

# Summary of the INT Workshop on The Hadronic Light-by-Light Contribution to the Muon Anomaly

## Abstract

The INT workshop on "The Hadronic Light-by-Light Contribution to the Muon Anomaly" was held Monday Feb. 28 through Friday March 4, 2011. It was convened to focus on one of the outstanding theoretical issues in interpreting the muon anomalous magnetic moment. 36 scientists from 14 countries (AUT, CAN, FRA, GER, GB, IN, IT, JAP, POL, RSK, RUS, SPA, SWE, USA) participated in 19 scientific talks addressing three important questions:

1. Can agreement on the individual and combined theoretical contributions to the hadronic light-by-light (HLbL) contribution be reached, based on the multitude of QCD-inspired models?
2. Can lattice calculations attain sufficient precision to check the models or to independently establish a HLbL value?
3. Which obtainable experimental data is essential to constrain the theoretical calculations at least in parts and what theoretical effort is required to connect this data to the model predictions?

The answers to these questions aim at narrowing the 20-45% theory error on the current HLbL prediction that is inadequate with respect to the goals of the new  $g-2$  experiment, which is expected to come online at Fermilab in the next 5 years. The HLbL uncertainty must be reduced to 10%, or better, as the goal for the new experimental error is 0.14 ppm compared to 0.54 ppm in the past experiment. In the following sections, we summarize the three major thrusts of the Workshop, give a short view on two followup publications of the model discussion and provide conclusions.

## 1 Models

Plenary sessions on Monday and Tuesday were dedicated to review the various models which have been used in the recent years to evaluate the HLbL contribution to the muon  $g-2$ : the Extended Nambu-Jona-Model (EJNL), the Chiral Approach and Resonance Dominance Models with Phenomenological Pion Form Factors (RDFF), Hadronic Models with Operator

Product Expansion constraints (OPE), Holographic QCD Approaches (HQCD) and a Dyson-Schwinger Approach (DSA). Discussions on the applicable range, various limitations and possible discrepancies followed each presentation. More technical issues on the various models have been discussed on Wednesday as well as in the parallel discussion sessions on Thursday and Friday.

Main conclusions which emerged from these presentations and discussions are as follows:

- Except for the DSA-approach, all the models give results which within their own estimated errors are in agreement. In particular, the new results from HQCD are in good agreement with the previous results on the dominant  $\pi^0$ -exchange contribution.
- Potential conceptual differences between the various RDFF-approaches and the OPE-approach, were substantially clarified.
- The disagreement with the DSA-result seems to arise from an enhanced Quark Loop Contribution in addition to the (in all other models) dominant  $\pi^0$ -exchange. It was pointed out that the DSA-result contradicts the simple picture which emerges from the Constituent Chiral Quark Model and that one has to be careful in singling out individual contributions from different model approaches and compare them. The discussion and various suggested tests to the DSA-approach resulted in two followup publications (see section Followups).
- The unique opportunity for a dialogue between theorists with expertise in QCD inspired models and theorists from the Lattice-QCD community led to important suggestions of various measurements of specific Green's functions in Lattice-QCD which are useful on their own but in addition also provide intermediate steps towards an eventual determination of the HLbL from first principles.

## 2 Lattice QCD

The lattice plenary session and the working group meetings were especially fruitful as ideas were crossfertilized between the participating experts in lattice QCD and models. These ideas, from the full calculation to smaller partial calculations, to check methods as well as the model calculations, are mentioned in the following in order of decreasing difficulty. Besides the daunting nature of the calculation of the HLbL, where the full hadronic four-point vector correlation function serves as input to perturbative QED two loop integrals, several tricks can be used to reduce the computational burden of two independent loop momenta. A calculation of the full amplitude in combined QCD+QED appears tractable with today's

computing resources and the use of several tricks for the momentum input, however. Extremely attractive as a model check as well as for confidence in the calculations underway, would be fiducial calculations of the four-point function for fixed momenta. Also three-point and lower correlation functions as well as other quantities (namely: the  $\pi^0 \rightarrow \gamma\gamma$  form factor, the axial-vector-vector-vector form factor, the  $\pi^0 \rightarrow e^+e^-$  amplitude, the hadronic contribution in the running of  $\theta_W$  and the quark condensate magnetic susceptibility) deemed to be useful in checking models and lattice methods as well as reducing their systematic errors. New, encouraging results on the HVP contribution were presented and discussed, their error successively approaches the ones coming from  $e^+e^- \rightarrow$  hadrons and tau decays.

