Last Name, First Name Key, Answer student id__________ score________

1) [5 points] An electron moves in a region of uniform electric field of magnitude $E = 3$ N/C which points to the right. What is the change in potential energy as the electron moves from point A to point C?

$\Delta V = -\int_A^C \vec{E} \cdot d\vec{\ell}$

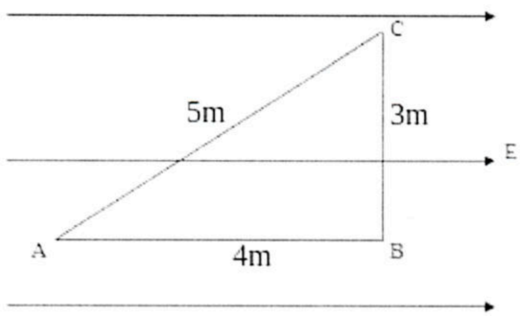
$= -E \times \ell_{\parallel}$ (component parallel to field)

$= -3\frac{N}{C} \times 4m$

$= -12 \frac{J}{C}$

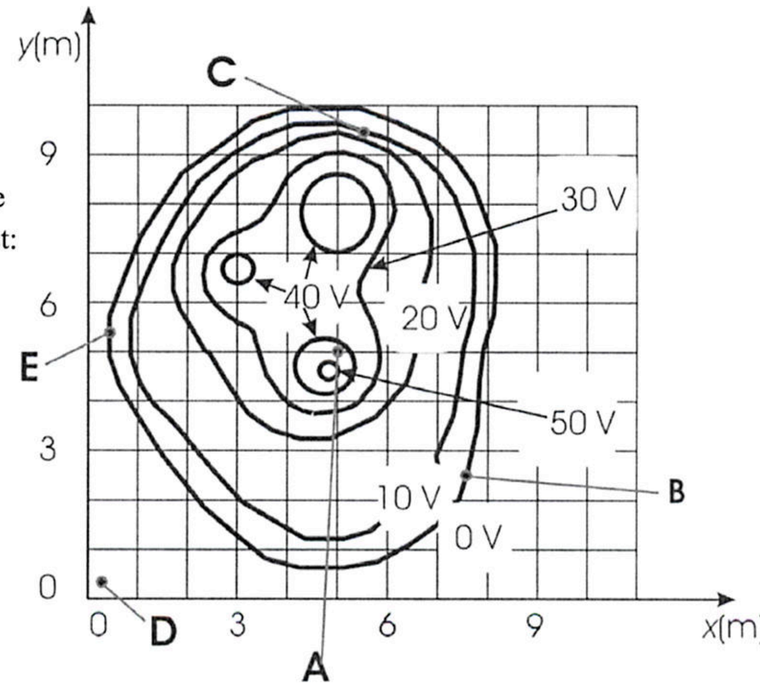
$\Delta U = -e\Delta V$

$= -1.6\times10^{-19} C \times (-12 \frac{J}{C}) = 1.92\times10^{-18}$ J



(A.) $+1.92\times10^{-18}$ J
B. -2.40×10^{-18} J
C. -1.92×10^{-18} J
D. $+2.40\times10^{-18}$ J
E. It depends on the path the electron takes

2) – 3) [10 points] refer to the electric potential diagram at right.



2) [5 points] Select the point where the magnitude of the electric field is largest:

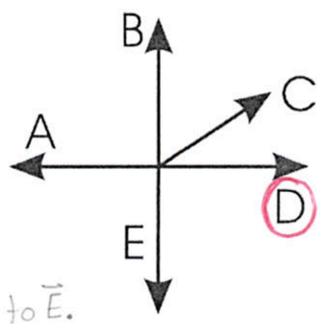
$\vec{E} = -\vec{\nabla} V$. $|\vec{E}|$ is maximized where V changes the most rapidly, i.e. the place with the highest density of potential contour lines

A. $(x,y) = 5.0, 5.0$ m
B. $(x,y) = 7.5, 2.5$ m
(C.) $(x,y) = 5.5, 9.5$ m
D. $(x,y) = 0.3, 0.3$ m
E. $(x,y) = 0.5, 5.5$ m

3) [5 points] If an electron starts from rest at point E with coordinates $(x, y) = (0.5, 5.5)$ m, in which direction will begin to move?

