Stat-491-Fall2014-Assignment-III

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October 31, 2014

NOTE: This assignment is due on November 5 2014.

1. (4 points). 3 white balls and 3 black balls are distributed in two urns in such a way that each urn contains 3 balls. At each step we draw one ball from each urn and exchange them. Let X_n be the number of white balls in the left urn at time n. (a) Compute the transition probability matrix and its stationary distribution.
   (b) Verify whether the Markov chain is reversible.

2. (4 points). A certain Markov chain has transition matrix

$$\left(\begin{array}{c|c|c} 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 2/3 \\ 1/3 & 1/3 & 1/3 \end{array}\right) \tag{1}$$

   (a) Compute the stationary distribution.
   (b) Verify whether the Markov chain is reversible.

3. (4 points). The transition matrix of a certain Markov chain on states A, B, C, D, is given below.

$$\left(\begin{array}{c|c|c|c|c} & A & B & C & D \\ A & 1/2 & 1/3 & 1/6 & 0 \\ B & 1/3 & 1/3 & 1/3 & 0 \\ C & 1/3 & 1/3 & 1/3 & 0 \\ D & 0 & 0 & 1 & 0 \end{array}\right) \tag{2}$$

   (a) Which states are recurrent and which, transient? (b) Compute the stationary distribution for the Markov chain.

4. (4 points) Consider the Markov chains in Figure 1
   (a) compute stationary distributions for the first two chains on A, B, C and A', B', C', D'
   (b) For the third Markov chain on $A, B, C, D, A', B', C', D'$ find a stationary distribution $\pi(\cdot)$ which takes value $1/12$ on A. Which states are recurrent and which transient?

5. (4 points) In each of the following cases examine whether the random variable T is a 'stopping time' by

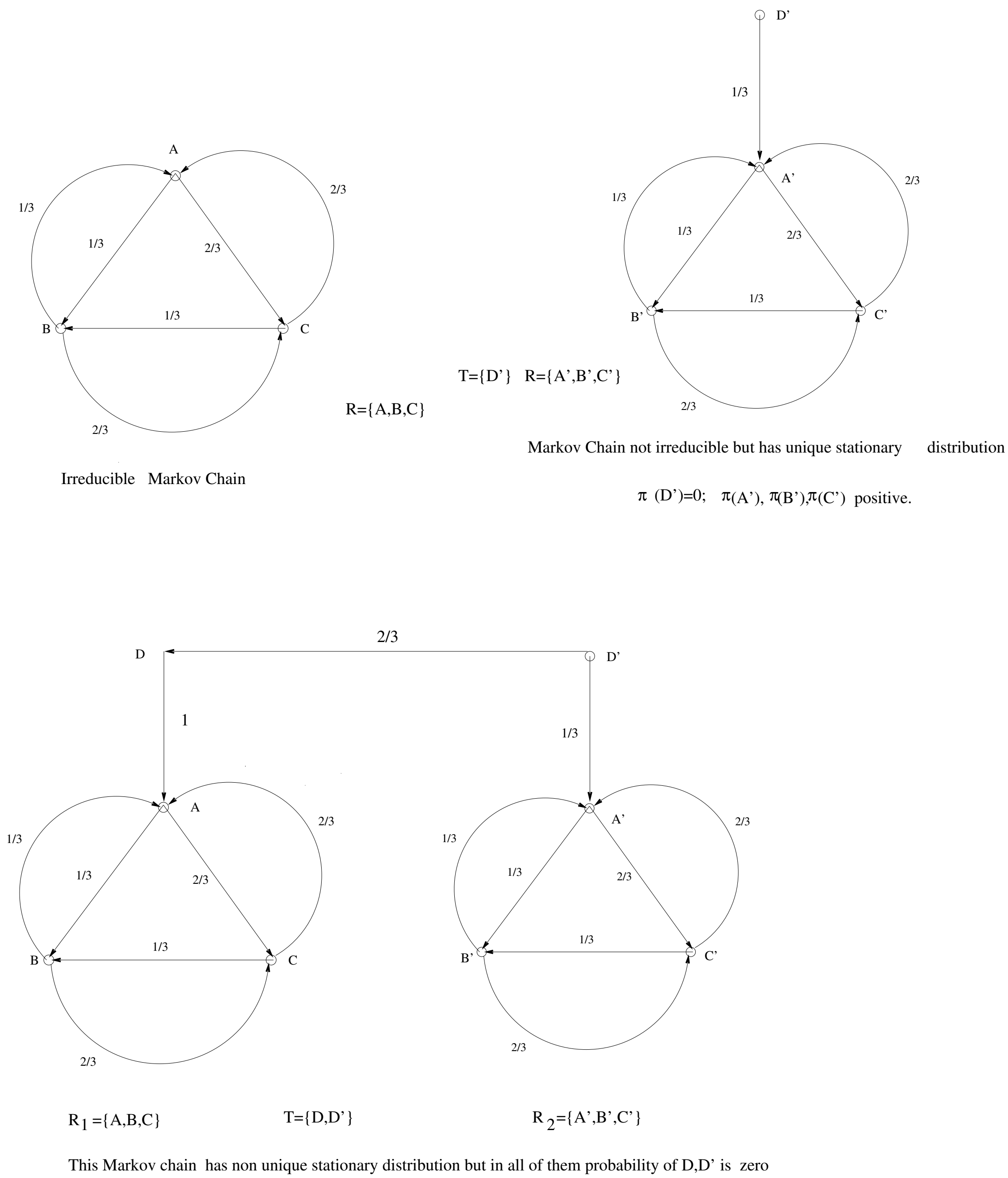

Figure 1: Markov Chains

(i) precisely describing the set of sequences of states which determines whether $T = n$ or not and (ii) justifying your conclusion about T being a stopping time.

(iii) Given that the initial state is chosen according to a probability distribution $\pi(\cdot)$, how would you determine $Pr\{T = n\}$?

(a) $T = n$ if $X_n = y$;

(b) $T = n$ if $X_n = y$ and for $0 \le i \le n, X_i = y$ exactly five times;

(c) $T = n$ if for $n \leq i \leq n + 10, X_i = y$ exactly five times;

(d) $T = n$ if $X_n = a\ state\ y$ such that $Pr\{X_{n+10} = z \mid X_n = y\}$ is 1/3. Here z is defined but y is specified only through the probability condition.