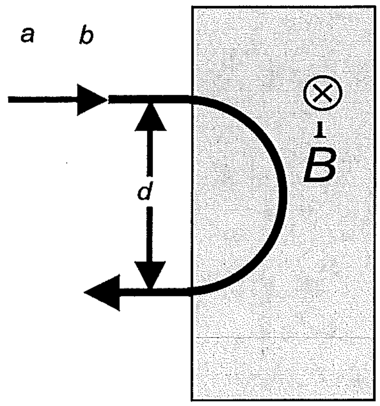
Answer Key

The next two questions pertain to the situation described below.

A charged particle is accelerated through a potential difference from *a* to *b*. The particle then enters a uniform magnetic field B directed into the plane of the paper. While in this magnetic field, the particle travels in a semicircle of diameter *d*.



1. If we were to double the magnitude of the potential difference between *a* and *b*, what would happen to the diameter *d*?

a. *d* would double
b. *d* would increase by a factor of $\sqrt{2}$
c. *d* would stay the same
d. *d* would decrease by a factor of $\sqrt{2}$
e. *d* would decrease by a factor of 2

Double potential difference

$\Rightarrow$ Double Kinetic energy $(\frac{1}{2}mv^2)$

$\Rightarrow$ Increase Velocity by factor of $\sqrt{2}$

$|\vec{F}_{mag}| = q|\vec{v}||\vec{B}| = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$

$\Rightarrow d = 2r \propto v \Rightarrow$ d increase by factor of $\sqrt{2}$

2. Let *T* be the amount of time the particle is in the shaded region that contains the magnetic field. If we were to double the particle velocity as it enters the field, what would happen to *T*?

a. *T* would double
b. *T* would increase by a factor of $\sqrt{2}$
c. *T* would stay the same
d. *T* would decrease by a factor of $\sqrt{2}$
e. *T* would decrease by a factor of 2

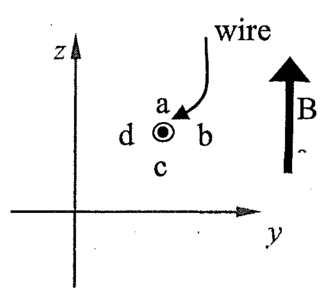
Path length:

$\frac{\pi d}{2} = \pi r = \frac{\pi m v}{qB}$

$T = \frac{\text{Path length}}{\text{speed}} = \frac{\pi m}{qB} = \text{constant}$

3. A long straight wire carrying current out of the page (along $+\hat{x}$) is sitting in a uniform external magnetic field $B = B_o\hat{z}$. At what point could the total magnetic field equal 0?

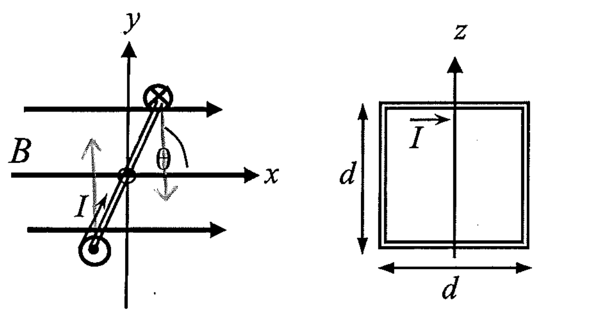
a. a
b. b
c. c
d. d
e. There is *no* possible location where the field could be zero



Point d is the only place where the wire's $\vec{B}$ points downward (RHR) and thus could cancel some or all of the external field.

The next two questions pertain to the situation described below.

A square loop carries a current I and pivots without friction about the z-axis. A uniform magnetic field $B = 2.4$ T points in the $+x$ direction, and the loop initially makes an angle $\theta = 70°$ with the x–z plane.



