Analysis of Longitudinal Data for Inference and Prediction



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Outline of Short Course

- Inference
 - ▶ How to handle "stochastic" measurements and/or events?
 - How to address time-dependent covariates?
- Prediction
 - How to measure ability of marker(s) to predict event time?
 - What are appropriate time-specific or overall summaries and how do these relate to medical decision making?

Some General Comments / Disclaimers

- Moderately advanced material
- Overview of major issues / depth only with select issues
- First time together / total (!)
- Theory into practice is my goal
- Course teaching ideas rather than SAS / STATA / R implementation

Analysis of Longitudinal Data for Inference and Prediction: Part I – Overview of Issues



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LDA Progress!

- During the last couple of decades statistical methods have been developed (ie. LMM, GEE) that can analyze longitudinal data with:
 - \triangleright Unequal number of observations per person (n_i)
 - \triangleright Unequally spaced observations (t_{ij})
 - \triangleright Time-varying covariates (x_{ij})
- Regression questions:

$$\mu_i(t) = E[Y_i(t) \mid X_i(t)]$$

• Q: When should we directly apply these now standard longitudinal methods to data with the features listed above?

Session One Outline

- Examples
 - Cystic Fibrosis Foundation (CFF)
 - Maternal Stress and Child Morbidity (MSCM)
 - United States Renal Data System (USRDS)
 - Collaborative Perinatal Project (CPP)
- Time-varying Covariate Processes
 - Exogenous
 - * Lagged covariates
 - Endogenous
 - * Fixed vs Dynamic exposure

(*) Session One Outline (continued)

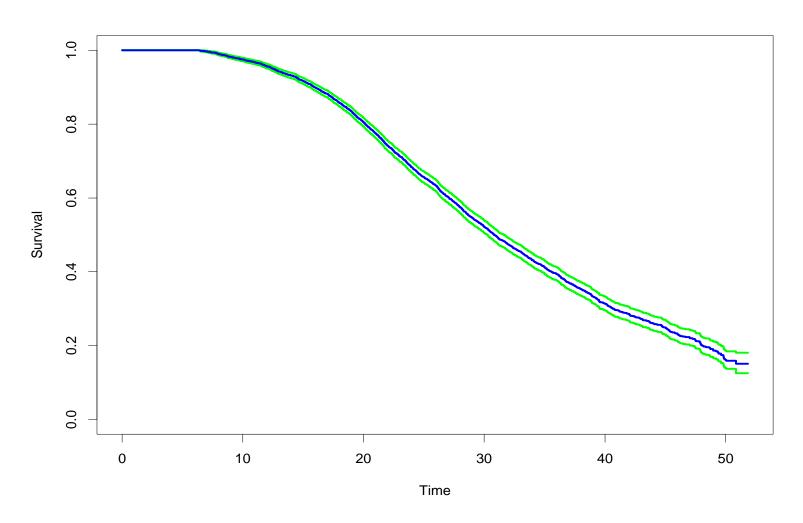
- Analysis with Drop-out
- Analysis with Death
 - Specification of model
 - Inference
- Analysis with "drop-in" (scheduling, birth)

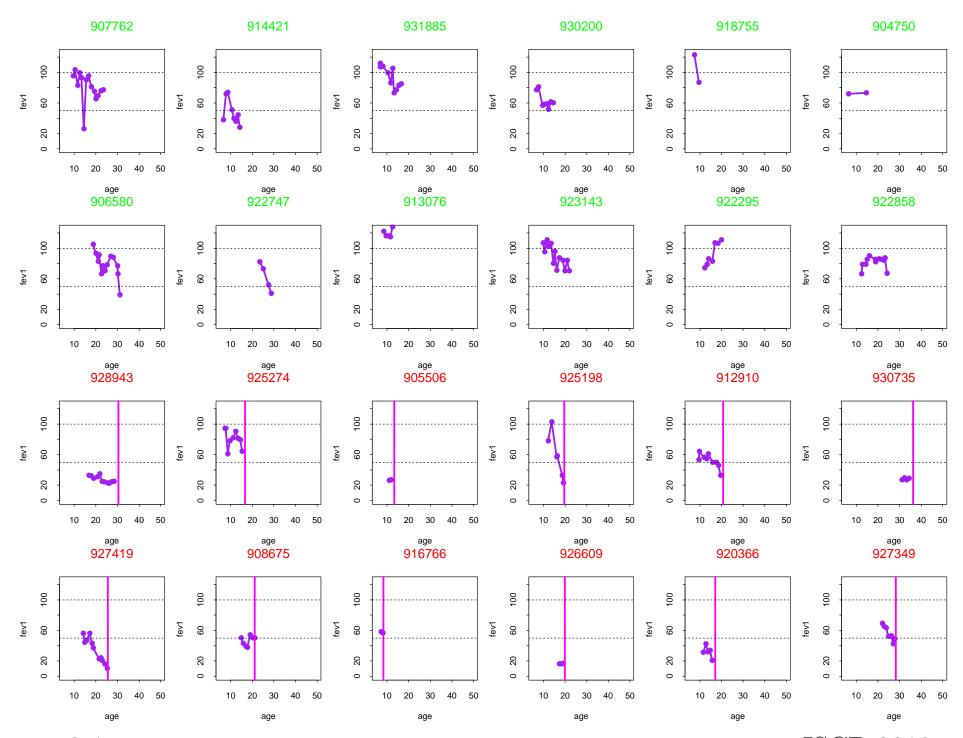
Repeated Measures Data

Cystic Fibrosis Data

- N = 23,530 subjects, 4,772 deaths, 1986-2000
- n = 160,005 longitudinal observations
- Longitudinal measurements: FEV1, weight, height
- Goal: identify factors associated with decline in pulmonary function.
- (Another Goal: predict mortality; transplantation selection)

