1

2.

(a) .

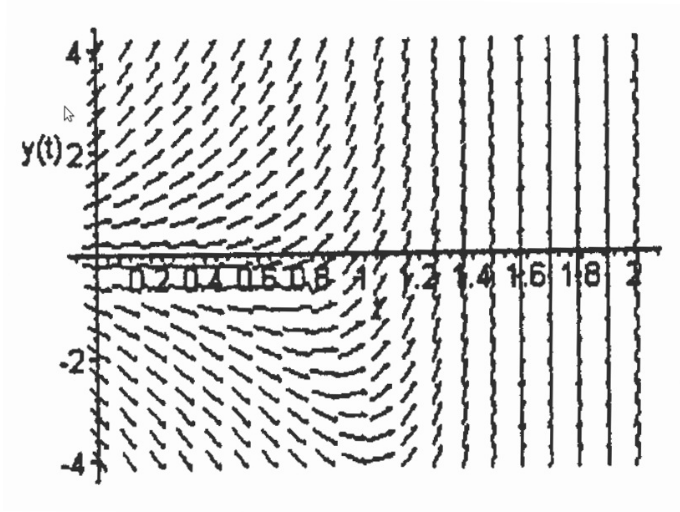

Figure 1:

(b) All slopes eventually become positive, hence all solutions will increase without bound.

(c) The integrating factor is $\mu(t) = e^{-2t}$, and hence $y(t) = t^3e^{2t}/3 + ce^{2t}$. It is evident that all solutions increase at an exponential rate.

6.

(a) .

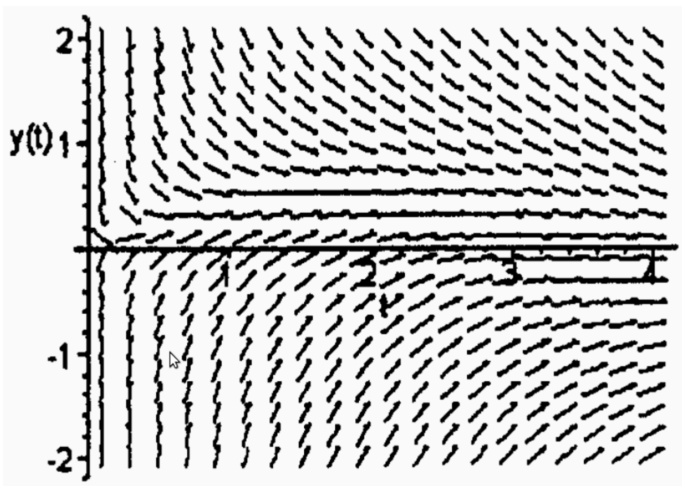

Figure 2:

(b) All solutions seem to converge to the function $y_0(t) = 0$.

(c) The integrating factor is $\mu(t) = t^2$ and hence the general solution is

$$y(t) = -\frac{cos(t)}{t} + \frac{sin(t)}{t^2} + \frac{c}{t^2} \tag{1}$$

in which c is an arbitrary constant. As t becomes large, all solutions converge to the function $y_0(t) = 0$.

14. The integrating factor is $\mu(t) = e^{2t}$. After multiplying both sides by $\mu(t)$, the equation can be written as $(e^{2t}y)' = t$. Integrating both sides of the equation results in the general solution $y(t) = t^2e^{-2t}/2 + ce^{-2t}$. Invoking the specified condition, we require that $e^{-2}/2 + ce^{-2} = 0$. Hence $c = -1/2$, and the solution to the initial value problem is $y(t) = (t^2 - 1)e^{-2t}/2$.

19. After writing the equation in *standard form*, we find that the integrating factor is $\mu(t) = \exp(\int \frac{4}{t}dt) = t^4$. Multiplying both sides by $\mu(t)$, the equation can be written as $(t^4y)' = te^{-t}$. Integrating both sides results in $t^4y(t) = -(t+1)e^{-t} + c$. Letting $t = -1$ and setting the value equal to zero gives $c = 0$. Hence the specific solution of the initial value problem is $y(t) = -(t^{-3}+t^{-4})e^{-t}$.

2

2. For $x \neq -1$, the differential equation may be written as $ydy = [x^2/(1 + x^3)]dx$. Integrating both sides, with respect to the appropriate variables, we obtain the relation $y^2/2 = \frac{1}{3}ln|1 + x^3| + c$. That is, $y(x) = \pm\sqrt{\frac{2}{3}ln|1 + x^3| + c}$.

3. The differential equation may be written as $y^{-2}dy = -sinxdx$. Integrating both sides of the equation, with respect to the appropriate variables, we obtain the relation $-y^{-1} = cosx + c$. That is, $(C - cosx)y = 1$, in which C is an arbitrary constant. Solving for the dependent variable, explicitly, $y(x) = 1/(C - cosx)$.

8. Write the differential equation as $(1 + y^2)dy = x^2dx$. Integrating both sides of the equation, we obtain the relation $y + y^3/3 = x^3/3 + c$, that is, $3y + y^3 = x^3 + C$.