$B = 2.4$ T
$d = 0.2$ m
$\theta = 70°$

top view of loop　　　plane view of loop

4. What is the direction of the torque on the loop at the time shown?

a. $-\sin\theta\hat{x} + \cos\theta\hat{y}$
b. $+\sin\theta\hat{x} - \cos\theta\hat{y}$
c. $-\hat{z}$
d. +x
e. +y

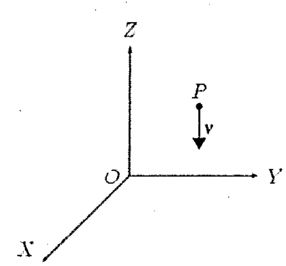
Forces are as shown in red (from $I\vec{l} \times \vec{B}$)
↪ loop rotates clockwise (from top view)
↪ torque is into the page in accordance w/ RHR

5. The magnitude of the torque on the loop exerted by the magnetic field at the time shown is measured to be r=1.2 N m. What is the magnitude of the current I in the loop?

a. I=12.5 A
b. I=0.5 A
c. I=1.3 A
d. I=36.5 A
e. I=2.9 A

$$\vec{\tau} = 2\,\vec{r} \times \vec{F} = 2\,\frac{d}{2}|\vec{F}|\sin(90-\theta)$$
$$= d \times (d\,I\,B)\sin(20°)$$
$$\Rightarrow I = \frac{\tau}{d^2 B \sin(20°)} = \frac{1.2\ \text{N}\cdot\text{m}}{(0.2\,\text{m})^2 \cdot 2.4\,\text{T} \cdot \sin(20°)}$$

6. An electron has a velocity in the negative Z direction at point P. The magnetic force on the electron at this point has a component in the negative Y direction. Which one of the following statements about the magnetic field at point P can be determined from this information?



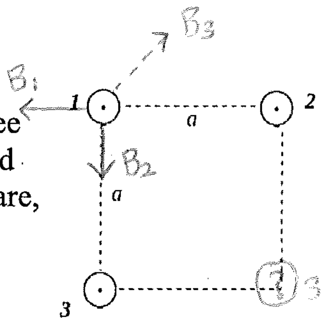
a. Bx is positive
b. By is negative
c. Bx is negative
d. Bz is positive
e. By is positive

$\vec{F} = q\vec{v} \times \vec{B}$

electron is negatively charged, so $(q\vec{v})$ points ↑ in positive Z-direction.

$\hat{z} \times (-\hat{x})$ yields $-\hat{y}$ by RHR.

7. Three long wires carrying current I out of the page are placed at three corners of a square of radius a, as shown. Determine the magnitude and direction of the current in a wire placed at the fourth corner of the square, such that the total force on wire 1 vanishes.



a. I into the page
b. √2 I into the page
c. √2 I out of the page
d. 2I into the page
e. 2I out of the page

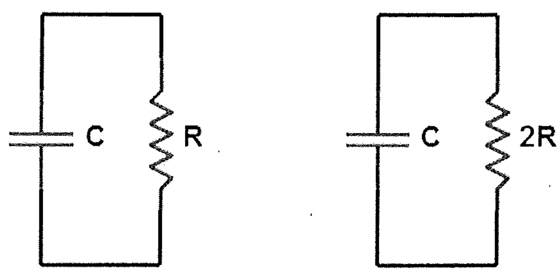
Need $\vec{B}_{tot} = 0$ at wire 1,
so $\vec{B}_3 = -\vec{B}_1 - \vec{B}_2$.

$\Rightarrow |\vec{B}_3| = \sqrt{2}\,|\vec{B}_1|$

$\Rightarrow \frac{\mu_0 I_3}{2\pi(\sqrt{2}\,a)} = \sqrt{2}\,\frac{\mu_0 I}{2\pi a} \Rightarrow I_3 = 2I$

Into the page, to produce a $\vec{B}_3$ in the correct direction.

The following two questions concern the two circuits shown below.
The two circuits contain identical capacitors that hold the same charge at t = 0. Circuit 2 has twice as much resistance as circuit 1.



Circuit 1 Circuit 2

For an RC circuit,

$Q(t) = Q_0 e^{-t/\tau}$

For circuit 1, $\tau = RC$
circuit 2, $\tau = 2RC$

8. Which circuit has a larger power dissipation at t=0?

a. Circuit 1
b. Circuit 2
c. They have the same nonzero power dissipation
d. They both have zero power dissipation

$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau}$.

at $t=0$, $I_0 = \frac{Q_0}{\tau}$.