CFF Survival





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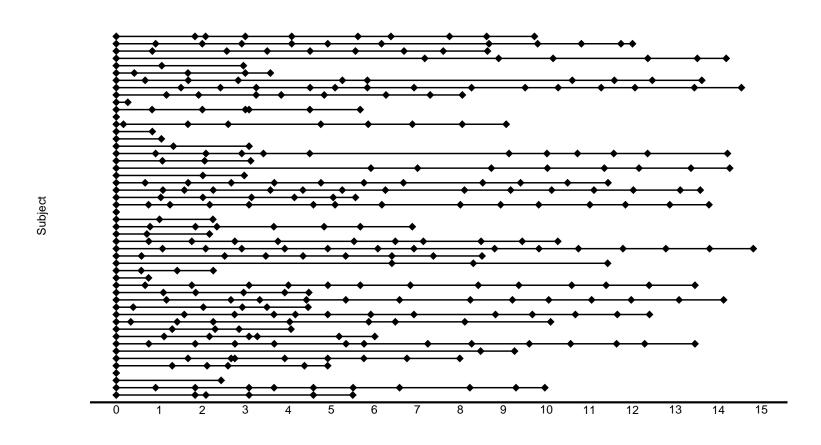
Example: Scientific Goals & CF

Parad RB, Gerard CJ, Zurakowski D, Nichols DP, Pier GB
 "Pulmonary outcome in cystic fibrosis is influenced primarily by
 mucoid Pseudomonas aeruginosa infection and immune status and
 only modestly by genotype."

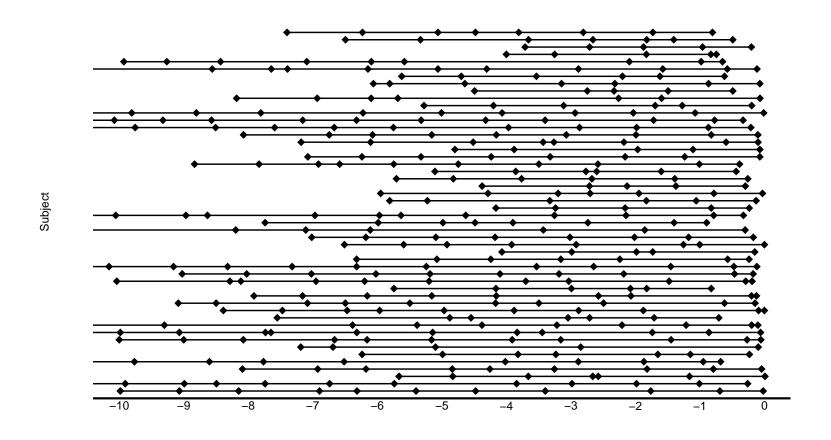
Infect. Immun., 67(9): 4744-50, 1999.

Variables:

- \triangleright Measurement time: t_{ij}
- \triangleright Pulmonary function: $Y_i(t_{ij})$
- \triangleright Time-dependent covariate: $X_i(t_{ij})$ infection status
- \triangleright Death: $D_i(t)$ counting process for T_i



Age-Age0



Age-AgeD

Maternal Stress and Child Morbidity

Example 2: Time-dependent covariates

- daily indicators of stress (maternal), and illness (child)
- primary outcome: illness, utilization
- covariates: employment, stress
- Q: association between employment, stress and morbidity?
- Q: Does stress cause morbidity?

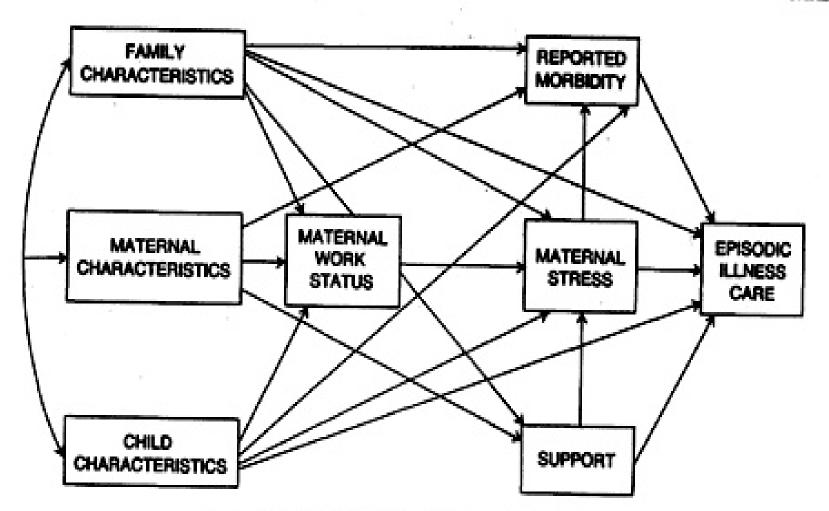
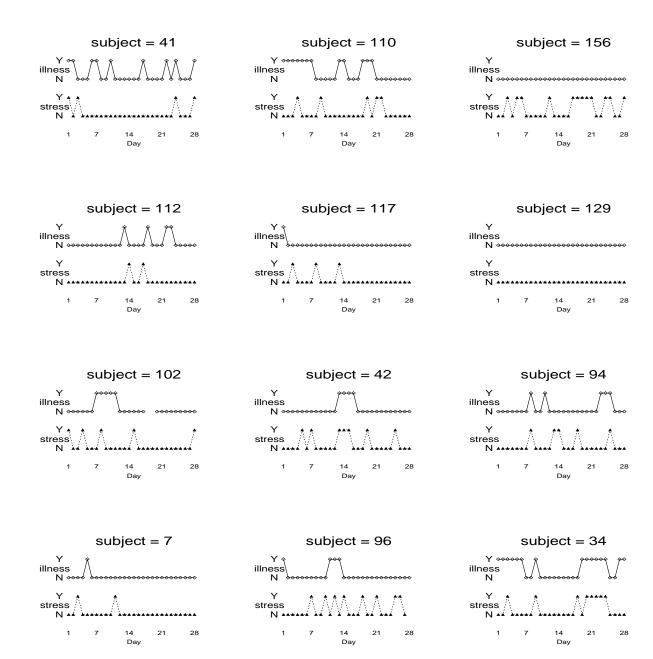


Fig. 1. Determinants of episodic illness care utilization.



