give results with experimental errors only (columns 2 and 3) and including theory errors (columns 4 and 5). The effect of the additional information on the accuracy of the parameter determination is clearly visible. It is particularly significant for $\tan \beta$, which is not well determined by the measurements of kinematic endpoints at the LHC. The best source of information on $\tan \beta$ is the light MSSM Higgs mass [109], but this observable strongly relies on the assumed minimal structure of the Higgs sector, on the knowledge of many other MSSM parameters, and on the estimate of the theory errors due to higher orders. Because of a lack of complementary measurements (for example $A_t$) a change in $\tan \beta$ can always be compensated by an appropriate change in other MSSM parameters, leaving the value of all LHC observables unchanged. Additional sources of a $\tan \beta$ measurement are the production rate for heavy Higgs bosons [110] and rare decays like $B_s \rightarrow \mu^+\mu^-$, which we study elsewhere in this volume, but both of them only work for large enough values of $\tan \beta$.

The $(g-2)$ prediction has a leading linear dependence on $\tan \beta$. Therefore, the improvement of the $\tan \beta$ errors by more than a factor of two can be easily understood. This improved accuracy of $\tan \beta$ influences those parameters which must be re-rotated when $\tan \beta$ is changed to reproduce the same physical observables. Correlations and loop corrections propagate the improvement over almost the complete parameter space.
tan \beta determinations from Les Houches 2007 Report, M.M. Nojiri, T. Plehn, G. Polesello

- \(a_\mu\) linear, requires \(\chi, \tilde{\mu}\) masses
- lightest Higgs mass \(M_h\): depends on minimality of Higgs sector, many other SUSY parameters (\(A_t!\))
- production rate of heavy Higgs bosons: only if heavy Higgs can be discovered, cross section measured! Then \(\mathcal{O}(10\%)\) possible.
- \(B_s \rightarrow \mu\mu \propto \tan^6 \beta/M_A^4\): problems
  (1) non-perturbative theory uncertainty \(\mathcal{O}(30\%)\),
  (2) requires precise \(M_A!\) Can hope to obtain \(\mathcal{O}(10 \ldots 20\%)\) for \(\tan \beta > 30\).
- Note, heavy Higgses difficult unless \(\tan \beta\) large

**Graph:**

**ATLAS Preliminary (Simulation)**

- lep/had Discovery Limit
- lep/had Theoretical Uncertainty
- lep/had Uncorrelated Systematics
- lep/had No Systematics
- lep/lep Discovery Limit
- lep/lep +10% \(\sigma(t\bar{t})\) Uncertainty

\[\sqrt{s}=14\text{ TeV}, L=30 fb^{-1}\]

\[\tan \beta \text{ vs. } m_A [\text{GeV}]\]

**Note:**

Magnetic moment \((g - 2)_\mu\) and new physics

Backup on LHC, fits, tan \(\beta\)
Complementarity to $B \rightarrow \tau \nu, B_S \rightarrow \mu \mu$

How sensitive to NP are observables?

[Isidori, Paradisi ’06]:
“The observables $B \rightarrow \tau \nu, B_S \rightarrow \mu \mu$ and $a_\mu$ can be considered as the most promising low-energy probes of the MSSM scenario with heavy squarks and large $\tan \beta$.”

Which aspects of NP are determined by observables?

- $a_\mu$: $\propto \tan \beta$, no flavour-parameters
- $B_u \rightarrow \tau \nu$: tree-level ($H^\pm$) $\sim \tan^2 \beta / M_{H^\pm}^2$
- $B_S \rightarrow \mu \mu$: loop-induced, $\propto \tan^6 \beta / M_A^4$
- $b$-decays sensitive to non-MFV-parameters!

Recall, heavy Higgses difficult at LHC

Use complementarity (assume masses known (LHC)):

- $a_\mu \rightarrow t_\beta, b$-decays $\rightarrow t_\beta / M_{H,A}$/SUSY-flavour structure
- If MFV is assumed/established: use of $b$-decays: if $M_A,H$ known $\rightarrow$ alternative $\tan \beta$-measurement if not, use $t_\beta$ from $a_\mu \rightarrow t_\beta$
- Note: $(t_\beta)^{a_\mu}$: Higgsino-coupling, $(t_\beta)^b$: Higgs-coupling

Crucial test of SUSY!
Impact of small non-MFV on $b \rightarrow s\gamma$

Illustration: dependence on small non-MFV parameters

- Minimal Flavour Violation at which scale?
- loops induce FCNC (=non-MFV) Gluino-couplings
- $b \rightarrow s\gamma$-prediction depends on $\mu_{MFV}$!
- similar if there are generic non-MFV contributions