$P_0 = I_0^2 R = \frac{Q_0^2}{RC^2}$ $R_2 > R_1$
$P_2 < P_1$

9. Which of the following statements best describes the charge remaining on each of the the two capacitors for any time after t = 0?

a. Q1<Q2
b. Q1>Q2
c. Q1=Q2
d. Q1<Q2 at first, then Q1>Q2 after a long time
e. Q1>Q2 at first, then Q1<Q2 after a long time

$Q(t) = Q_0 e^{-t/\tau}$

Higher τ → Charge dissipates more slowly → more charge remains on capacitor

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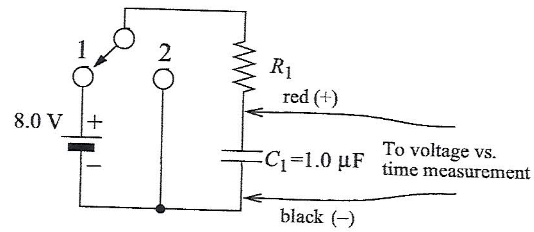
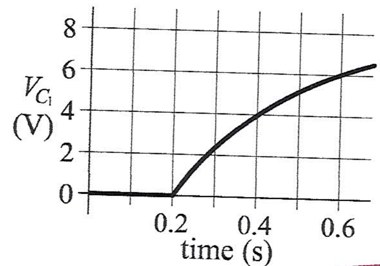
II. Lab questions [12 pts] ANSWER THESE ON YOUR SCANTRON SHEET.

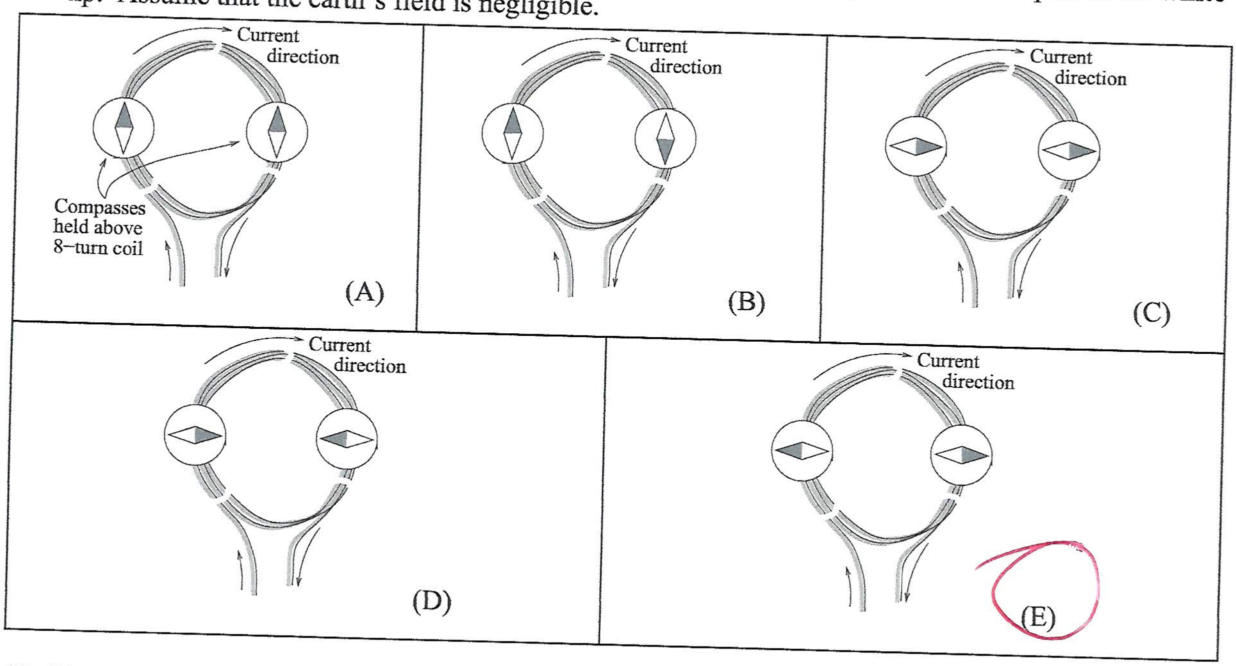
10. [4 pts] An *RC* circuit is connected as shown in the diagram below. The switch is originally left in position 2 for a long time, and then at $t = 0.2$ s it is flipped to position 1 while the voltage across the 1.0 μF capacitor C_1 is measured and plotted as a function of time (also shown). The resistance R_1 is not known prior to this measurement. From the information given in the circuit diagram and the graph, what is the **resistance** R_1?



