Initial Review of *Maximum Likelihood Estimation of Misspecified Models* by Halbert White

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April 17, 2012

Bibliography Stuff

Maximum Likelihood Estimation of Misspecified Models

by Halbert White

Econometrica, Vol. 50, No. 1 (Jan., 1982), pp. 1-25

Obligatory Demotivational Quote

"This is a very frustrating paper." - V. Minin, 2012

Why Should You Care?

Correct model specification = efficient inference

Especially helpful for small sample size

But how do you **know**?

White's Contribution

Answers questions about the following in a unified framework:

- does MLE converge? (interpretation?)
- if yes, is MLE asymptotically normal?
- can properties of MLE determine model truth?

That Which Hath Come Before

Consistency of MLE estimator

Interpretation of such

Asymptotic normality

Inference

So Really...

White's real contributions:

- A unified approach
- More restrictive, yet more intuitive and easily checked conditions
- Some new statistics and tests

A First Theorem

Theorem

Given assumptions, $\sqrt{n}(\hat{\theta}_{MLE} - \theta_{KLIC}) \rightarrow_d N(0, \mathbf{C}(\theta_{KLIC}))$

- $\hat{\theta}_{MLE}$ converges to the Kullback-Leibler Information Criterion-minimizing parameter
- Form of the asymptotic covariance matrix?

A Sandwich Estimator!

Covariance Matrix Formula

$$
\begin{aligned}
U_t &= \text{data vectors} \\
\theta &= \text{parameter vector of model family} \\
f(U_t, \theta) &= \text{model density function} \\
\mathbf{A}(\theta) &= E\left[\frac{\partial^2 log(f(U_t,\theta))}{\partial\theta_i \partial\theta_j}\right] \\
\mathbf{B}(\theta) &= E\left[\frac{\partial log(f(U_t,\theta))}{\partial\theta_i}\frac{\partial log(f(U_t,\theta))}{\partial\theta_j}\right] \\
\mathbf{C}(\theta) &= \mathbf{A}(\theta)^{-1}\mathbf{B}(\theta)\mathbf{A}(\theta)^{-1}
\end{aligned}
$$

Another Theorem

Theorem

Given some assumptions, the elements of $\mathbf{A}$ *and* $\mathbf{B}$ *can be used to derive a statistic that indicates model misspecification*

Idea: In a correct model $\mathbf{A} + \mathbf{B} = \mathbf{0}$.

The further you are from this, the more likely your model is misspecified.

Sensitive to misspecifications which invalidate usual inference.

Some Mathy Specifics

θ is a p-dimensional vector.

$$
\begin{aligned}
d_l(U_t,\theta) &= \partial log(f(U_t,\theta))/\partial\theta_i \cdot \partial log(f(U_t,\theta))/\partial\theta_j \\
&\quad + \partial^2 log(f(U_t,\theta))/\partial\theta_i\partial\theta_j \\
dim(d) &= q \times 1 \text{ with } q \le p(p+1)/2 \\
\nabla D(\theta) &= E[\partial d(U_t,\theta)/\partial\theta_k] \\
W &= d(U_t,\theta) - \nabla D(\theta)\mathbf{A}(\theta)^{-1}\nabla log(f(U_t,\theta)) \\
\mathbf{V}(\theta) &= E[W \cdot W^T]
\end{aligned}
$$

Theorem

Given some assumptions, $\sqrt{n}D_n(\hat{\theta}_n) \rightarrow_d N(0,\mathbf{V}(\theta_0))$; $\mathbf{V}_n(\hat{\theta}_n) \rightarrow_{a.s.} \mathbf{V}(\theta_0)$; $\mathfrak{I}_n = nD_n(\hat{\theta}_n)'\mathbf{V}_n(\hat{\theta}_n)^{-1}D_n(\hat{\theta}_n) \rightarrow_d \chi^2_q$

Two Other Theorems

Two more tests for model misspecification.

Sensitive to inconsistent estimator misspecifications.

I'm still working these out.

What Do We Do With All This???

THE METHOD

- Step 1: Perform first test.
- Step 2a: If you "do not reject", MLE away!
- Step 2b: If you "reject", perform the other two tests
- Step 3a: If you "do not reject", use sandwich inference
- Step 3b: If you "reject", reconsider your model choice.

The Master Plan

Review Proofs

Simulation Study

Study with Actual Data??

The End

(What the title says)