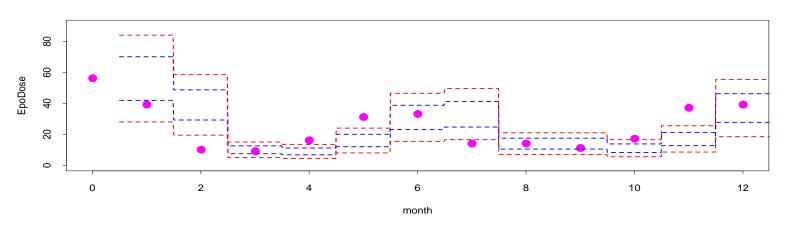
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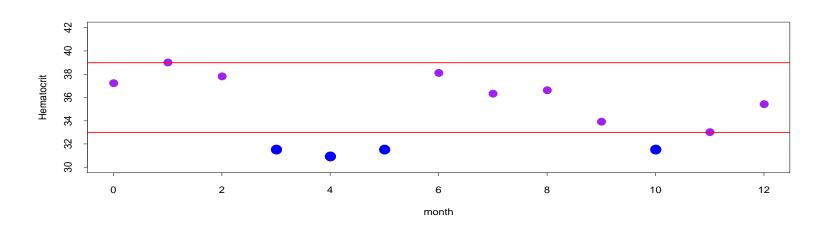
USRDS Data: Safety of ESAs?

- End Stage Renal Disease (ESRD)
 - Poor kidney function
 - Dialysis
 - ▶ Fail to stimulate formation of red blood cells
- Epoetin
 - Anemia treatment
 - ▶ \$3 billion Medicare / year
- Studies show an association between high dose and risk of death
 - Adverse outcomes?
 - Confounding by indication?

USRDS Dialysis Data



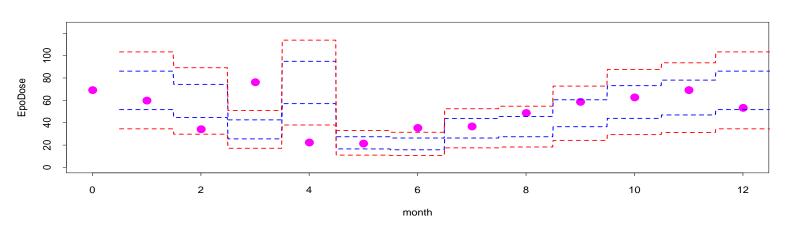


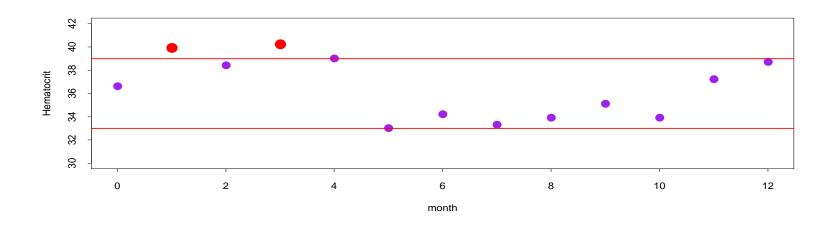


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USRDS Dialysis Data







"Repeated Measures" Data

CPP Data

- Subset of N=8467 women
- A total of 8467 + 1960 + 527 births
- Recruitment during 1959-1965
- Prospective measurements: smoking, weight, birth outcomes.
- Goal: risk factors for poor pregnancy outcome?
 - Exposures: Infection, Drugs, Smoking.
 - Outcomes: Weight, SGA, malformations.
- Multiple births/woman recorded if within 1959-1965.

The Processes

Primary Process

 $Y_i(t)$ The response process

Secondary Processes

- $X_i(t)$ The covariate process
- $S_i(t)$ The scheduling process (not today)
- $R_i(t)$ The recording process (not today)
- $D_i(t)$ The death process (not today)
- $B_i(t)$ The birth process (not today)

LDA and Regression

Most statistical representations focus on discussion of

$$\mu_i(t) = E[Y_i(t) \mid X_i(t)]$$

• But what about the other processes? Do we mean:

CFF :
$$E[Y_i(t) \mid X_i(t), X_i(s), S_i(t) = 1, R_i(t) = 1, D_i(t) = 0]$$

USRDS :
$$E[Y_i(t) | X_i(t), X_i(s), R_i(t) = 1, D_i(t) = 0]$$

MSCM :
$$E[Y_i(t) | X_i(t), X_i(s), R_i(t) = 1]$$

CPP :
$$E[Y_i(t) \mid X_i(t), X_i(s), dB_i(t) = 1, R_i(t) = 1]$$

Time-dependent Covariates

- Exogenous $Y_i(t)$ doesn't predict $X_i(s)$, s > t.
 - \triangleright Appropriate lags: $E[Y_i(t) \mid X_i(s), s < t]$
 - Weighted estimation and bias (DHLZ section 12.3)
- Endogenous $Y_i(t)$ does predict $X_i(s)$, s > t.
 - Causal inference targets
 - Model covariate process MSMs
 - See: Robins, Hernán & Brumback (2000)
 - * Model response process G-comp
 - See: Robins, Greenland, & Hu (1999)
 - Association
 - * Model covariate process
 - See: Miglioretti & Heagerty (2004)

Motivation: Hospitalization and EPO Dose?

- Background:
 - NEJM − November 2006
 - * RCTs target high versus low hemoglobin
 - ∗ Higher target → higher Epo dose
 - Higher target associated with AEs
 - ▶ FDA March 2007

Issued a "black box warning" which indicated that aggressive use of erythropoiesis-stimulating agents to raise hemoglobin to a target of 12 g/dL or higher was associated with "serious and life-threatening side-effects and/or death."

- General Question:
 - Q: Are higher doses of EPO associated with greater rates of adverse events such as hopitalization?