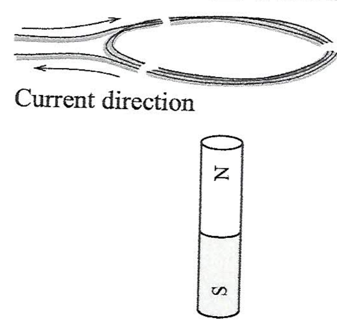


(A) 0.2 Ω (B) 0.3 Ω (C) 20 kΩ (D) 40 kΩ (E) 300 kΩ

11. [4 pts] An 8-turn coil (like that used in the lab) is lying on a nonmagnetic table. A large current is flowing through it in the direction shown. If two compasses are held directly above the coil wires, which picture shows the direction of the compass needles? The compass needle's **N** pole is the **white** tip. Assume that the earth's field is negligible.



12. [4pts] A coil is held directly above a bar magnet as shown at right. Current is flowing through the coil in the direction indicated. Which of the following is experienced **by the magnet** due to this current?
   (A) A force directly toward the center of the coil (up).
   (B) A force directly away from the center of the coil (down).
   (C) A force toward the left or right, but not up or down.
   (D) There is no net force, but the magnet experiences a torque about its center.
   (E) There is no net force or torque on the magnet from this coil.



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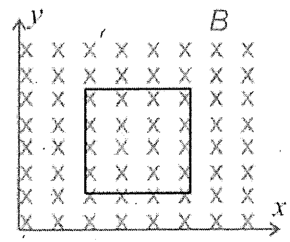
III [25 points] A square loop of wire of side a and electrical resistance R lies in a plane perpendicular to a magnetic field as shown at right. The field is given by $\vec{B}(t) = -\hat{k}\, B_0 \sin \omega t$. Calculate the following in terms of the variables given and explain your results.



i. [4 points] At what time does the induced EMF become zero for the first time after $t = 0$ (that is for $t > 0$)?

EMF is zero when magnetic flux isn't changing, i.e. when $\frac{d\vec{B}}{dt} = 0$. This first occurs when $\omega t = \frac{\pi}{2}$ $\Rightarrow$ $\boxed{t = \frac{\pi}{2\omega}}$

ii. [4 points] What is the direction of the EMF at time $t = 0$ (clockwise, ccw, zero)?

At $t=0$, magnetic flux increasing into the page. According to Lenz's law, induced EMF counteracts this by generating an induced field out of the field. This is achieved by $\boxed{\text{CCW}}$ current.

iii. [4 points] Give an expression for the EMF as a function of time. (Use clockwise as positive).

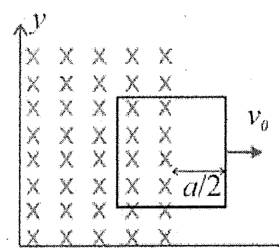
$\Phi_B = a^2 B_0 \sin(\omega t)$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \boxed{-\omega a^2 B_0 \cos(\omega t)}$$

iv. [5 points] Assume the answer to part iii is given by $\mathcal{E}(t)$. Give expressions for the current, net force, and net torque on the square loop.

Current: $\mathcal{E}(t)/R$
Net Force: } zero
Net Torque: }

For the next two parts the field is constant in time, $B(t) = -\hat{k}\, B_0$ and the square is moving to the right with constant velocity v_0 at a time when half of the square has left the region with magnetic field.



v. [4 points] Deduce an expression for the force on the square. Give the direction of the force as well.

$\Phi_B = B_0 \times$ Area of field through loop.

$d\Phi_B/dt = B_0 \frac{dA}{dt} = B_0 a v_0 \Rightarrow I = \frac{B_0 a v_0}{R}$ clockwise.