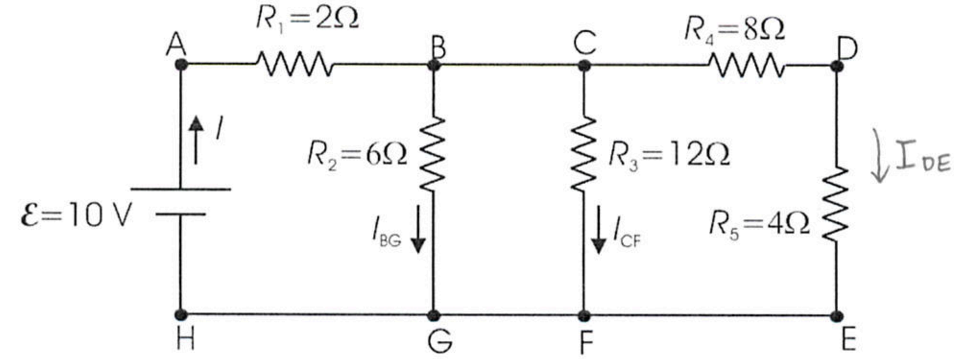
Electric field points perpendicular to equipotential contours, towards areas of lower potential ("down the potential gradient"), i.e. leftwards.

Electron has a negative charge, so it feels a force rightwards, opposite to $\vec{E}$.



Last Name, First Name______________________ student id_____________score_________

4)-6) [13 points] Five resistors are connected as shown. The battery has zero internal resistance. The circuit elements have the following values: $\mathcal{E}$=10 V, R_1=2 Ω, R_2=6 Ω, R_3=12 Ω, R_4=8 Ω, and R_5=4 Ω.



4) [3 points] Compare the magnitude of the current I to $|I_{BG}+I_{CF}|$ and the magnitude of the potential difference between points EF, $|V_{EF}|$, to that between points DE, $|V_{DE}|$.

A. $I = |I_{BG}+I_{CF}|$ and $|V_{EF}| = |V_{DE}|$
B. $I = |I_{BG}+I_{CF}|$ and $|V_{EF}| > |V_{DE}|$
C. $I = |I_{BG}+I_{CF}|$ and $|V_{EF}| < |V_{DE}|$
D. $I > |I_{BG}+I_{CF}|$ and $|V_{EF}| = |V_{DE}|$
E. $I > |I_{BG}+I_{CF}|$ and $|V_{EF}| < |V_{DE}|$

I is distributed between 3 branches
↪ $I = I_{BG} + I_{CF} + I_{DE}$.
V_{EF} is zero; V_{DE} is nonzero.

5) [5 points] Calculate the current I in amperes.
A. 12 A
B. 10 A
C. 5 A
D. 3 A
E. 2 A

$R_4 + R_5$ in series → $R_{45} = 12\,\Omega$
$R_3 + R_{45}$ in parallel → $R_{345} = 6\,\Omega$
$R_2 + R_{345}$ in parallel → $R_{2345} = 3\,\Omega$
$R_1 + R_{2345}$ in series → $R_{tot} = 5\,\Omega$

$I = \frac{V}{R_{tot}} = \frac{10\text{ V}}{5\,\Omega} = 2\text{ A}$

6) [5 points] Which of the following is ***incorrect***?
A. The sum of the power dissipated by all resistors is $P = I^2 R_1$
B. The power dissipated by R_2 is $P_2 = I_{BG}^2 R_2$
C. The power dissipated by R_2 equals that dissipated by R_3, R_4 and R_5: $P_2 = P_3+P_4+P_5$
D. The power dissipated by R_5 is smaller than that dissipated by R_4, $P_5 < P_4$.

This is only the power dissipated by resistor 1.

7) [5 points] Two cylindrical resistors are made of Nichrome wire. R_1 is twice as long, but has half of the diameter of R_2. How do their resistances compare?
A. $R_1 = R_2/8$
B. $R_1 = R_2/2$
C. $R_1 = R_2$
D. $R_1 = 2\,R_2$
E. $R_1 = 8\,R_2$

$R = \frac{\rho L}{A}$

$\frac{R_1}{R_2} = \frac{\rho L_1}{\pi r_1^2} \cdot \frac{\pi r_2^2}{\rho L_2} = \frac{L_1}{L_2}\left(\frac{r_2}{r_1}\right)^2$

$= 2 \cdot (2)^2 = 8$

8) [5 points] A 10 kW heater is designed to operate at 120 volts. Assume that its resistance is constant. What is the power it delivers if the heater is used with 240 volts?

A. $P = 2.5$ kW
B. $P = 5$ kW
C. $P = 10$ kW
D. $P = 20$ kW
(E.) $P = 40$ kW

$P = \frac{V^2}{R}$ $\frac{P_{240}}{P_{120}} = \frac{(240\,V)^2}{(120\,V)^2} \times \frac{R}{R} = 4$

$\hookrightarrow P_{240} = 4 \times 10$ KW

9) [5 points] Two positive point charges are separated by a distance *R*. If the distance between the charges is decreased to *R*/2, what happens to the total electric potential energy of the system?

