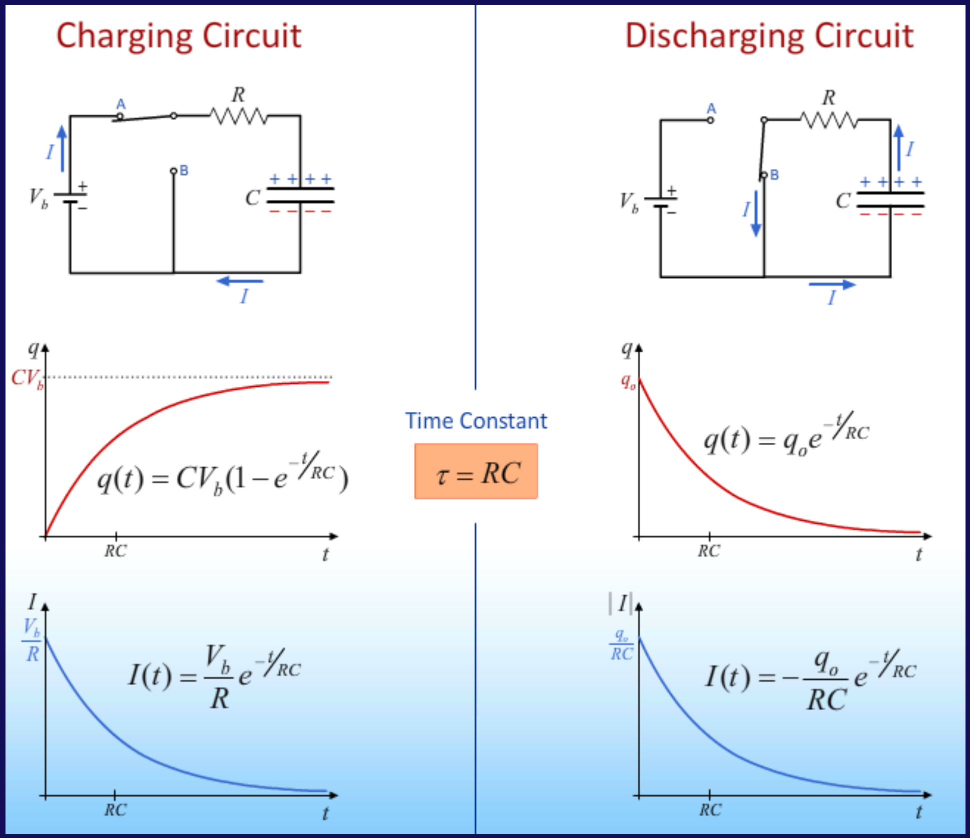
RC Cola



I was really confused about the calculus that lead to the current equation. Could you derive this in class? **- We'll do that today**

Phys 122 Lecture 15
G. Rybka

Business…

- Exam Thursday ! (no RC circuits on this exam)
- Practice exam solutions posted on Wed.
- I'll email you your seats
- Wed will be a review, with a few problems and a review of the key points of this unit

Lecture Material on Exam:

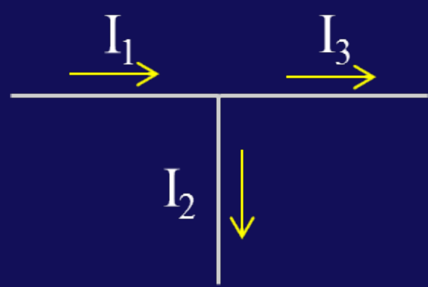
- **Lectures 7 - 14; From Electrical Potential Energy through Current, Voltage and Resistance, as listed on the syllabus**

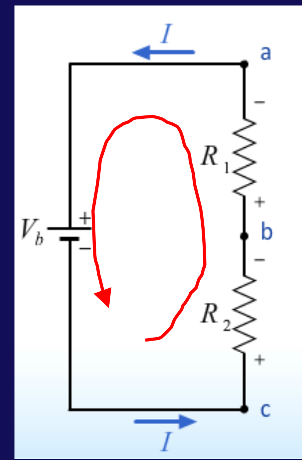
"I feel as if I'm starting to get this material, but with the midterm coming up, nothing new is going to stick."

Kirchoff Rules

1) Label all currents

Choose any direction





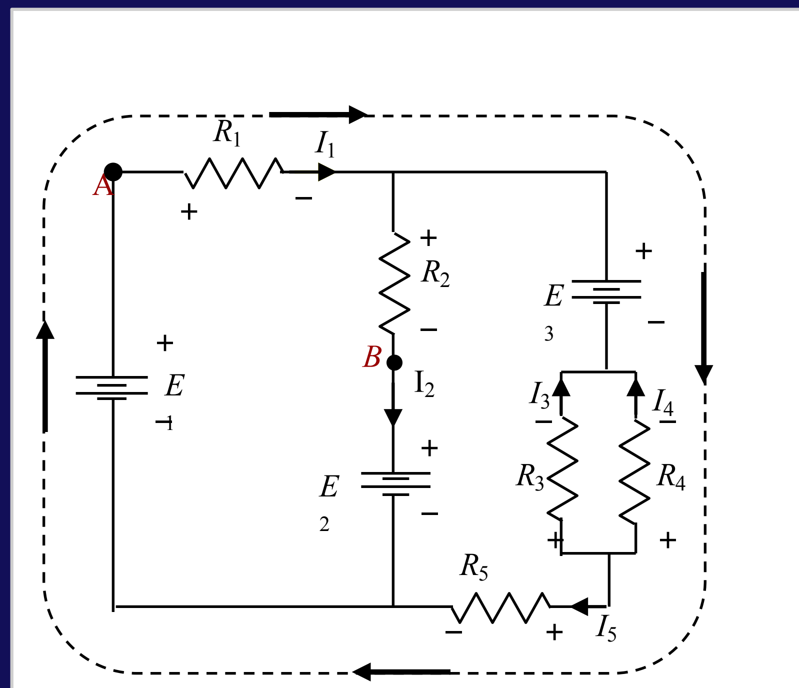
2) Label +/– for all elements

Current goes + to – (for resistors)

or remember to track direction of loop vs direction of current

3) Choose loop and direction

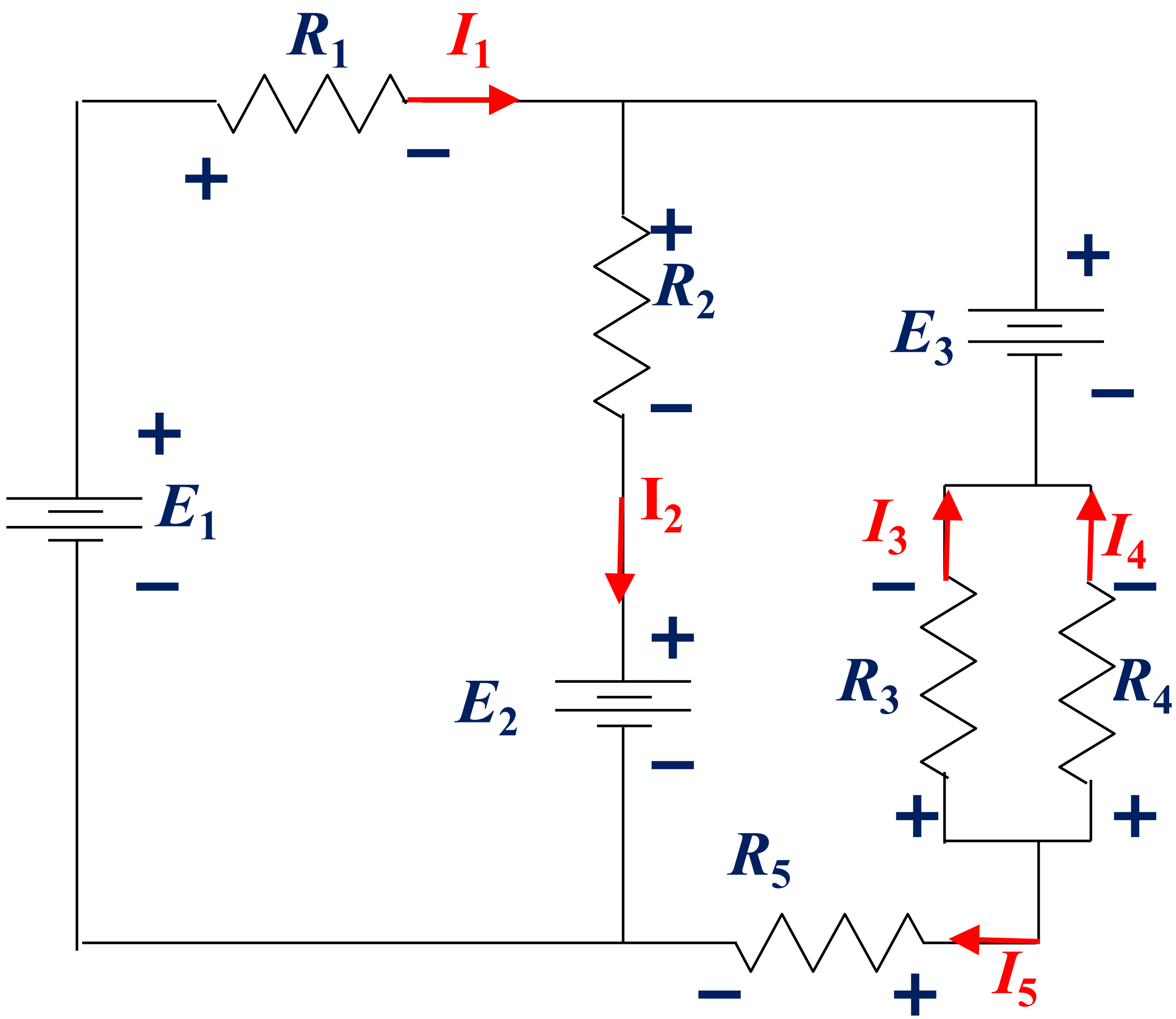
Must start on wire, not element.



