Math 307D First Final Exam

March 20, 2013

Instructions: There are ten problems, with the value of each problem indicated, for a total of 160 points. You are allowed the use of one page of handwritten notes, front and back, on standard sized paper. You are also allowed use of a scientific calculator (but graphing calculators and other calculational devices are not allowed)
The next page of the exam contains a table of Laplace transforms, which you may use in your solutions.

- Work the problems in the space provided. If you need more space, use the back of the page, and clearly indicate that you are doing so.
- Neatness counts! A well—organized solution, even with mistakes, will get more partial credit than a haphazard collection of unrelated calculations.
- Put the answer you want considered in the $\boxed{\text{BOX}}$ provided.
- You **MUST** show all your work and reasoning to receive credit. If in doubt, ask for clarification.
- Turn off all cell phones and pagers.

Problem 1	15 points	
Problem 2	15 points	
Problem 3	15 points	
Problem 4	15 points	
Problem 5	15 points	
Problem 6	20 points	
Problem 7	15 points	
Problem 8	15 points	
Problem 9	15 points	
Problem 10	20 points	
Total	160 points	

1. (15 points). Solve the initial value problem

$$y' = \frac{x+1}{x^2(2y+1)}, \qquad y(1) = 0$$

Answer:

2. (15 points) Solve the initial value problems

$$(t^2 + 1)y' + (2t)y = te^t, \qquad y(0) = 2.$$

Answer: $\boxed{}$

3. (15 points) A population of bacteria increases at a rate proportional to the square root of the current population. At time $t = 0$ days the population is 100, and at time $t = 4$ the population is 900.

   (a) Find a formula for the population $P(t)$ at time t days.

   (b) At what time does the population reach 3600?

Answer:

4. (15 points) Solve the initial value problem

$$y'' + 2y' + 5y = 0, \qquad y(0) = 2, y'(0) = 4.$$

Answer:

5. (15 points) Find the general solution to

$$y'' + y' - 2y = e^t + \sin t.$$

Answer: $\boxed{}$

6. (20 points) A 10 lb weight stretches a spring 2 ft. Suppose the Weight is pulled down an additional foot and given a downward velocity of 2 ft/sec. There is no damping, nor are there external forces. Determine the amplitude of the subsequent motion.

Answer:

7. (15 points) Let $f(t)$ be a function whose Laplace transform is $F(s)$. Define a new function

$$g(t) = e^{-2t} f(3t).$$

Determine the Laplace transform $G(s)$ of $g(t)$ in terms of $F(s)$.

Answer: $\boxed{}$

8. (15 points) Find the inverse Laplace transform of

$$F(s) = \frac{2s-3}{s^2+2s+10}$$

Answer: $\boxed{}$

9. (15 points). Use Laplace transforms to solve the initial value problem

$$y'' - y' - 6y = 0, \quad y(0) = 1, y'(0) = -1.$$

You can *check your answer*!

Answer:

10. (20 points) Let $f(t)$ be the forcing function defined by $f(t) = 1$ if $0 \le t \le 1$, and $f(t) = 0$ if $t > 1$. Solve the initial value problem

$$y'' + y = f(t), \qquad y(0) = 0, y'(0) = 0.$$

Answer: