Stat-491-Fall2014-Assignment-IV

Hariharan Narayanan

November 10, 2014

Note: This assignment is due on 17 November 2014.

1. (a) Suppose one of the states of an irreducible Markov chain has a self loop. Show that the chain has period 1.

   (b) Let N be a collection of integers closed under addition. Let 100 be the first integer in N for which the immediate successor is also in N. Find an integer after which every integer is present in N.

   (c) Using Euclid's Algorithm, find a, b such that $100a + 81b = 1$.

2. At a local 2 year college, 2/3 of freshmen become sophomores, 1/4 remain freshmen, and 1/12 drop out. Two thirds of sophomores graduate, 1/4 remain as sophomores and 1/12 dropout. Take the states to be 'F' for freshmen,'S' for sophomore, 'D' for dropout and 'G' for graduate. Let $q(i, R)$ denote the probability that from state i, we will eventually reach the absorbing state R. Let $l(i, R)$ denote the expected time to reach absorbing state R from state i.

   (a) Write the transition matrix for the Markov chain with rows and columns in the order F, S, G, D.

   (b) Write the set of equations for $q(i, G)$, $q(i, D)$.

   (c) Write the set of equations for $l(i, G \cup D)$.

   (d) What fraction of new students eventually graduate?

   (e) What is the expected time of graduation or dropout for a sophomore student?

3. A certain Markov chain has transition matrix

$$\begin{pmatrix} & A & B & C & D & E \\ A & 0 & 0 & 1/3 & 1/3 & 1/3 \\ B & 1/3 & 1/3 & 1/3 & 0 & 0 \\ C & 0 & 1/3 & 1/3 & 1/3 & 0 \\ D & 0 & 0 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

   (a) Which are the transient and which, the recurrent states? Are there any absorbing states?

   (b) Write linear equations for $q(i, k), l(i, k)$, where k is an absorbing state.

4. A certain Markov chain has its states partitioned into T, R_1, R_2, R_3. Let $x \in T$.

   (a) How will you compute the probability of starting from x and reaching either R_1 or R_2?

   (b) Let $r \in R_1$. How will you compute the probability of starting from x and reaching r *before* any other recurrent state?

5. Let P be a square matrix with nonegative entries. Suppose all rows of P have row sum strictly less than 1. Show that $I - P$ is invertible.

6. A certain branching process has the following one step probability for number of progeny: $p_0 = a, p_1 = b, p_2 = c$. For the following cases

   - $a = 1/2, b = 1/4, c = 1/4$;
   - $a = 1/4, b = 1/4, c = 1/2$;

   (a) Write down the equation the probability of extinction satisfies;

   (b) Examine the probability of extinction.