4) Write down voltage drops

First sign you hit is sign to use.

5) Write down node equation $I_{in} = I_{out}$



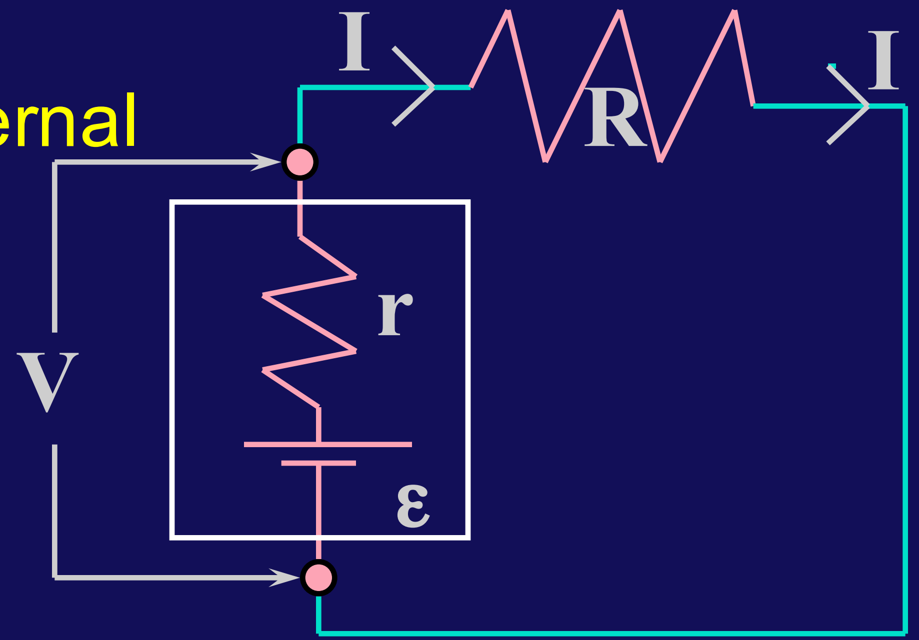
R_1
I_1
+
−
+
R_2
−
+
E_3
−
+
E_1
−
I_2
I_3
I_4
−
−
+
E_2
−
R_3
R_4
+
+
R_5
−
+
I_5

Okay, a brief summary

- **Capacitors:**
  - We learned how they add in series, parallel and how a dielectric increases the capacitance
- **Resistors:**
  - We learned how they add in series, parallel and how the material they are made of affects the resistance
- **Networks**
  - Voltage Loop: Drops (+); Gains (-); Label currents; Label the +/- in dir of I; Sum of Drops/Gains around loop = 0
  - Current rule: Sum of currents in junction = sum out
- **Power**
  - From battery: $P = IV$
  - In resistors, $P = IV = I^2R = V^2/R$
  - In general, look at sum of VI's for total

Batteries
(non-ideal)

- Parameterized with "internal resistance"



$$V = \varepsilon - Ir$$

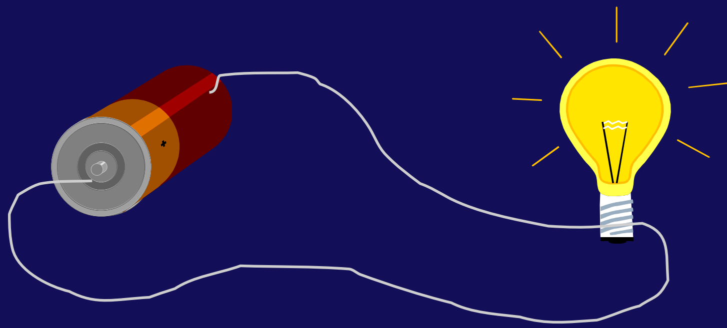
$$IR - \varepsilon + Ir = 0$$

$$I = \frac{\varepsilon}{R + r} \quad þ \quad V = \varepsilon \frac{R}{R + r}$$

Power

Batteries & Resistors

Energy expended



chemical
to electrical
to heat

Rate is: $$\frac{\text{energy}}{\text{time}} = \text{power}$$

What's happening?

Charges per time

Assert: $$P = VI$$

Energy "drop" per charge

For Resistors: $$P = (IR)I = I^2R$$

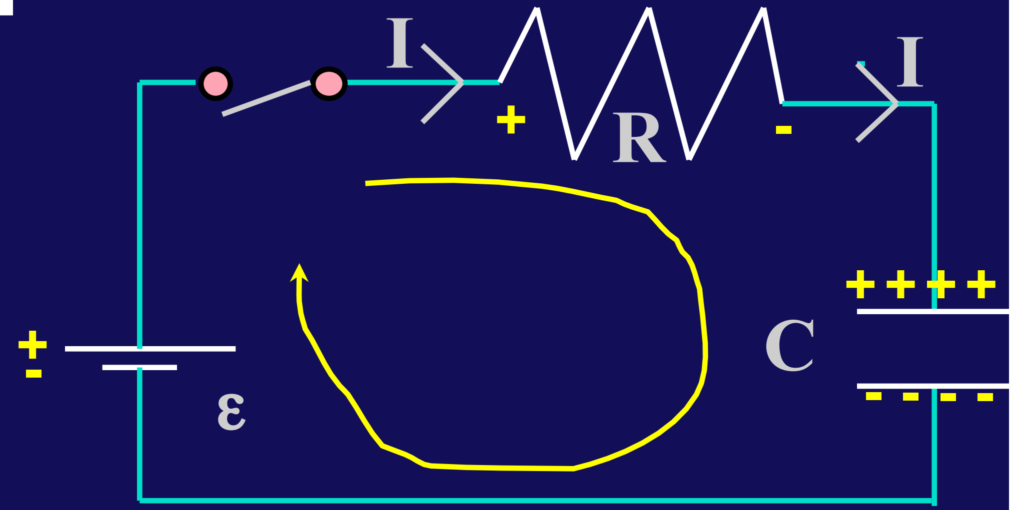
Units okay? $$\frac{\text{Joule}}{\text{Coulomb}} \times \frac{\text{Coulomb}}{\text{second}} = \frac{\text{J}}{\text{s}} = \text{Watt}$$

More complex now...

Add a Capacitor to circuit

Recall voltage "drop" on C is

$$V = \frac{q}{C}$$



Write KVL:

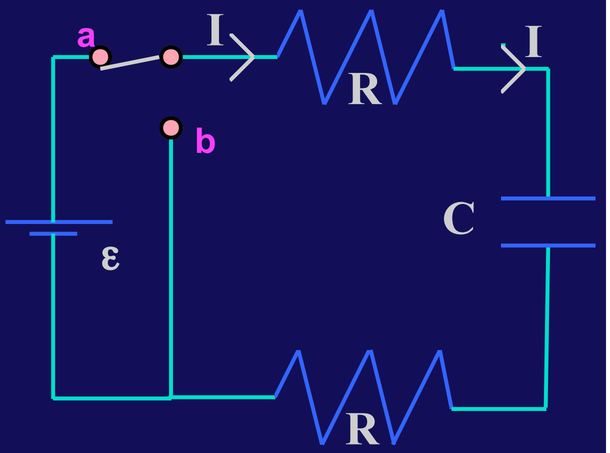
$$IR + \frac{q}{C} - \varepsilon = 0$$

Consider that $I = \frac{dq}{dt}$ and substitute. Now eqn has only "q"

$$R\frac{dq}{dt} + \frac{q}{C} - \varepsilon = 0$$

Differential Equation !

Clicker



At t = 0 the switch is thrown from position b to position a in the circuit shown: The capacitor is initially <u>uncharged</u>.

– What is the value of the current I_0 just after the switch is thrown?