[Degrassi,Gambino,Slavich '06]
Complementarity to CLFV

- different sensitivities
- $a_\mu$ from $\tilde{B}$-diagram could be sensitive to multiple FC-insertions:
  \[ \Delta m^2_{\tilde{\mu}\tilde{\tau}} L \times \Delta m^2_{\tau\tilde{\mu}} R \]
- only possibility for large FC-effects in $a_\mu$: $\Delta m^2_{\tilde{\mu}\tilde{\tau}} L$ and $\Delta m^2_{\tilde{\mu}\tilde{\tau}} R$ both large.
- However $\tilde{B}$-diagram anyway suppressed, $a_\mu$ dominated by chargino exchange
- Hence: even if $\Delta m^2 \approx 0.2 m^2$, only 10% correction to $a_\mu$
- [Moroi '95]
- $a_\mu$ clean probe of flavour-diagonal parameters

Magnetic moment $(g - 2)_{\mu}$ and new physics Backup on complementarity to flavour-changing processes
Gravity propagates in extra dimension

each KK-Graviton contributes equally, weakly, **no decoupling**

theory breaks down at scale $\sim \Lambda_\pi$, $n_c$ KK-gravitons up to that scale

$$a_\mu^{RS} \sim \frac{5n_c}{16\pi^2} \frac{m_\mu^2}{\Lambda_\pi^2}$$

**potential enhancement** $\propto n_c = \mathcal{O}(1 \ldots 100 \sim 1/coupling)$

- feels all KK-gravitons
- very sensitive to UV-completion of theory
$g - 2$ and Randall-Sundrum models

Complementarity: LHC
- lowest KK-gravitons
- determines model parameters
- $a_\mu$ tells us what the cutoff is
- hint to full 5-dim dynamics
- guides model building of full theory

potential enhancement $\propto n_c = \mathcal{O}(1 \ldots 100 \sim 1/\text{coupling})$
- feels all KK-gravitons
- very sensitive to UV-completion of theory
Alternative models

- Hidden sector coupling to muon [McKeen '09]

\[ L_{\text{int}} = \lambda_L X \bar{Y}_R \mu_L + \ldots \]
\[ \lambda_L = 0.1 \]

- Dark matter $\Rightarrow \alpha_\mu$, leptogenesis, neutrino masses simultaneously

[Hambye, Kannike, Ma, Raidal '06]

(C very large, $M \sim 1\text{TeV}$ possible)
Numerical results

General MSSM for $\tan \beta = 50$, all parameters $< 3$ TeV [DS '06]

- SUSY with $M_{\text{SUSY}} = 200 \ldots 600$ GeV fits well
- large parameter regions already excluded

Constrained MSSM [Ellis, Olive, et al, update K. Olive]

Complementary constraints:

- $a_\mu$, dark matter, $b \rightarrow s\gamma$
\[ a^\text{SUSY}_\mu \approx 130 \times 10^{-11} \tan \beta \ \text{sign}(\mu) \left( \frac{100\text{GeV}}{M_{\text{SUSY}}} \right)^2 \]

Why enhanced? \( g - 2 = \) chirality-flipping interaction

- In SUSY, chirality-flips governed by \( \lambda_\mu \) and \( m_\mu = \lambda_\mu \langle H_1 \rangle \)
- Two Higgs doublets: \( \tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle} \), \( \mu = H_2 - H_1 \) transition

\[ \Rightarrow \text{all terms } \propto \lambda_\mu \text{ but two options:} \]

\[ \propto \lambda_\mu \langle H_1 \rangle = m_\mu \]

\[ \propto \lambda_\mu \mu \langle H_2 \rangle = m_\mu \mu \tan \beta \]
$\mu$ and SUSY

\[
a^\text{SUSY}_\mu \approx 130 \times 10^{-11} \, \tan \beta \, \text{sign}(\mu) \left( \frac{100\text{GeV}}{M_{\text{SUSY}}} \right)^2
\]

where

\[
\lambda_\mu \langle H_2 \rangle = m_\mu \, \mu \, \tan \beta
\]

potential enhancement $\propto \lambda_\mu \propto \tan \beta = 1 \ldots 50$ (and $\propto \text{sign}(\mu)$)

- sensitive to muon mass generation mechanism
- structure of Higgs sector
SUSY without prejudice - compare observables

- $a_\mu$
- $b \to s\gamma$
- dark matter

have highest selective power and are complementary:

$tan \beta = 40, \mu > 0$

$m_0 = 114$ GeV

$m_{\chi^\pm} = 104$ GeV

Figure 16: Distributions of predictions for several observables as well as $tan \beta$ for our model sample subject to the constraints discussed in the text. The blue and green dashed lines show the SM predictions as well as the current central values obtained by experiment, respectively.

Magnetic moment $(g - 2)_\mu$ and new physics  
Backup on SUSY