Motivation: Full Data History

• Regression:

$$E[\mathsf{Hosp}(t) \mid \mathsf{Dose}(t-1), \ \mathsf{Dose}(t-2), \ldots, X]$$

- Statistical Issues:
 - What aspects of exposure history are associated with current hosp?
 - What is the role of the **outcome history** $\mathsf{Hosp}(t-1),\ \mathsf{Hosp}(t-2),\ldots$?
 - What is the role of intermediate history Hem(t-1), Hem(t-2),...?

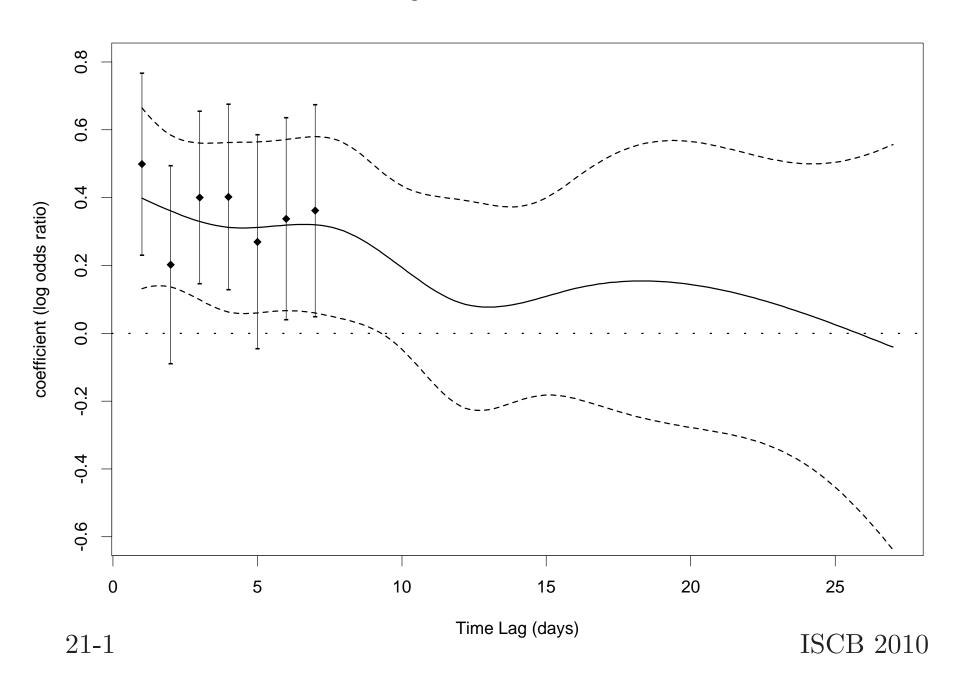
Time-dependent Covariates: Lagged Covariates

 Exogenous – future covariates are not influenced by current / past outcomes.

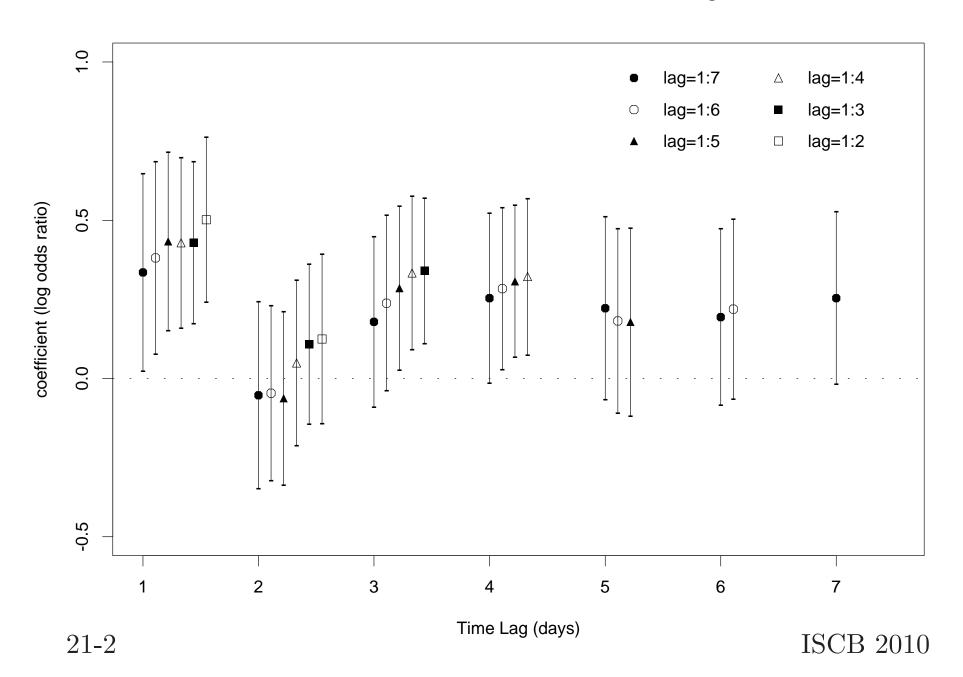
$$[X(t+1) \mid Y(t), X(t)] \sim [X(t+1) \mid X(t)]$$

- Analysis Issues:
 - Include single lagged covariates (current, cumulative)
 - * MSCM: $E[Sick(t) \mid Stress(t-k)]$
 - * **USRDS**: $E[\mathsf{Hosp}(t) \mid \mathsf{Dose}(t-k)]$
 - Include multiple lagged covariates
 - * MSCM: $E[Sick(t) \mid Stress(t-1), Stress(t-2)]$
 - * **USRDS**: $E[\mathsf{Hosp}(t) \mid \mathsf{Dose}(t-1), \mathsf{Dose}(t-2)]$

Lag Coefficient Function



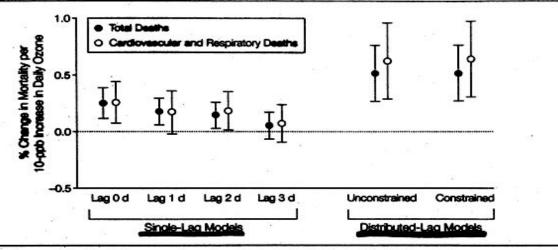
Multivariate models with different lags



Bell et al. *JAMA* (2004)