### 3 The tie-in to data

In contrast to the HVP, the HLbL contribution can be connected to experimental data only indirectly. Since evaluation of the HLbL is possible by theory only, experimental results which constrain the models in all possible ways are very important. As modeled, the HLbL is dominated by various single particle (especially pseudoscalar) exchange contributions. The experimental focus therefore is in measuring single and pair particle production in two-photon processes. Besides on-shell measurements, important constraints come from processes involving virtual gammas. A experiment providing important constraints is  $\gamma^*\gamma \rightarrow \pi^0, \eta, \eta'$  where the most recent one by BaBar exhibits a high energy behaviour contradictory with theoretical expectations (BaBar puzzle). Also in progress are measurements of  $\gamma^*\gamma^* \rightarrow \pi^0, \eta$  and  $\gamma\gamma \rightarrow \pi\pi \rightarrow \sigma$  form factors at Frascati, where no data in the low energy regime is available up to now. Also puzzling is the large branching fraction for the pseudovector decay  $f_1(1285) \rightarrow \gamma\rho^0$  where improved measurements are important. Furthermore important are studies of Dalitz-decays like  $\rho, \omega, \phi \rightarrow \pi^0(\eta)e^+e^-$  at Novosibirsk, NA60, JLab, Mainz, Bonn, Jülich and BESIII. Commonly used in  $\gamma\gamma$  experiments is the Monte Carlo eventgenerator EKHARA, where the next step of the development is the implementation of radiative corrections.

### 4 Followup activities

In continuation of the discussion on the DS-approach, two papers [1],[2] have been submitted recently. Boughezal and Melnikov (BM) find that the HLbL prediction of the Constituent Quark Loop model is stable under inclusion of QCD radiative corrections. Within their approach BM do not find large dressing effects of the quark-photon vertex which would enhance the HLbL relative to the HVP contribution. Fischer, Goecke and Williams (GFW) submitted a paper to demonstrate the calculation of the HVP within the DS-approach as requested

in workshop discussions. In the paper GFW present in detail their model and its features, especially the dressing effects of the quark-photon vertex that lead to the comparably high value of HLbL in the DSA. In their work GFW demonstrate the power of the DS-approach with dynamical quark masses in qualitative agreement with lattice QCD, reproduction of vector meson bound-states poles and good agreement with the Adler function obtained from dispersion relations. Despite the hope that the followup discussion after the workshop will help to resolve the deviations of DSA and the other models, the very good agreement of the DSA to the established HVP value and the predictive power of this approach hints that there is more work to follow to resolve this issue.

## 5 Conclusion

The models discussed incorporate specific features of QCD in different ways. An important outcome of the discussion was to clarify how the different approaches are complementary but not contradictory. Besides the DSA, all models, including the more recent HQCD models, give results which agree within their own estimated errors. The early stages of the lattice calculation appears promising. Most important, in addition to increasing hardware resources, is that a critical mass of researchers is forming to take on this very important problem. Also the smaller sidesteps in calculating aforementioned form factors will happen in near future and give another important check for models. In addition to that, more and more experiments are able to improve and extend measurements of two-photon processes with light hadrons, which again deliver indispensable constraints for the hadronic models. These experimental programs then themselves are supported by theorists for providing event-generators.

## References

- [1] Boughezal, Melnikov (2011) <http://arxiv.org/abs/1104.4510>
- [2] Goecke, Fischer, Williams (2011) <http://arxiv.org/abs/arXiv:1107.2588>