(a) $I_0 = 0$

(b) $I_0 = \varepsilon/2R$

(c) $I_0 = 2\varepsilon/R$

Just after the switch is thrown, the capacitor still has no charge

→ therefore the voltage drop across the capacitor = 0

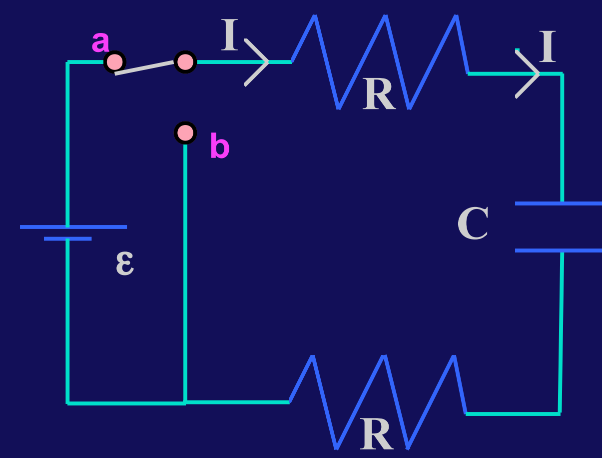
→ it acts like a "wire" in the circuit

Applying KVL to the loop at t = 0,

→ $IR + 0 + IR - \varepsilon = 0 \Rightarrow I = \varepsilon/2R$

Can you go over why charge flows through the capacitor right when the switch is flipped? Essentially, why does it act as a wire immediately after the switch is thrown? Thanks.

Clicker



What is the value of the current $I_{¥}$ after a very long time?

(a) $I_{¥} = 0$ **(b) $I_{¥} = ε/2R$** **(c) $I_{¥} > 2ε/R$**

As the current continues to flow, the charge on the capacitor continues to grow.

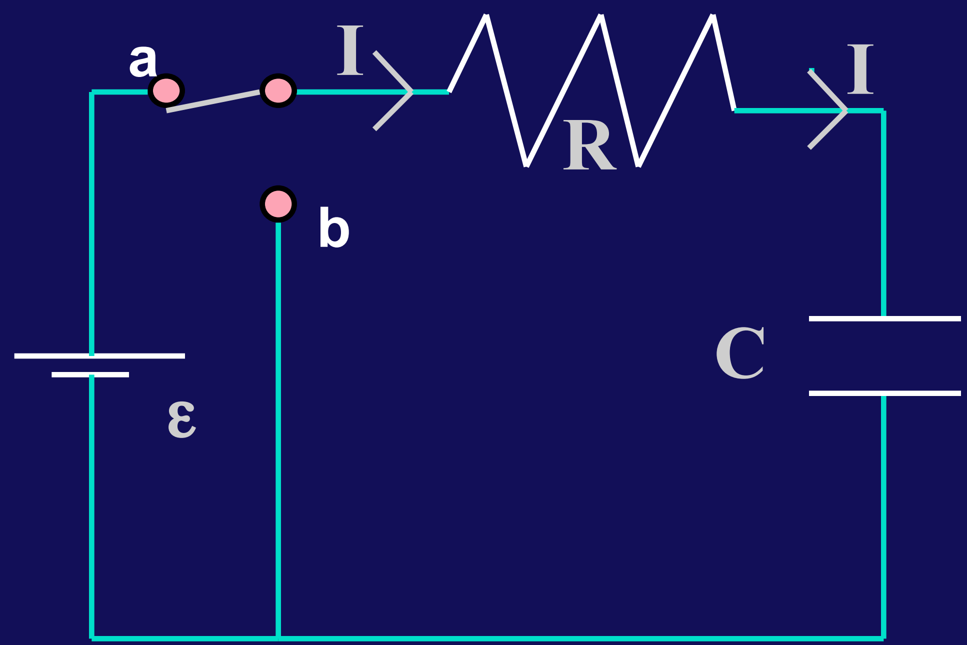
As the charge on the capacitor continues to grow, the voltage across the capacitor will increase.

The voltage across the capacitor is limited to ε; **the current goes to 0.**

RC Circuit Time Dependence: *Charging*

- **Loop equation**

$$\varepsilon = R\frac{dq}{dt} + \frac{q}{C}$$



- **Guess solution:**

$$q = C\varepsilon\left(1 - e^{-t/RC}\right)$$

- **Check that it is a solution:**

$$\frac{dq}{dt} = -C\varepsilon e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$\Rightarrow \quad R\frac{dq}{dt} + \frac{q}{C} = \varepsilon e^{-t/RC} + \varepsilon\left(1 - e^{-t/RC}\right) = \varepsilon \quad !$$

Note that this "guess" incorporates the boundary conditions:

$$t = 0 \Rightarrow q = 0$$

$$t = \infty \Rightarrow q = C\varepsilon$$

RC Circuit Time Dependence: *Charging*

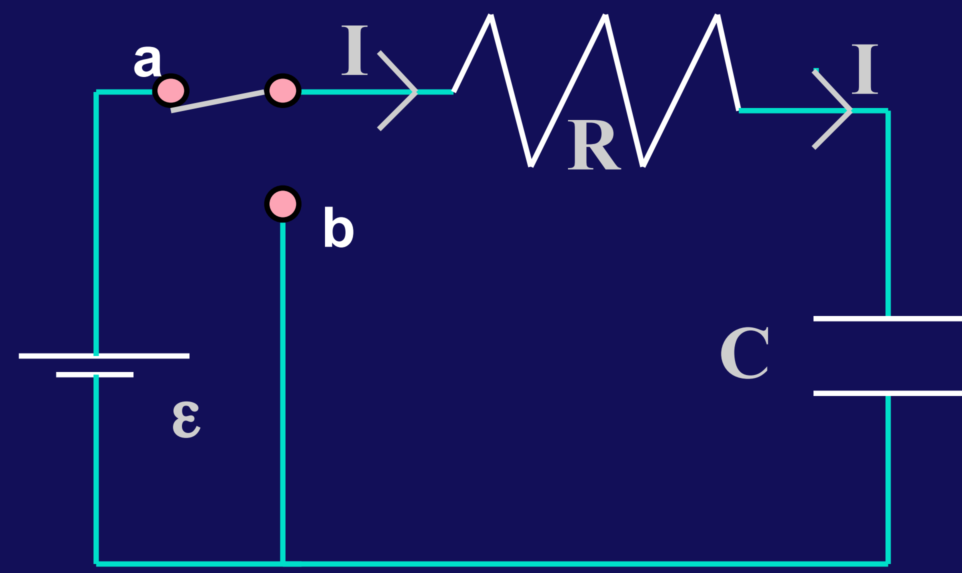
- **Charge capacitor:**

$$q = C\varepsilon\left(1 - e^{-t/RC}\right)$$

- **Current is found from differentiation:**

$$I = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

Þ



Conclusion:

- **Capacitor reaches its final charge ($Q=C\varepsilon$) exponentially with time constant τ = RC.**
- **Current decays from max ($I_{max} = \varepsilon/R$) with same time constant.**