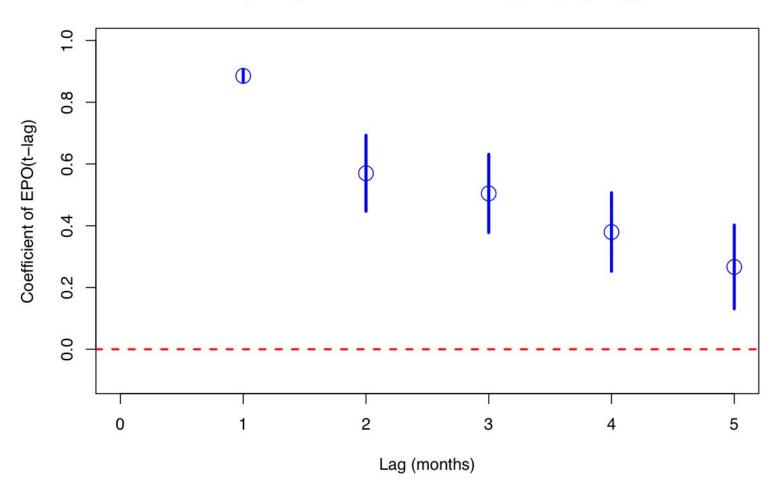
OZONE AND MORTALITY IN US URBAN COMMUNITIES

Figure 1. Percentage Change in Daily Mortality for a 10-ppb Increase in Ozone for Total and Cardiovascular Mortality, for Single-Lag and Distributed-Lag Models

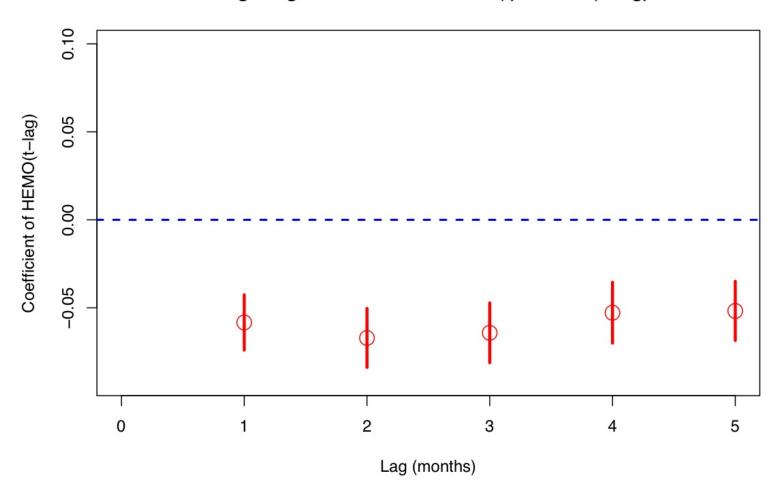


The single-lag model reflects the percentage increase in mortality for a 10-ppb increase in ozone on a single day. The distributed-lag model reflects the percentage change in mortality for a 10-ppb increase in ozone during the previous week. Error bars indicate 95% posterior intervals.

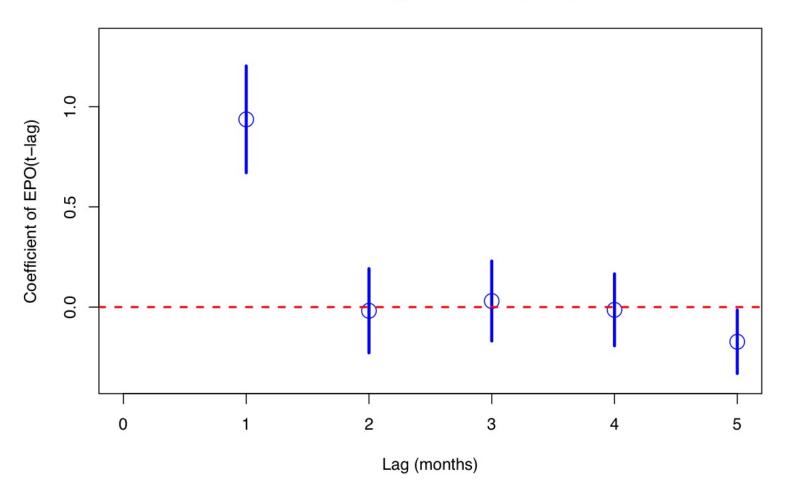
Single lag coefficients for Death(t) \sim Epo(t-lag)



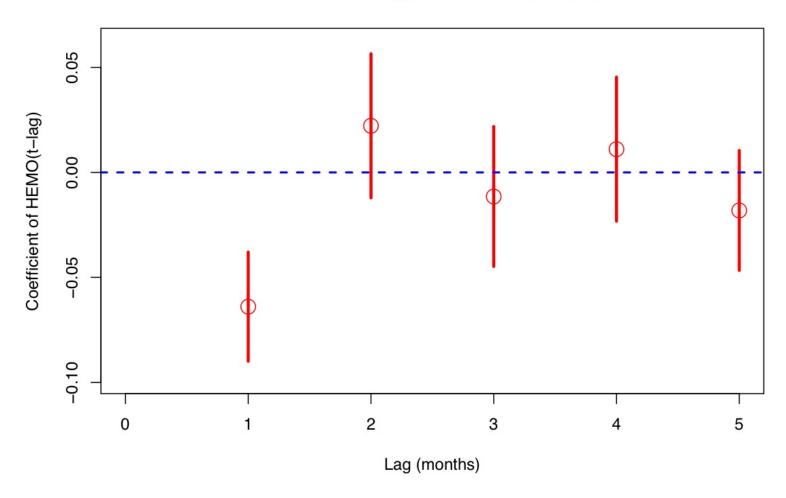
Single lag coefficients for Death(t) \sim Hemo(t-lag)



Coefficients for Death(t) \sim Hemo(t-1) + Epo(t-1) + ...



Coefficients for Death(t) \sim Hemo(t-1) + Epo(t-1) + ...



