Formulas for Biostat 513 Midterm Exam

Pearson's χ^2 statistic:

$$X^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Pearson's χ^2 statistic for a 2×2 table:

$$X^2 = \frac{N(ad - bc)^2}{n_1 n_2 m_1 m_2}$$

95% Confidence interval for log relative risk:

$$\begin{aligned} \log(\widehat{RR}) &= \log\{\widehat{p}_1/\widehat{p}_2\} \\ \log(\widehat{RR}) &\pm 1.96\sqrt{\frac{1-\widehat{p}_1}{n_1\widehat{p}_1} + \frac{1-\widehat{p}_2}{n_2\widehat{p}_2}} \\ \log(\widehat{RR}) &\pm 1.96\sqrt{\frac{b}{a(a+b)} + \frac{d}{c(c+d)}} \end{aligned}$$

95% Confidence interval for log odds ratios (Woolf):

$$\begin{aligned} \log(\widehat{OR}) &= \log\left\{\frac{\widehat{p}_1/(1-\widehat{p}_1)}{\widehat{p}_2/(1-\widehat{p}_2)}\right\} \\ \log(\widehat{OR}) &\pm 1.96\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}} \end{aligned}$$

McNemar's Test:

$$Z = \frac{n_{01} - M\frac{1}{2}}{\sqrt{M/4}}$$

Mantel-Haenszel Test:

$$\begin{aligned} O &= \sum_i O_i = \sum_i a_i \\ E &= \sum_i E_i = \sum_i \frac{(a_i + b_i)(a_i + c_i)}{N_i} \\ V &= \sum_i V_i = \sum_i \frac{(a_i + b_i)(c_i + d_i)(a_i + c_i)(b_i + d_i)}{N_i^2(N_i - 1)} \end{aligned}$$

$$X^2_{MH} = \frac{(|O-E| - \frac{1}{2})^2}{V}$$

Mantel-Haenszel Estimate:

$$\widehat{\Psi}_{MH} = \frac{\sum_i a_i d_i / N_i}{\sum_i b_i c_i / N_i}$$

Mantel-Haenszel Test-Based 95% CI:

$$\log(\widehat{\Psi}_{MH}) \pm 1.96 \left(\frac{\log(\widehat{\Psi}_{MH})}{\sqrt{X^2_{MH}}} \right)$$

Woolf's Test of Homogeneity:

$$\begin{aligned} w_i &= \frac{1}{s_i^2} \\ \overline{\beta} &= \frac{\sum_i w_i \widehat{\beta}_i}{\sum_i w_i} \\ X^2_H &= \sum_i w_i (\widehat{\beta}_i - \overline{\beta})^2 \end{aligned}$$

Chi-square Test for Trend:

$$\begin{aligned} A &= \sum_i n_i (\widehat{p}_i - \overline{p})(S_i - \overline{S}) \\ B &= \overline{p}(1-\overline{p}) \left\{ \left(\sum_i n_i S_i^2 \right) - \left(\sum_i n_i S_i \right)^2 / N \right\} \\ X^2_T &= A^2/B \end{aligned}$$

The logistic function:

$$f(z) = \frac{\exp(z)}{1+\exp(z)}$$

Wald statistic:

$$Z = \widehat{\beta}_j / (\widehat{s.e.})$$