(A.) It is doubled
B. It is reduced to half its original value
C. It is quadrupled
D. It is reduced to one fourth its original value
E. It remains the same

$U = \frac{Kq_i q_2}{R}$ $R \rightarrow \frac{R}{2}$

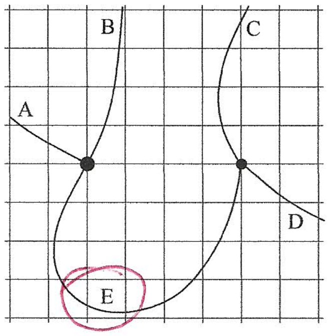
$U \rightarrow 2U$

Name ______________________________ **Student ID** _____________ **Score**______

last *first*

II. Lab questions [12 pts] ANSWER THESE QUESTIONS ON YOUR SCANTRON SHEET.

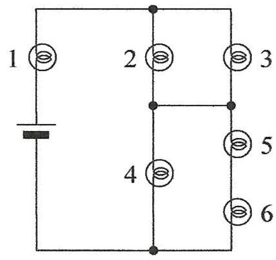
10. [4 pts] Two charges of **unequal magnitude** and **unknown sign** are drawn on a grid, as shown at right by the black dots. Five possible electric field lines have been drawn (but the field-line direction is not shown and the diagram is incomplete). Every field line **except one** is **correct**. Which field line **is inconsistent** with the others?



The next three questions concern the circuit shown below right. It is made of bulbs, sockets and a power supply **identical** to what you used in the lab.

11. [2 pts] Which **pairs** of bulbs are connected in **series**?

(A) 1&2 only
(B) 1&2 and 2&4
(C) 5&6 only
(D) 2&4 and 5&6
(E) 2&4 and 3&5 and 5&6



12. [2 pts] Which **pairs** of bulbs are connected in **parallel**?

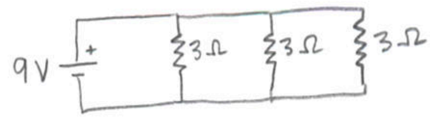
(A) 2&3 only
(B) 1&2 and 2&3
(C) 2&3 and 4&5 and 4&6
(D) 1&2 and 2&3 and 4&5 and 4&6
(E) None are connected in parallel.

13. [4 pts] It is observed that all bulbs are glowing. Then bulb 5 is **unscrewed** from its socket. The table below lists possible changes in brightness for each of the remaining bulbs. Which row shows the correct changes?

	Brighter	No Change	Dimmer
(A)		4	1, 2, 3, 6
(B)	4	2, 3	1, 6
(C)			1, 2, 3, 4, 6
(D)	4		1, 2, 3, 6
(E)	6	1, 2, 3	4

Last Name, First Name Key, Answer student id____________ score___________

1) **[9 pts]** You are given a three resistors with resistances of 3 ohms each, and a 9 volt battery.
   a. **[5 pts]** Draw the arrangement of these components that maximizes the power dissipated by the circuit. Explain why this arrangement maximizes power or show that other arrangements dissipate less power.



$P = \frac{V^2}{R_{eff}}$.

this configuration minimizes the effective resistance

   b. **[4 pts]** Calculate the power dissipated by the circuit you drew in a)

Parallel resistances:

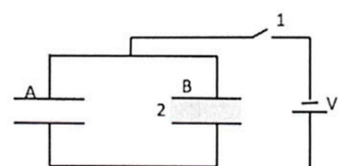
$\frac{1}{R_{eff}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$

$\rightarrow R_{eff} = \frac{R}{3} = 1\,\Omega$

$P = \frac{V^2}{R_{eff}} = \boxed{81\ W}$

For questions 2 and 3, consider a system of connected capacitors A, and B, with capacitances C_1 and C_2 respectively. Both capacitors are initially unfilled.

2) **[8 pts]** Suppose the two capacitors are, as shown in the figure, first connected to a battery with voltage V, and then disconnected (1). After the battery is disconnected, capacitor B is filled with a dielectric with dielectric constant κ (2). Calculate the final charge on each capacitor Q_A, and Q_B in terms of C_1, C_2, κ, V, and ε_0.