Charging Capacitor

Charge on C

$$q = C\varepsilon\left(1 - e^{-t/RC}\right)$$

Max = $C\varepsilon$

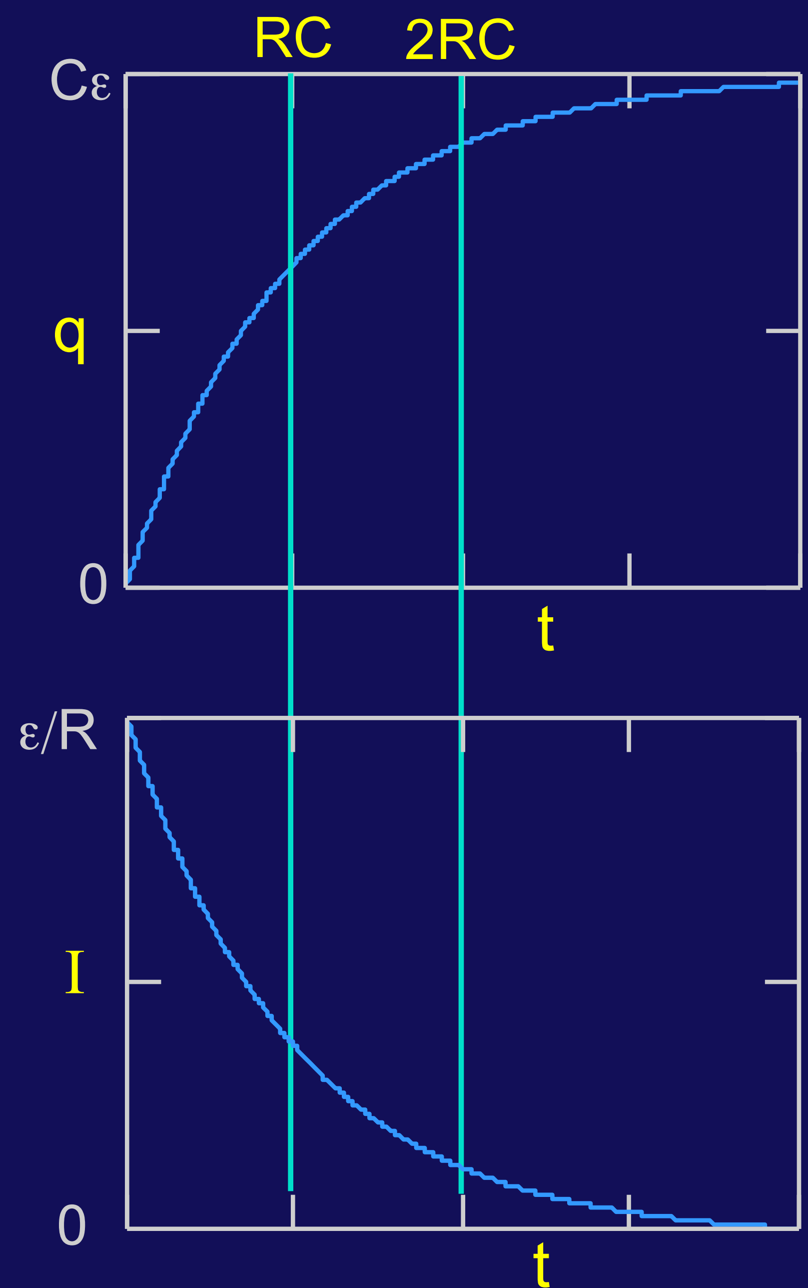
63% Max at t=RC

Current

$$I = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-t/RC}$$

Max = ε/R

37% Max at t=RC



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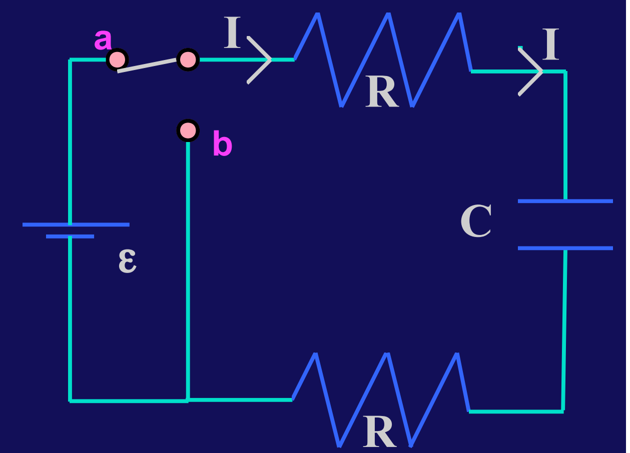
Hint: think graphically!



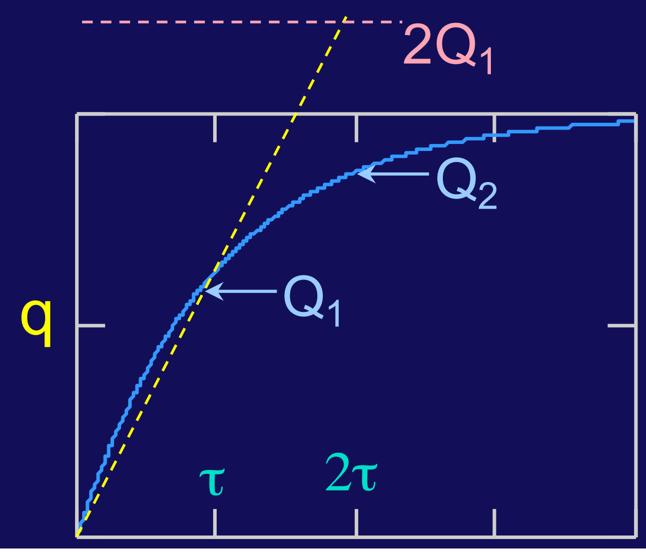
- At t = 0 the switch is thrown from position b to position a in the circuit shown: The capacitor is initially uncharged.
  - At time $t = t_1 = \tau$, the charge Q_1 on the capacitor is (1-1/e) of its asymptotic charge $Q_f = C\varepsilon$.
  - What is the relation between Q_1 and Q_2 , the charge on the capacitor at time $t = t_2 = 2\tau$?

(a) $Q_2 < 2\,Q_1$ (b) $Q_2 = 2\,Q_1$ (c) $Q_2 > 2\,Q_1$

The charge q on the capacitor increases with time as: $$q = C\varepsilon\left(1 - e^{-t/2RC}\right)$$

The question is: how does this charge increase *differ from a linear increase?*

- See graph: the charge increase is not as fast as linear.
- The rate of increase is proportional to the current (dq/dt) which decreases with time.
- Therefore, $Q_2 < 2Q_1$.

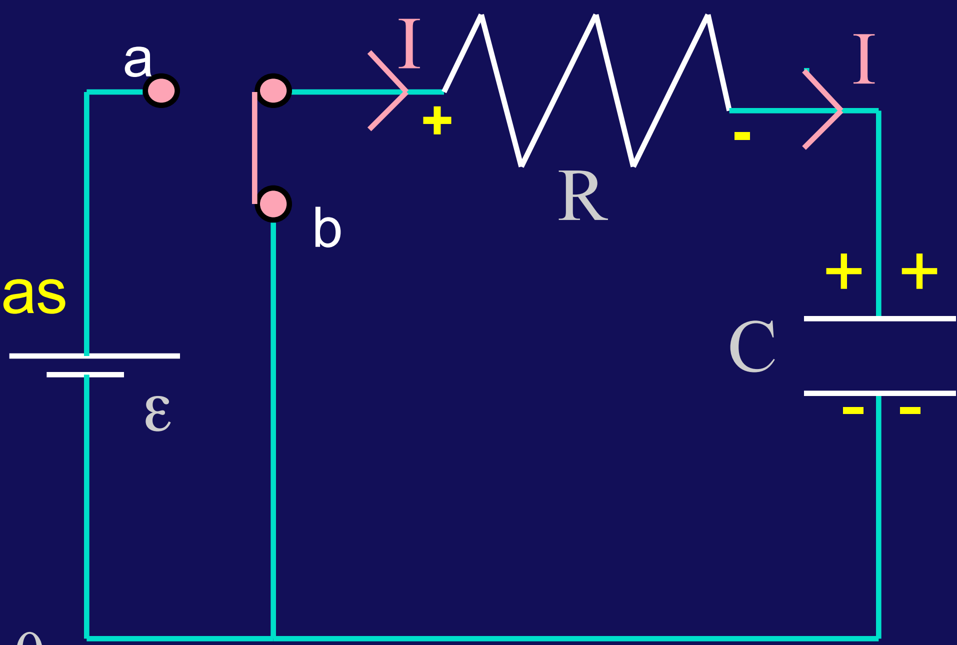


RC Circuit Time Dependence: *Discharging*

C initially charged with $Q = C\varepsilon$

Connect switch to b at $t = 0$.

Calculate current and charge as function of time.



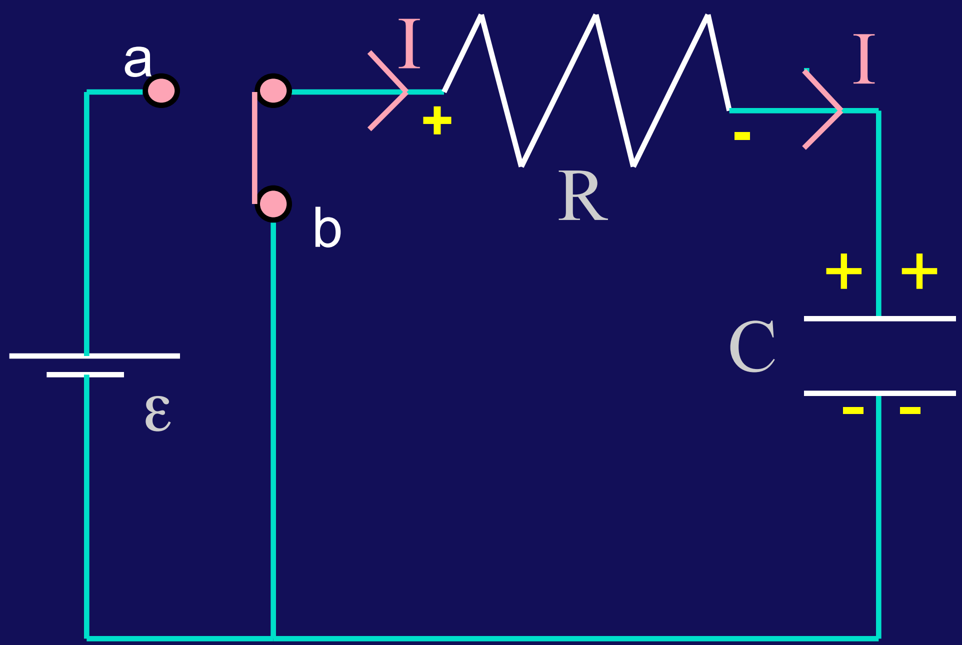
- Loop theorem Þ $IR + \frac{q}{C} = 0$
- Convert to differential equation for q:

$$I = \frac{dq}{dt} \quad Þ \quad R\frac{dq}{dt} + \frac{q}{C} = 0$$

RC Circuit Time Dependence: *Discharging*

- Discharge capacitor:

$$R\frac{dq}{dt}+\frac{q}{C}=0$$



- Guess solution:

$$q = C\,\varepsilon e^{-t/RC}$$

- Check that it is a solution:

$$\frac{dq}{dt}=C\varepsilon e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$\Rightarrow \quad R\frac{dq}{dt}+\frac{q}{C}=-\varepsilon e^{-t/RC}+\varepsilon e^{-t/RC}=0 \quad !$$