Endogenous: Analysis

Definition: – The covariate is influenced by past outcomes (or intermediate variables)

$$Y(t) \rightarrow X(t+1)$$

• Implication:

$$E[Y_i(t) \mid X_i(1), \dots, X_i(n)]$$

depends on $X_i(s)$ for s > t (future values of covariate).

- Role for causal inference concepts.
- See: DHLZ (2002) section 12.5 for introduction.

Causal Targets of Inference

Longitudinal Treatment

$$\operatorname{vec}(X_0) \equiv [X(1) = \mathbf{0}, X(2) = \mathbf{0}, \dots, X(n) = \mathbf{0}]$$

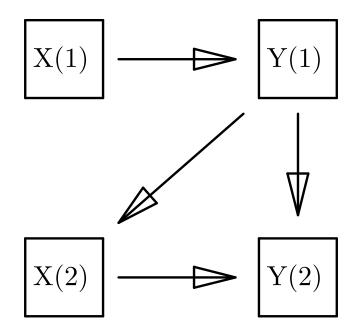
 $\operatorname{vec}(X_1) \equiv [X(1) = \mathbf{1}, X(2) = \mathbf{1}, \dots, X(n) = \mathbf{1}]$

- Population Means
 - \triangleright Mean of population if all subjects had X=1 at all times, and similar population mean if X=0 at all times.

$$\mu_0(n) \equiv E[Y(n) \mid \text{vec}(X_0)]$$

$$\mu_1(n) \equiv E[Y(n) \mid \text{vec}(X_1)]$$

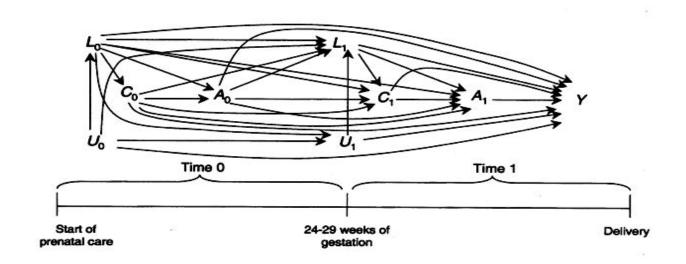
Endogenous Covariates



treatment / response exposure

Bodnar et al. AJE (1997)

MSMs for Causal Effects of Time-dependent Treatment



Model / Estimation

- G-computation
 - ▶ Model: model outcome given past outcomes / exposure.

$$P[Y(t) \mid X(t), \{Y(s), X(s)\} \; s < t] \quad : \quad \text{outcome}$$

$$P[X(t) \mid \{Y(s), X(s)\} \; s < t] \quad : \quad \text{exposure}$$

Compute: compute means of interest by allowing intermediate effects, Y(s), to occur naturally, but controlling exposure.

$$\mu_1(t) = E_t \{ E_s[Y(t) \mid \mathbf{X(t)=1}, \{Y(s), \mathbf{X(s)=1}\} \ s < t] \}$$

Model / Estimation

- Marginal Structural Models
 - ▶ Model: model exposure given past outcomes / exposure.

$$[X(t) | \{Y(s), X(s)\} | s < t]$$

Compute: compute a regression of the outcome using inverse probability weights (IPW) to control for exposure selection bias.

Table 1: Regression of stress, S_{it} , on illness, I_{it-k} k=0,1, and previous stress, S_{it-k} k=1,2,3,4+ using GEE with working independence.

est.	s.e.	Z
-1.88	(0.36)	-5.28
0.50	(0.17)	2.96
0.08	(0.17)	0.46
0.92	(0.15)	6.26
0.31	(0.14)	2.15
0.34	(0.14)	2.42
1.74	(0.24)	7.27
-0.26	(0.13)	-2.01
0.16	(0.12)	1.34
-0.19	(0.07)	-2.83
-0.09	(0.07)	-1.24
0.03	(0.12)	0.21
0.42	(0.13)	3.21
-0.16	(0.12)	-1.28
	-1.88 0.50 0.08 0.92 0.31 0.34 1.74 -0.26 0.16 -0.19 -0.09 0.03 0.42	-1.88 (0.36) 0.50 (0.17) 0.08 (0.17) 0.92 (0.15) 0.31 (0.14) 0.34 (0.14) 1.74 (0.24) -0.26 (0.13) 0.16 (0.12) -0.19 (0.07) -0.09 (0.07) 0.03 (0.12) 0.42 (0.13)

Table 2: MSM estimation of the effect of stress, S_{it-k} $k \geq 1$, on illness, I_{it} .

	est.	s.e.	Z
(Intercept)	-0.71	(0.40)	-1.77
S_{it-1}	0.15	(0.14)	1.03
S_{it-2}	-0.19	(0.18)	-1.05
S_{it-3}	0.18	(0.15)	1.23
$mean(S_{it-k}$, $k \geq 4)$	0.71	(0.43)	1.65
employed	-0.11	(0.21)	-0.54
married	0.55	(0.17)	3.16
maternal health	-0.13	(0.10)	-1.27
child health	-0.34	(0.09)	-3.80
race	0.72	(0.21)	3.46
education	0.34	(0.22)	1.57
house size	-0.80	(0.18)	-4.51

method	logOR
GEE cross-sectional association	0.66
GEE with seven days lagged	1.38
Transition model (direct effect)	0.50
G-computation	0.80
MSM	0.85

Summary of Endogenous

- Interest in exposure over time more than simply the acute (most recent) exposure.
- A variable (perhaps outcome) is both a **consequence** of exposure at early times, and a **cause** of exposure at later times.
- Intermediate and confounder.
- G-computation
- MSM
- Interest in outcomes under a controlled and static treatment plan.

EPO: November 2006 NEJM

- Drüeke | CREATE
 - Control Hemoglobin rather than fix the dose.
 - * Low group (11.0-12.5)
 - * Normal group (13.0-15.0)
- Singh CHOIR
 - Control Hemoglobin rather than fix the dose.
 - * Low group (11.3)
 - * Normal group (13.5)
- Research Question(s)

Q: What target hemoglobin should be used? How to use observational data to compare different targets and/or compare mortality experience to RCT data?