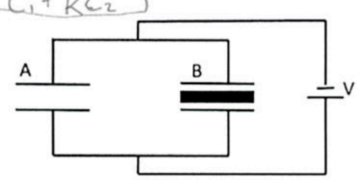


- Total charge on A and B is $Q_{tot} = C_{eff} V$

$\Rightarrow Q_A + Q_B = V(C_1 + C_2)$ ①

- Dielectric is added and C_2 becomes κC_2; charge redistributes.
- Same voltage across both capacitors $\Rightarrow V_{new} = \frac{Q_A}{C_1} = \frac{Q_B}{\kappa C_2}$ ②
- Solve ① & ② to find $\boxed{Q_A = \frac{VC_1(C_1+C_2)}{C_1+\kappa C_2},\ Q_B = \frac{V\kappa C_2(C_1+C_2)}{C_1+\kappa C_2}}$

3) **[8 pts]** Suppose instead the capacitors are left connected to a battery with voltage V. A conductor with thickness that is half the separation between the capacitor plates of B is placed between the plates of B. Calculate the equilibrium stored energy in each capacitor in terms of C_1, C_2, V, and ε_0 for both capacitor A and capacitor B.



Capacitor B effectively becomes two capacitors in series, with equal surface area and $\frac{1}{4}$ the plate spacing (and thus 4 times the capacitance).

$\frac{1}{C_{B,new}} = \frac{1}{4C_2} + \frac{1}{4C_2} = \frac{1}{2C_2} \rightarrow C_{B,new} = 2C_2$

Stored energy $U = \frac{1}{2}CV^2 \rightarrow \boxed{U_A = \frac{1}{2}C_1V^2}$

$\rightarrow \boxed{U_B = C_2V^2}$

Suggested Rubric

1. a) 3 pts. → correct diagram
   2 pts. → explanation : 'minimizes resistance'
   and/or
   'maximizes current'

   b) 2 pts. → calculates effective resistance
   2 pts. → correct power dissipated

2. a) + 2 pts → calculates total charge from initial configuration

   + 2 pts → realizes $C_{2,new} = \kappa C_2$

   + 2 pts → equates voltages across capacitors

   + 1 pt each → solves for correct expressions

   b) + 2 pts. → recognizes that B is now 2 capacitors; correctly calculates capacitance for series.

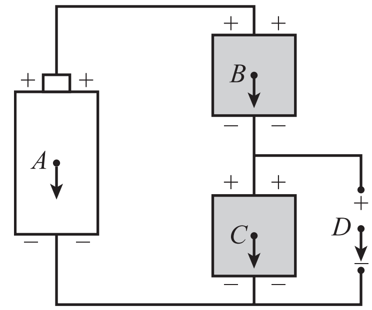
   + 2 pts → Correct relationship between capacitance and plate separation distance

   + 2 pts each → correct answers for U_A, U_B

Name _____________________________________ **Student ID** _______________ **Score**________
last *first*

IV. [20 points total] For the following questions, assume that the switch and all wires are ideal conductors. Also assume the battery is simple and ideal, with the positive terminal oriented toward the top of the page.

A. A battery is connected in series with two identical resistors. An open switch (represented by a gap) is connected in parallel to the lower resistor as shown.



i. On the diagram, draw an arrow to indicate the direction of the electric field at each point. If the electric field at any point is zero, state so explicitly. Explain your reasoning.

- [2 pts.] Point *A*, inside the battery.

  Chemical reactions in the battery push positive charges to the upper terminal and negative charges to the lower terminal, so the electric field is ***down****.*

- [2 pts.] Point *B*, inside the upper resistor.

  Charges move down through the resistor, which means the resistive force points up. Since charges move at constant speed, the electric field points ***down.***

- [2 pts.] Point *C*, inside the lower resistor.

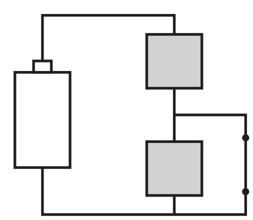
  Since no charges flow through the switch, all charges flow down through the resistor. With the same reasoning as the upper resistor, the field points ***down.***

- [2 pts.] Point *D*, between the ends of the switch.

  The potential difference across the lower resistor and switch are the same, so the field in each must point in the same direction. The field in the switch points ***down.***

ii. [4 pts] On the diagram above, sketch surface charge distributions on the battery, resistors, and switch that could create the electric fields you found above.

B. The switch is now closed and the circuit reaches equilibrium.



For each component, state whether or not the new charge distribution on each component differs from the charge distribution you drew above. Explain.

- [2 pts.] The battery.

  The electric field in the battery is unchanged, so the charge distribution is ***unchanged****.*

- [2 pts.] The upper resistor.

  There is no potential drop across the closed switch, so the potential drop across the upper resistor is equal to the entire voltage of the battery instead of half. The charge distribution is ***larger*** *than before.*

- [2 pts.] The lower resistor.

  No charges flow through this resistor, so the field and charge distributions are both ***zero.***

- [2 pts.] The switch.

  Since the switch is an ideal conductor, the charges on each end of the switch can flow through the switch to neutralize each other. The charge distribution on the ends of the closed switch is ***zero****.*