Force on left side of loop: $\vec{F} = I\vec{l} \times \vec{B} = \boxed{\frac{B_0^2 a^2 v_0}{R} \text{ left}}$ (Forces on top/ bottom sides cancel)

vi. [4 points] Give an expression for the power dissipation in the square and explain where this energy comes from.

$P = I^2 R = \frac{B_0^2 a^2 v_0^2}{R}$

This energy comes from whatever agent is applying the rightward force to the loop to keep it moving at a constant velocity.

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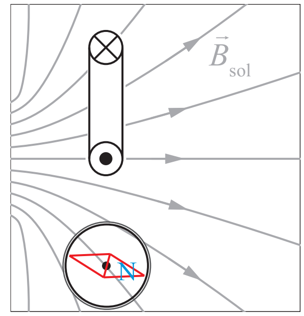
IV. [20 points total] Tutorial questions.

A circular loop carrying a current I_o is placed near the end of a solenoid as shown in the cross-sectional view at right. The field lines shown indicate the magnetic field of *the solenoid only*.

Note: The symbol ⊗ indicates current going into the page and the symbol ⊙ indicates current coming out of the page.

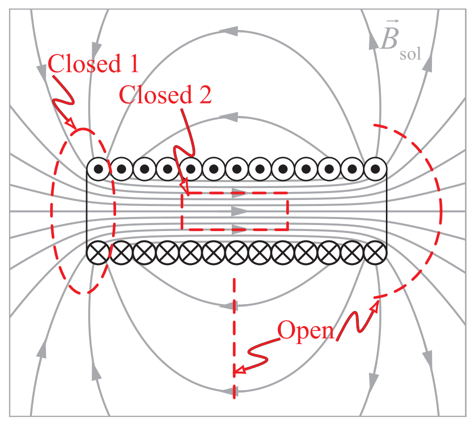


Field lines of solenoid (solenoid not shown)

A. [6 pts] Suppose a compass were placed below the current loop as shown at right. In the space provided, draw the most stable orientation of the compass needle. Explain your reasoning.

The compass needle will be stable when its poles are aligned from south to north along the net *magnetic field direction, the sum of the fields from the loop and solenoid. The contribution from* $\vec{B}_{sol}$ *is tangent to the field line at that point. From the RHR we developed in tutorial to treat the current loop like a small bar magnet, field lines for the loop will come out of the left side and go into the right side. This means the contribution for the loop will be directly to the right at this point, so the compass needle will be angled between the field line and the horizontal.*

B. The circular current loop and compass are removed. The magnetic field around the solenoid is shown at right.



i. [7 pts] On the diagram at right, draw and label a *closed path* for which the line integral of the magnetic field along that path is zero (*i.e.*, $\oint_{\text{path}} \vec{B} \bullet d\vec{l} = 0$). Explain.

There are many correct answers to this question. Two examples requiring different reasoning are the dashed ellipse and rectangle at right.

Closed 1: From Ampère's law, the line integral of magnetic field along any closed path that encircles net zero current will be zero.

Closed 2: The magnetic field lines show an approximately uniform field in the center of the solenoid. Choosing the counterclockwise path, the dot product $\vec{B} \bullet d\vec{l}$ *is zero where the vectors are perpendicular (left and right sides),* Bdl *when they are parallel (bottom), and* $-Bdl$ *when they are antiparallel (top). Since* $\vec{B}$ *has the same magnitude everywhere in this region and the top and bottom segments are equal length, the line integral along the entire path will sum to zero.*

ii. [7 pts] On the diagram at right, draw and label an *open path* for which the line integral of the magnetic field along that path is zero (*i.e.*, $\int_{\text{path}} \vec{B} \bullet d\vec{l} = 0$). Explain your reasoning.

As with the previous question, there are many possible correct answers. The straight and curved open paths on the diagram are two examples requiring the same reasoning.

Ampère's law only applies to closed paths, so we can't use it directly here. Instead, the simplest path we can choose is one where $\vec{B} \bullet d\vec{l}$ *is zero at every point on the path, so* $\vec{B}_{sol}$ *and* $d\vec{l}$ *need to be perpendicular for the entire path.*