Note that this "guess" incorporates the boundary conditions:

$$t=0 \Rightarrow q=C\varepsilon$$

$$t=\infty \Rightarrow q=0$$

RC Circuit Time Dependence: *Discharging*

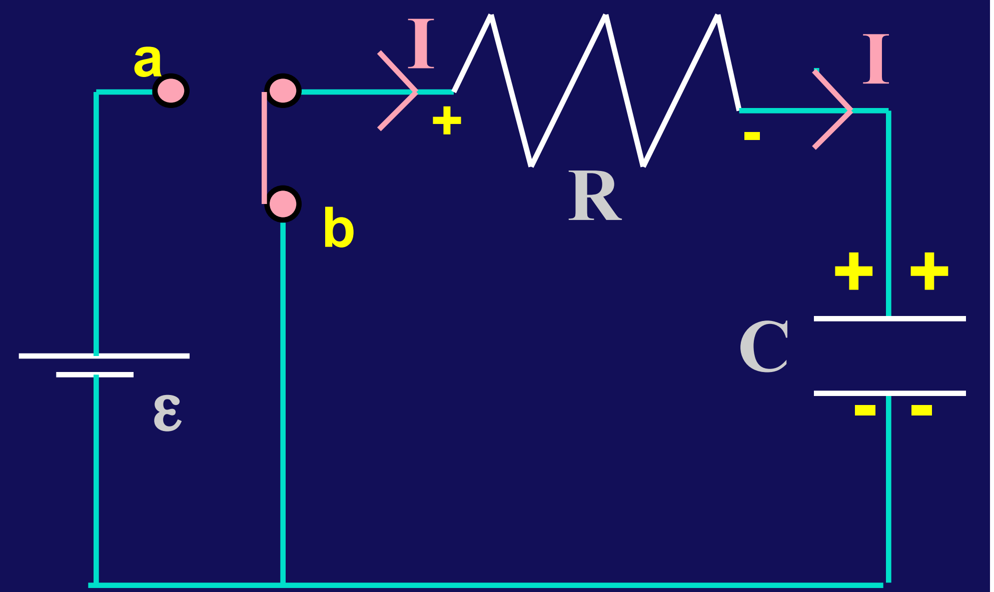
- **Discharge capacitor:**

$$q = C\,\varepsilon e^{-t/RC}$$

- **Current is found from differentiation:**

$$I = \frac{dq}{dt} = -\frac{\varepsilon}{R} e^{-t/RC}$$

Þ



"Does current really "flow backwards" when the capacitor is discharged? I think it would help me have a better understanding of what's going on if you explained this process in terms of the flow of electrons."

Conclusion:

- Capacitor discharges exponentially with time constant $\tau = RC$
- Current decays from initial max value ($= -\varepsilon/R$) with same time constant