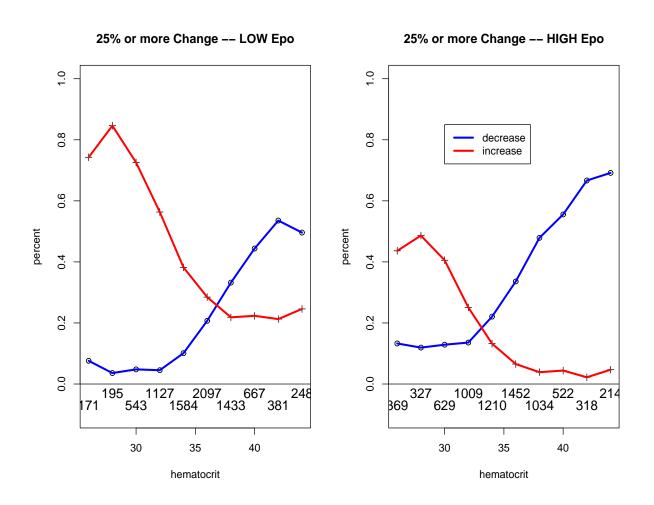
Analysis of Dynamic Treatment

• Note: The guidelines for Epo do not suggest a static dose be administered. Rather, dose is driven by the state of the intermediate (Hb):

$$X(t+1) = \begin{cases} 1.25 \times X(t) & \text{if } Z(t) \le 11 \\ X(t) & \text{if } 11 < Z(t) \le 13 \\ 0.75 \times X(t) & \text{if } Z(t) > 13 \end{cases}$$

- This corresponds to a dynamic treatment guideline, \mathcal{G}_1 .
- Q: How to formulate DOSE questions in this setting?
 - \triangleright \mathcal{G}_1 corresponds to correction of $\pm 25\%$ at Hb=(11,13).
 - \triangleright Compare to a \mathcal{G}_2 which uses alternative target Hb threshold(s).

USRDS Data (2003 sample)



Some Literature on Dynamic Treatment

- Hernán et al. (2006) "Comparison of dynamic treatment regimes via inverse probability weighting" $Basic \ \ \ Clin \ Pharm \ \ \ Tox$, 98; 237-242.
- Robins, Orellana, and Rotnitzky (2008) "Estimation and extrapolation of optimal treatment and testing strategies" *Statistics in Medicine*, 27: 4678-4721.
- Cotton and Heagerty (2010) "Inference for the comparison of dynamic treatment regimens with application to epoetin dosing strategies" (submitted)

(*) Drop-out

- Extensive literature has been developed. See summaries:
 - ▶ Book: DHLZ (2002) Chapter 13
 - \triangleright Book: (Geert)² Chapters 14-21(!)
- Main issue is selection bias (Little & Rubin, 1987):

$$E[Y_i(t) \mid X_i, R_i(t) = 1] \neq E[Y_i(t) \mid X_i]$$

- Main approaches to analysis with MAR, (NI sensitivity):
 - Inverse Probability Weighting (IPW)
 - * See: Robins, Rotnitzky & Zhao (1995)
 - Correctly specified likelihood
 - * See: Laird (1988)

(*) LDA with Death

- Different than drop-out
- With Drop-out:

$$E[Y_i(t) \mid X_i] = E[Y_i(t) \mid X_i, R_i(t) = 1] \times P[R_i(t) = 1 \mid X_i] + E[Y_i(t) \mid X_i, R_i(t) = 0] \times P[R_i(t) = 0 \mid X_i]$$

- Linear Mixed Models (LMM) applied to the observed data where $R_i(t)=1$ can validly estimate parameters in the mean $E[Y_i(t)\mid X_i]$ when data are MAR.
- With Death:

$$E[Y_i(t) \mid X_i] = E[Y_i(t) \mid X_i, D_i(t) = 0] \times P[D_i(t) = 0 \mid X_i] + E[Y_i(t) \mid X_i, D_i(t) = 1] \times P[D_i(t) = 1 \mid X_i]$$

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(*) LDA with Death: Analysis

- Analysis conditional on death information:
 - ▶ Full (future) stratification:

$$E[Y_i(t) \mid X_i(t), \mathbf{T_i} = \mathbf{s}] \ s > t$$

- * See: Pauler, McCoy & Moinpour (2003)
- Partial (current status) conditioning:

$$E[Y_i(t) \mid X_i(t), \mathbf{T_i} > \mathbf{t}]$$

- * See: Kurland and Heagerty (2004)
- Conditional on principal strata (potential status):

$$E[Y_i(t \mid 1) - Y_i(t \mid 0) \mid \{\mathbf{T_i(0)} > \mathbf{t}, \mathbf{T_i(1)} > \mathbf{t}\}]$$

* See Frangakis and Rubin (2002), Rubin (2007)

(*) LDA with Death: Comments on Analysis

- Full stratification using $[T_i = s]$ s > t
 - \triangleright Compares groups defined by X_i comparable in terms of death.
 - Conditions on future (not yet observed) information.
- Partial (current status) conditioning: $[T_i > t]$
 - Conditions on observed vital status.
 - \triangleright Compares groups defined by X_i after selection by death.
- Principal stratification: $[\{T_i(0) > t, T_i(1) > t\}]$
 - \triangleright Compares subgroups defined by X_i comparable in terms of death.
 - Conditions on unobservable potential status.

(*) Measurement Process

- Health status (current) may influence the scheduling, $S_i(t)$, of administrative data collection.
- Does past/current/future outcome (health status) predict time-until next visit?

$$\lambda_k(t \mid t_{i1}, \dots, t_{ik-1}, Y_i(t), X_i)$$

 Potential for bias if visit intensity process is not independent of response process.

(*) Measurement Process: Analysis

- Estimating Equations: Lin, Scharfstein & Rosenheck (2004)
 - Assume λ_k independent of current/future $Y_i(t)$, but may depend on observed past (including auxiliary variables).
 - \triangleright Model λ_k and use inverse intensity weights (IIW) GEE.
 - Semiparametric.
- Likelihood: Lipsitz et al. (2002)
 - Assume λ_k only depends on $Y_i(t-)$ and not on t_{i1}, \ldots, t_{ik} (given Y).