17.

(a) $y(x) = -5/2 - \sqrt{x^3 - e^x + 13/4}$.

(b) .

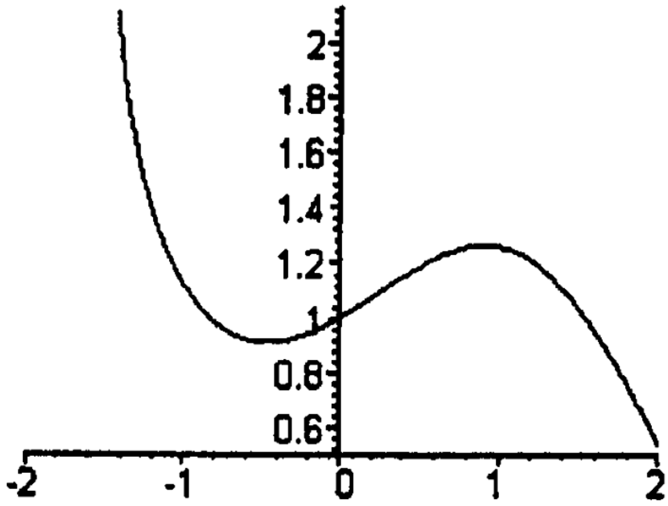

Figure 3:

(c) The solution is valid for $x > -1.45$ and $x < 4.6297$. This value is found by estimating the root of $4x^3 - 4e^x + 13 = 0$.

3

5.

(a) Let Q be the amount of salt in the tank. Salt enters the tank of water at a rate of $2\frac{1}{4}(1 + \frac{1}{2}sint) = \frac{1}{2} + \frac{1}{4}sint\ oz/min$. It leaves the tank at a rate of $2Q/100\ oz/min$. Hence the differential equation governing the amount of salt at any time is

$$\frac{dQ}{dt} = \frac{1}{2} + \frac{1}{4}sint - Q/50. \tag{2}$$

The initial amount of salt is $Q_0 = 50\ oz$. The governing ODE is *linear*, with integrating factor $\mu(t) = e^{t/50}$. Write the equation as $(e^{t/50}Q)' = e^{t/50}(\frac{1}{2} + \frac{1}{4}sint)$. The specific solution is $Q(t) = 25 + [12.5sint - 625cost + 63150e^{t/50}]/2501\ oz$.

(b) .

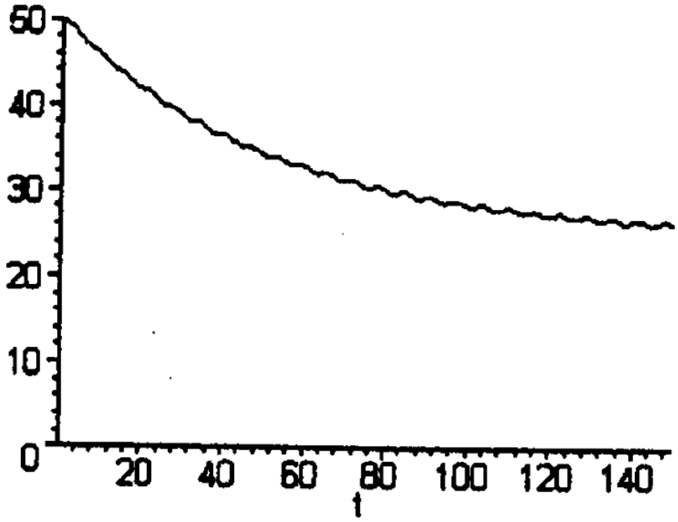

Figure 4:

(c) The amount of salt approaches a *steady* state, which is an oscillation of amplitude 1/4 about a level of 25 *oz*.

8.

(a) The equation governing the value of the investment is $dS/dt = rS$. The value of the investment, at any time, is given by $S(t) = S_0e^{rt}$. Setting $S(T) = 2S_0$, the required time is $T = ln(2)/r$.

(b) For the case $r = 7\% = .07$, $T \approx 9.9yrs$.

(c) Referring to Part(a), $r = ln(2)/T$. Setting $T = 8$, the required interest rate is to be approximately $r = 8.66\%$.

13. Let $P(t)$ be the population of mosquitoes at any time t. The rate of *increase* of the mosquito population is rP. The population *decreases* by 20, 000 *per day*. Hence the equation that models the population is given by $dP/dt = rP - 20,000$. Note that the variable t represents *days*. The solution is $P(t) = P_0e^{rt} - \frac{20,000}{r}(e^{rt} - 1)$. In the absence of predators, the governing equation is $dP_1/dt = rP_1$, with solution
$P_1(t) = P_0e^{rt}$. Based on the data, set $P_1(7) = 2P_0$, that is, $2P_0 = P_0e^{7r}$. The growth rate is determined as $r = ln(2)/7 = .09902$ *per day*. Therefore the population, including the *predation* by birds, is $P(t) = 2 \times 10^5e^{.099t} - 201,997(e^{.099t} - 1) = 201,997.3 - 1977.3e^{.099t}$.