Discharging Capacitor

Charge on C

$$q = C\,\varepsilon e^{-t/RC}$$

Max = $C\varepsilon$

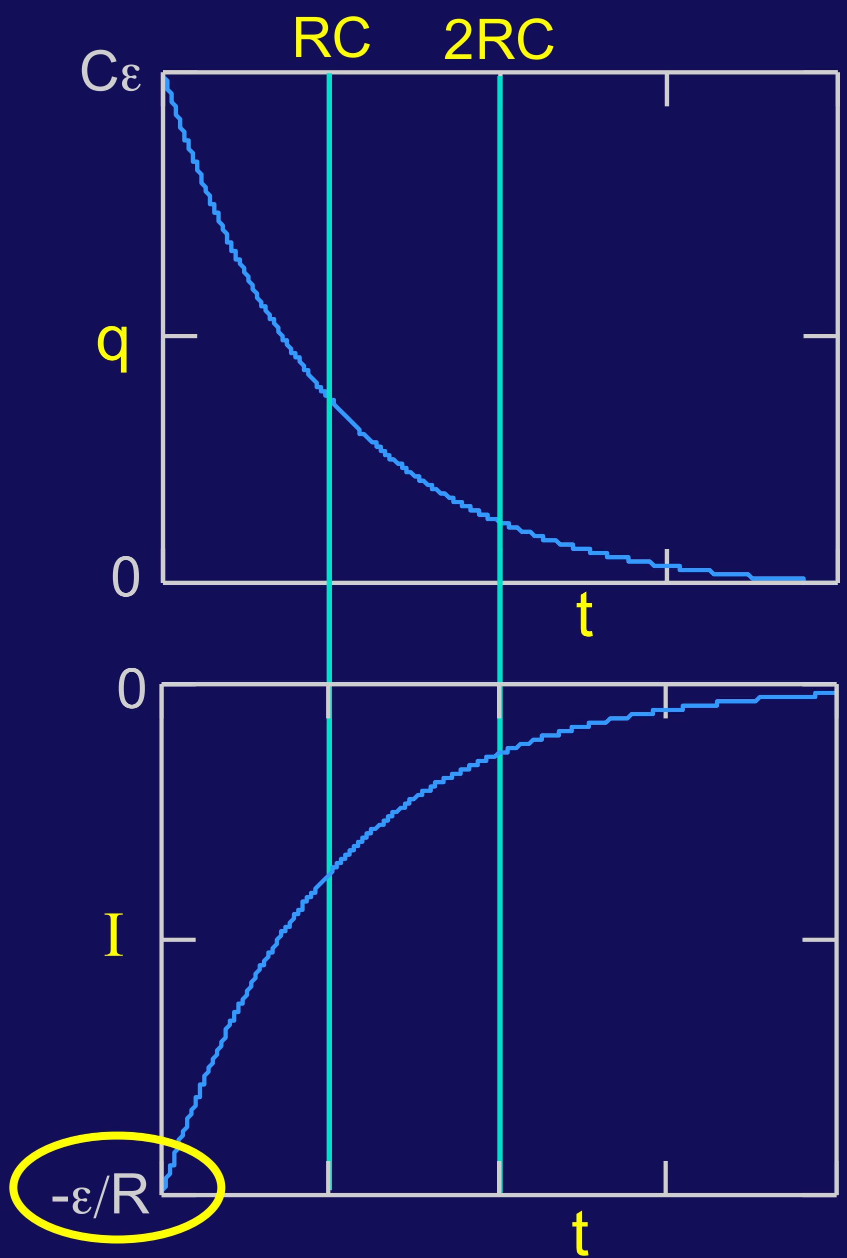
37% Max at t=RC

Current

$$I = \frac{dq}{dt} = -\frac{\varepsilon}{R} e^{-t/RC}$$

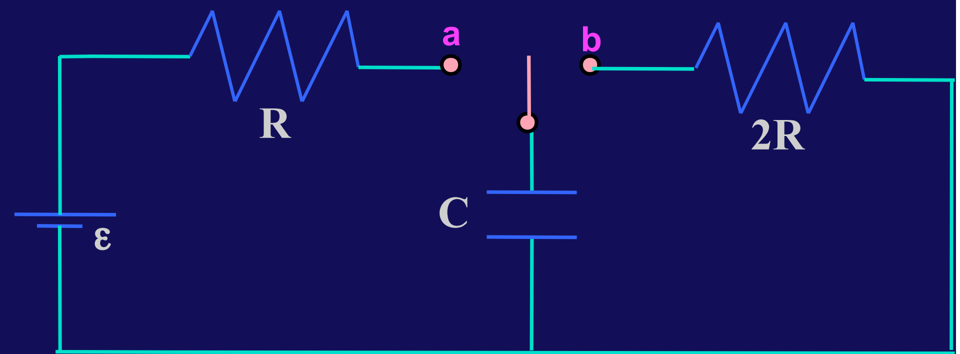
Max = $-\varepsilon/R$

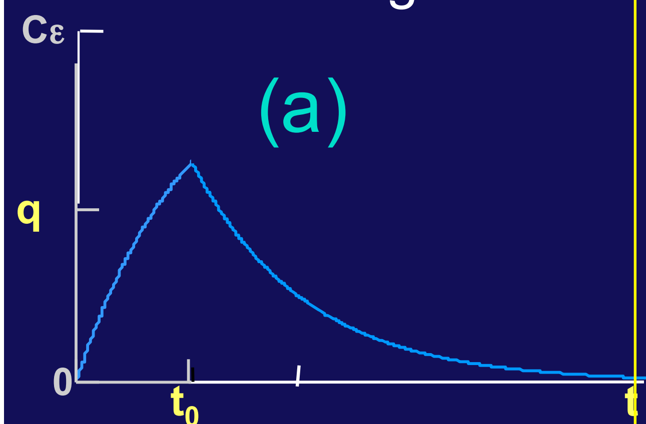
37% Max at t=RC

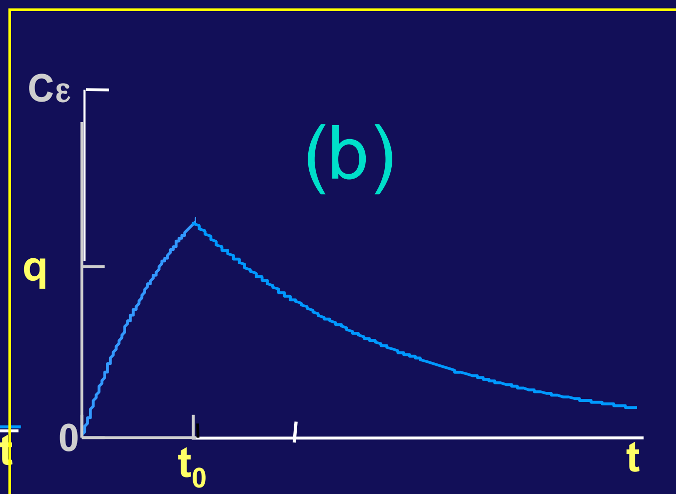


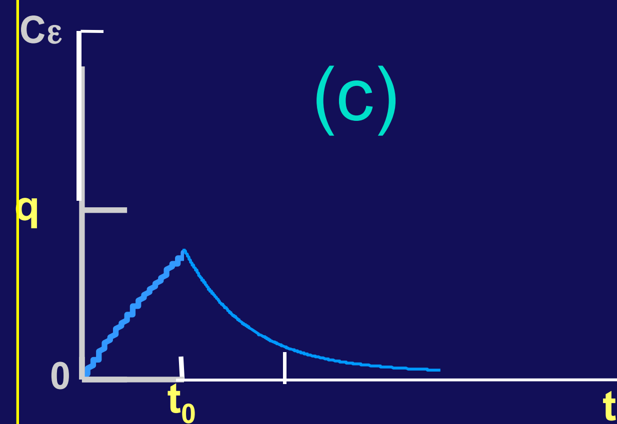
Clicker

- At $t = 0$ the switch is connected to position a in the circuit shown: The capacitor is initially uncharged.
  - At $t = t_0$, the switch is thrown from position a to position b.
  - Which of the following graphs best represents the time dependence of the charge on C?



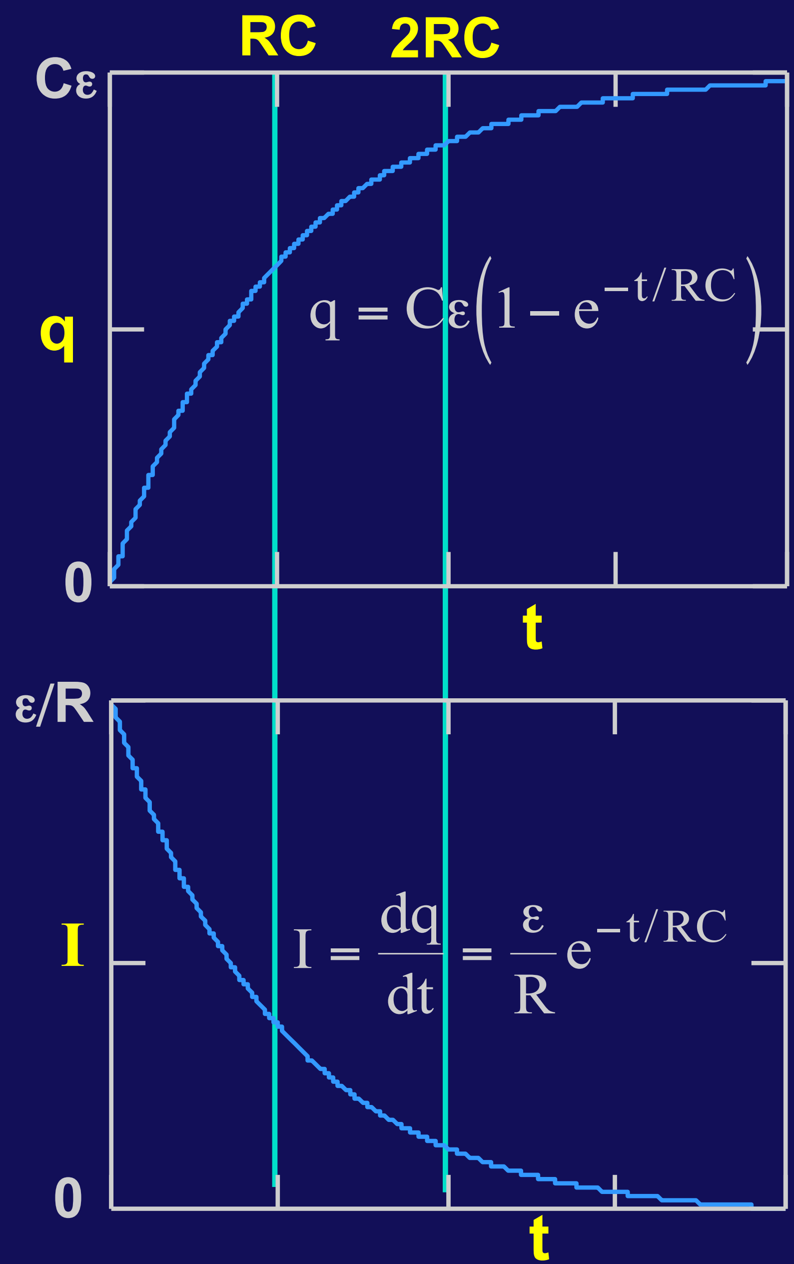




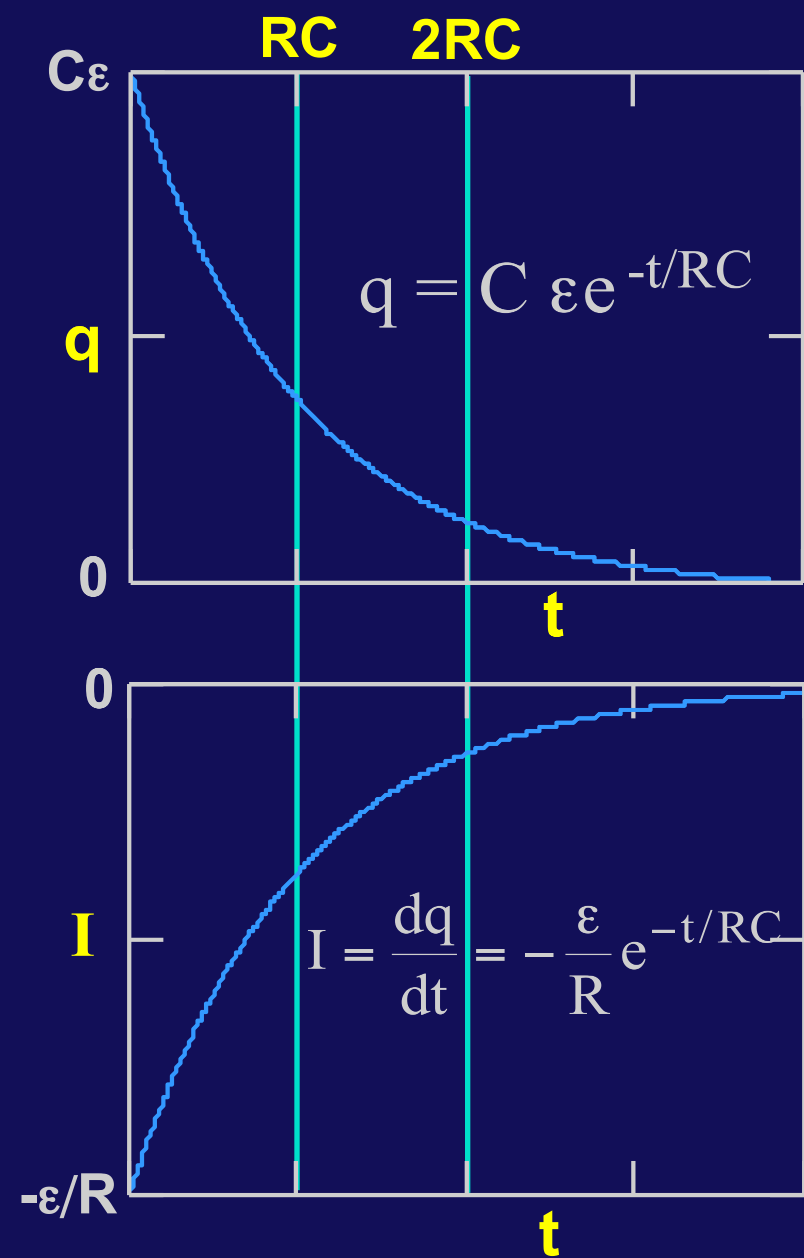


- For $0 < t < t_0$, the capacitor is charging with time constant $\tau = RC$
- For $t > t_0$, the capacitor is discharging with time constant $\tau = 2RC$
  - (a) has equal charging and discharging time constants
  - (b) has a larger discharging τ than a charging τ
  - (c) has a smaller discharging τ than a charging τ