 - Parametric.

(*) Birth Processes

- When discussion of missing data or measurement processes we assume $Y_i(t)$ is measurable at any time t.
- For the Collaborative Perinatal Project (CPP) research focuses on measurements that are associated with a (recurrent) event time.
- Birth Process:
 - \triangleright B(t) is counting process for number of births for subject i through time t.
 - The birth outcome, $Y_i(t)$, is <u>only</u> defined at the time of birth: when dB(t)=1.
- Regression analysis for (recurrent) marked point process data

$$E[Y_i(t) \mid X_i(t), dB_i(t) = 1]$$

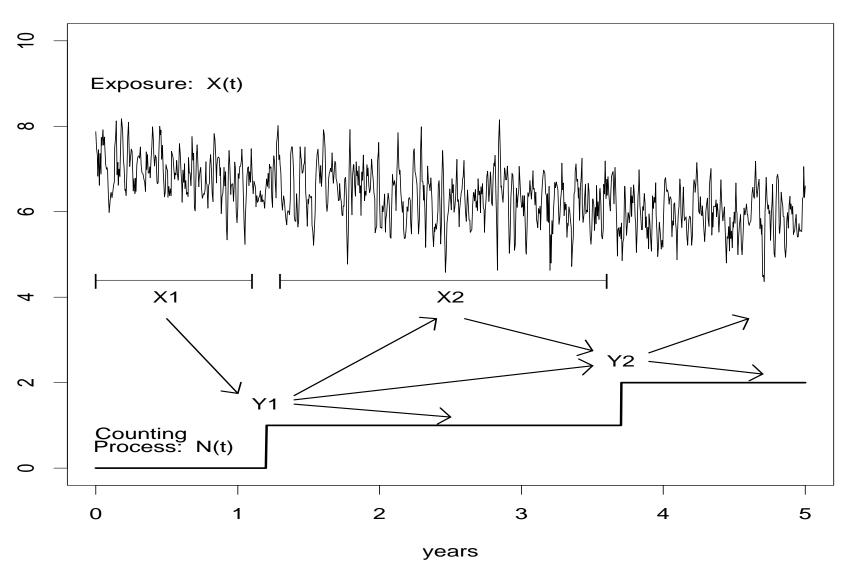
(*) Birth Processes: Mark Regression Analysis

 For some analyses the past response is an <u>intermediate</u> outcome and interest is in marginal means:

$$E[Y_i(t) \mid X_i(t), dB_i(t) = 1, H_i^X(t-), H_i^B(t-)]$$

- If $Y_i(t)$ predicts time-until future birth, $B_i(t+)$, then this endogeneity implies standard non-independence GEE or LMM will give biased estimates.
- If endogeneity then IEE valid characterization of association between birth outcome and prior exposure (and parity).
- French and Heagerty (2009)

Birth Data and Exposure



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(*) Birth Processes: simulations

• Scenario = probability of the next birth depends on the last birth outcome if there was one. Babies per mom is still between 1 and 5 with mean 2-3.

Estimation	Intercept est.		Slo	pe est.	
method	mean	mean (std dev)		(std dev)	
OLS using first	1.048	(0.352)	1.019	(0.181)	
born					
GEE with working	1.014	(0.041)	1.004	(0.038)	
independence					
Linear Mixed ML	0.741	(0.063)	0.914	(0.039)	
random intercept only					

(*) Inclusion: CPP Data

Cox model for time-until-second birth

covariate	est.	exp(est.)	s.e.(est.)	Z	р
weight(2)	0.1649	1.179	0.0861	1.914	0.056
weight(3)	0.1825	1.200	0.0798	2.285	0.022
weight(4)	-0.0152	0.985	0.1501	-0.101	0.920
site(2)	-0.3497	0.705	0.0895	-3.906	< 0.001
site(3)	-0.6911	0.501	0.0841	-8.218	< 0.001
site(4)	-0.0943	0.910	0.0847	-1.114	0.271
race(black)	0.0885	1.093	0.0844	1.048	0.293
race(other)	-0.0289	0.972	0.1501	-0.192	0.851
smoke	0.0594	1.061	0.0462	1.287	0.200

	First (n=8	born 3467)		d born 1960)	_	d born =527)		oirths 0954)	All b (n=10	oirths 0954)
	Linear Regn		Linear	r Regn	egn Linear Regn		GEE		Mixed Model	
	est.	s.e.	est.	s.e	est.	s.e.	est.	s.e.	est.	s.e.
Intercept SMOKE site(2) site(3) site(4) race(black) race(other) parity(2)	3280.5 -161.6 33.7 45.9 -84.8 -212.6 -180.3	(11.7) (13.1) (23.6) (19.6) (25.5) (24.5) (40.7)	3372.8 -179.5 -124.7 -36.2 -68.4 -221.9 -200.1	(23.2) (26.3) (51.1) (51.6) (56.5) (56.4) (88.6)	3334.0 -179.2 113.1 96.4 -146.3 -110.2 -248.7	(44.7) (49.7) (136.7) (105.7) (113.5) (113.3) (197.9)	3284.4 -164.7 10.9 36.6 -85.4 -208.3 -184.3 65.5	(10.9) (12.3) (21.1) (17.8) (22.7) (22.6) (36.3) (14.8)	3281.7 -157.6 17.1 39.8 -88.9 -210.1 -179.7 64.1	(11.1) (11.8) (22.4) (18.8) (24.2) (24.2) (38.7) (12.3)
parity(2)							63.7	(26.5)	81.9	(23.5)

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Some recommendations

- In applications we should identify factors that influence the secondary stochastic processes and choose appropriate statistical techniques in order to validly answer the scientific question.
- In statistical research reports we should be explicit about the assumptions we are making regarding the secondary stochastic processes.
- For time-dependent covariates ask about associations with both past and future covariate values – consider the factors that drive the covariate.