Charging

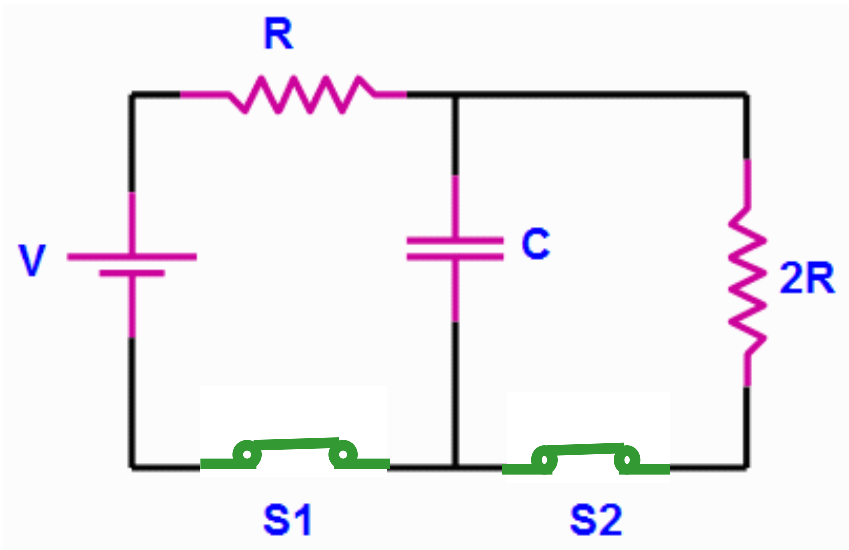


Discharging



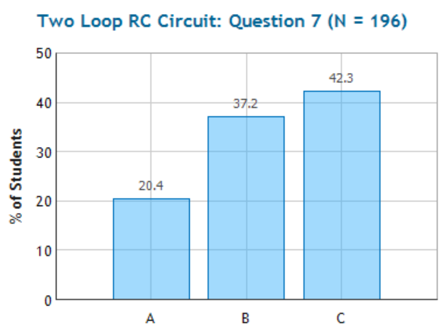
This is the tough one — CheckPoint 8

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



Now suppose both switches are closed. What is the voltage across the capacitor after we wait for a very long time.?

A ○ $V_c = 0$
B ○ $V_c = V$
C ○ $V_c = 2V/3$



After both switches have been closed for a long time

The current through the capacitor is zero

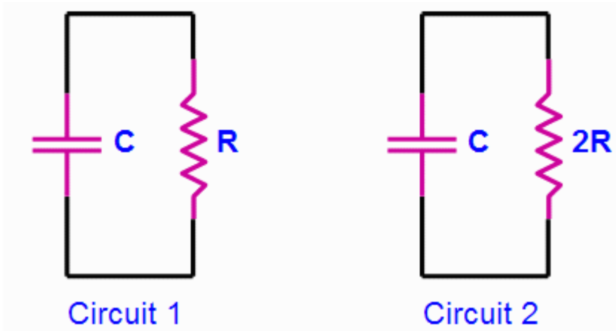
The current through R = current through $2R$

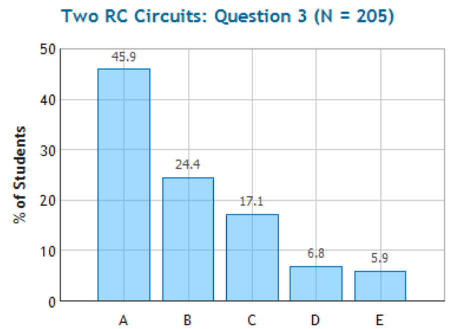
$$V_{capacitor} = V_{2R}$$

$$V_{2R} = 2/3\ V$$

CheckPoint 10 & 12

Same charge initially





Which circuit has the largest time constant?

A) Circuit 1

B) Circuit 2

C) Same

$$\tau = RC$$

Relationship of Q_1 to Q_2 at any time after t = 0 ?

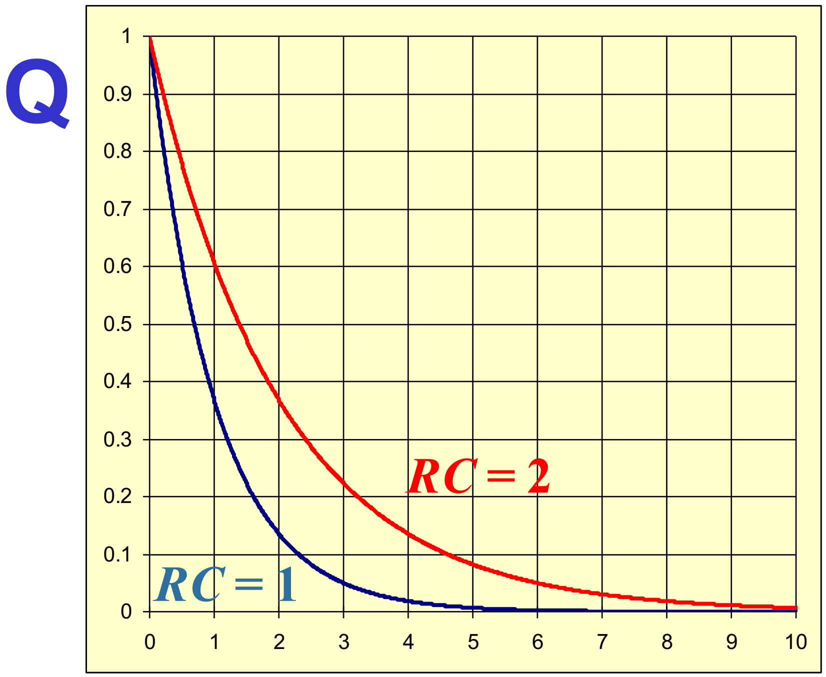
A) $Q_1 > Q_2$

B) $Q_1 > Q_2$

C) $Q_1 = Q_2$

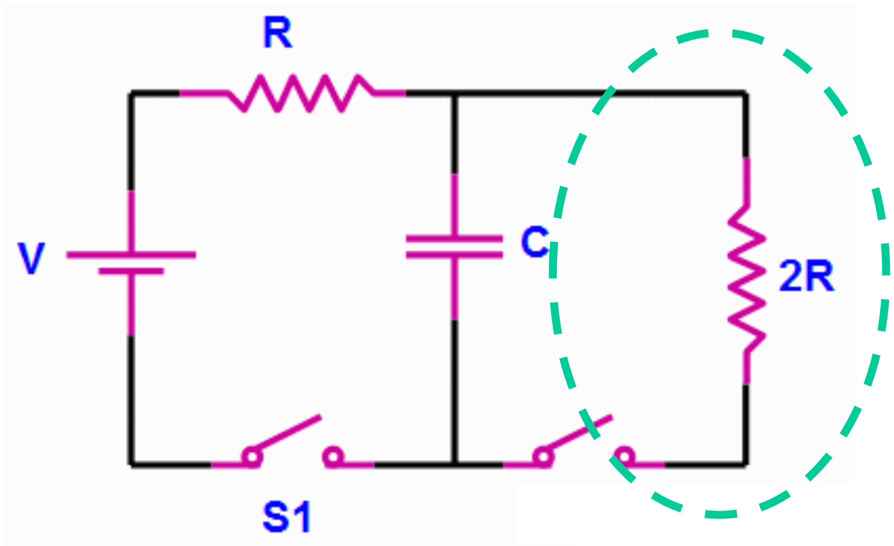
D) Something wrong …

E) Something else wrong …



CheckPoints 2 & 4

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



Not involved for these questions

Close $S1$,

V_1 = voltage across C immediately after

V_2 = voltage across C a long time after

A) $V_1 = V \quad V_2 = V$

B) $V_1 = 0 \quad V_2 = V$

C) $V_1 = 0 \quad V_2 = 0$

D) $V_1 = V \quad V_2 = 0$

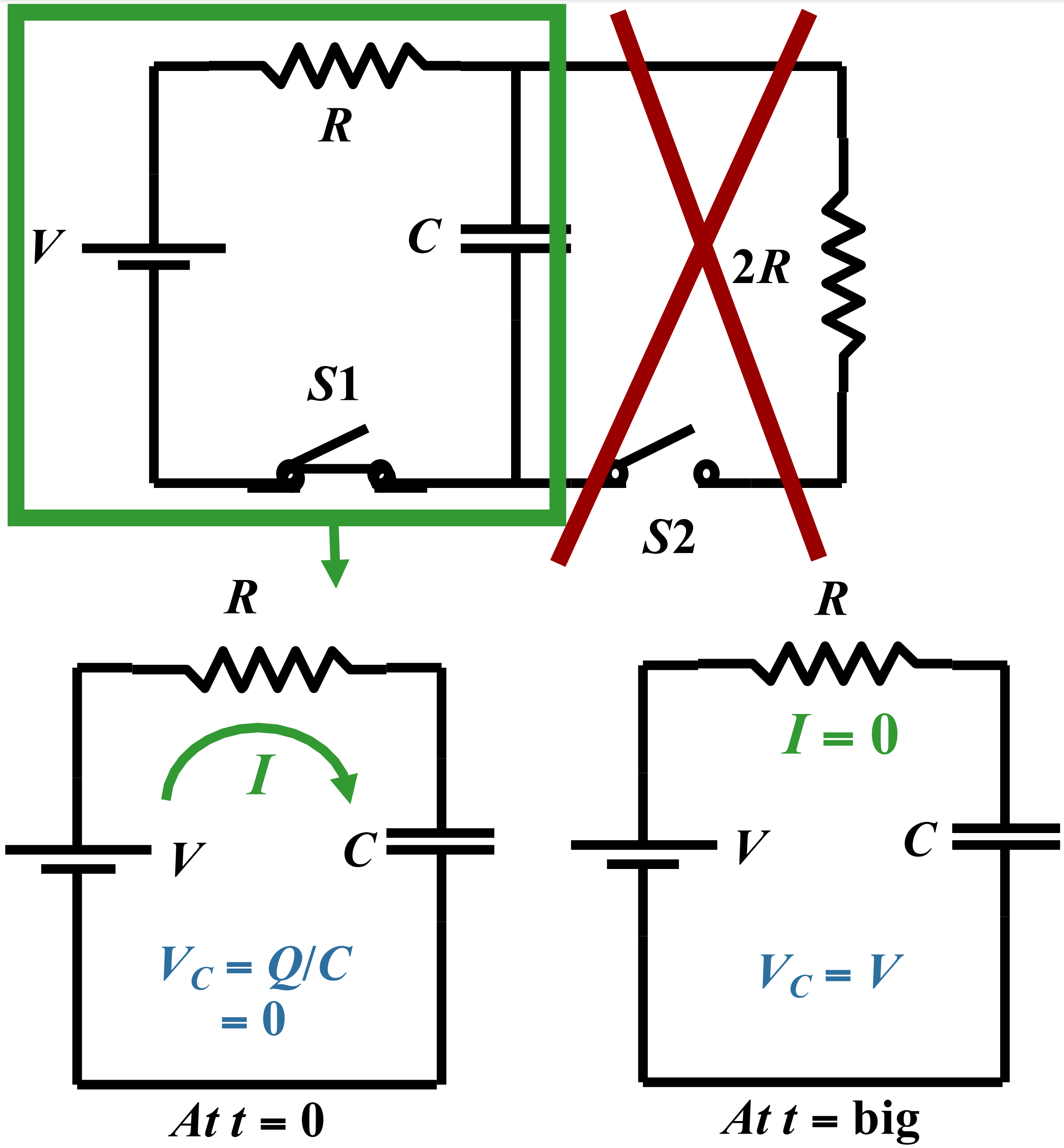
Immediately after the switch S_1 is closed:

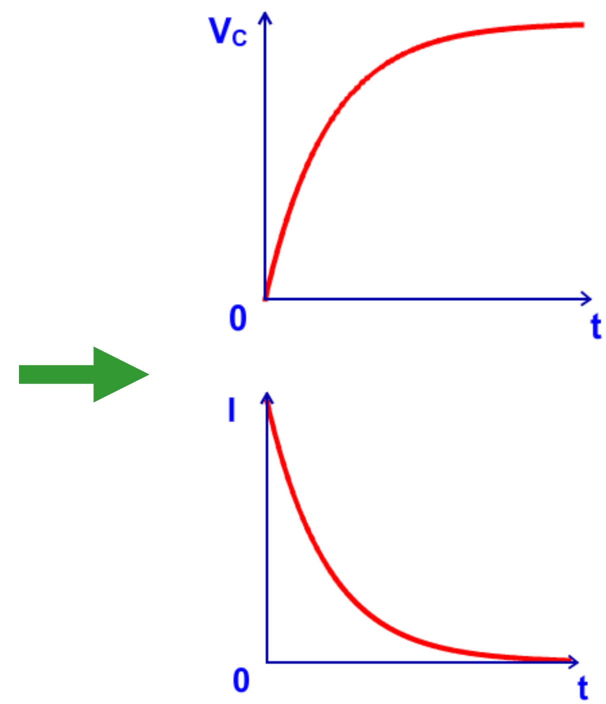
$Q = 0$ → $V = Q/C$ → $V_1 = 0$

After the switch S_1 has been closed for a long time

$I = 0$ → $V_R = 0$ → $V_2 = V$

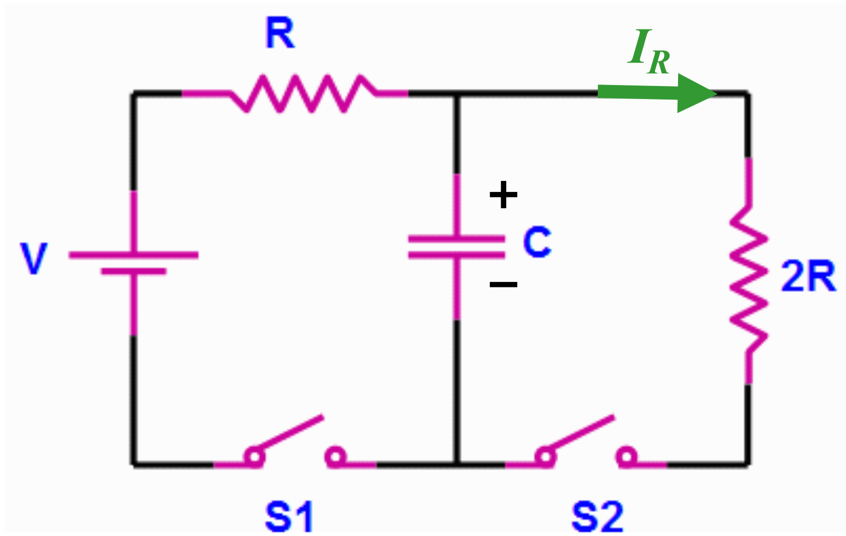
Close S_1 at $t = 0$
(leave S_2 open)





CheckPoint 6

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



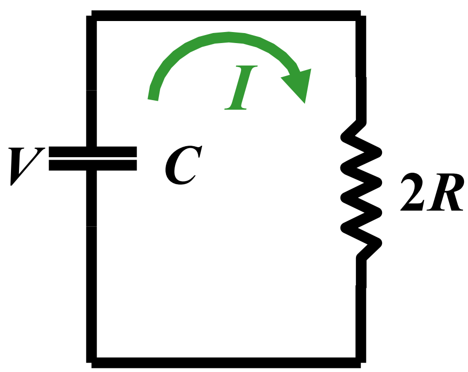
6) After being closed a long time, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after the switch 2 is closed?

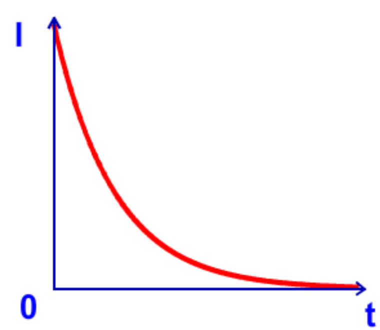
A ○ $I_R = 0$

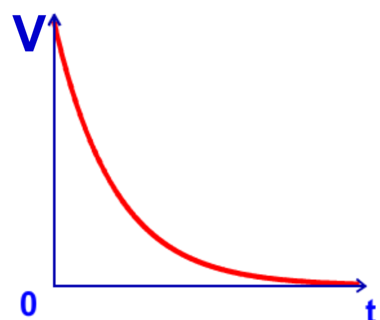
B ○ $I_R = V/3R$

C ○ $I_R = V/2R$

D ○ $I_R = V/R$







Open $S1$ at t = big
and close $S2$