Waveguide Bends for the Pedantic

A Background in Integrated Photonics and a Method to Reduce Loss

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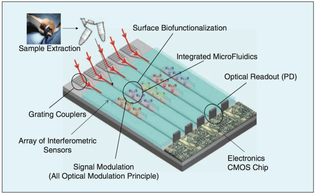
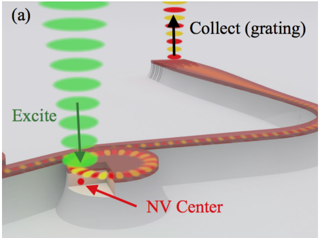
Part 1: Background

Photonics Now

(a) Computing/Communication (e.g. Internet)[1]



(b) Sensing[2]



(c) Quantum

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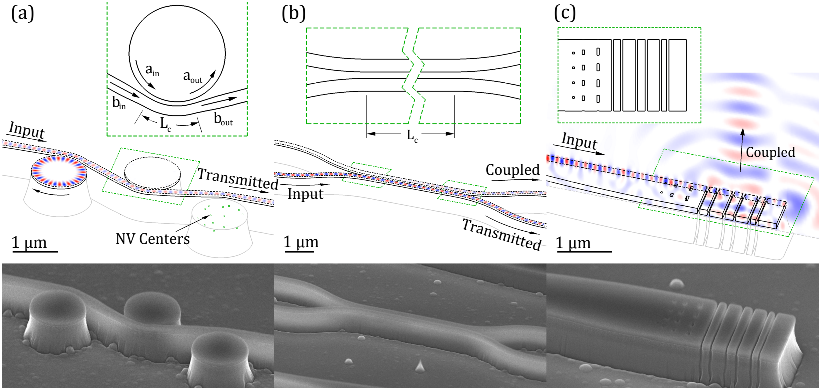
[1] Luxtera

[2] D. Duvall

Highlight: Quantum



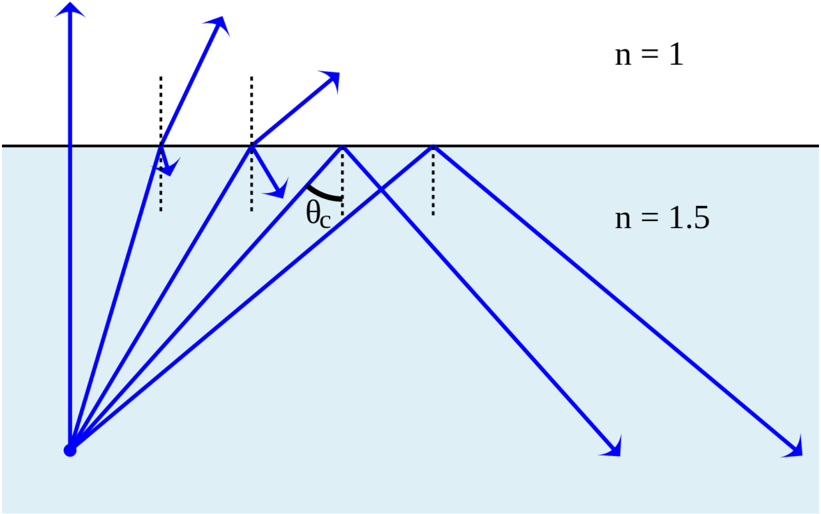
We can do cool stuff with integrated photonics...

Important: Loss Is Bad

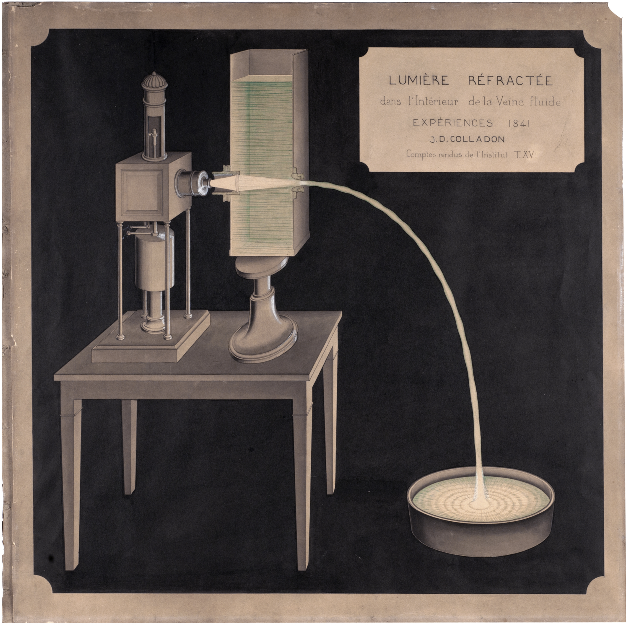
- In computing and communication, need stronger signal
  - More heat generated
  - More energy used
- In quantum, working in single-photon level
  - Photon loss implies, for instance, reduced entanglement efficiency.
  - ‘Every photon counts’

Part 1a: History

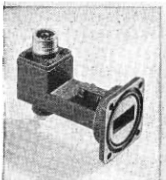
Snell’s Law (1630)



Daniel Colladon (1842)



Conducting Waveguides (1930s-40s, e.g. radar)


Crystal Mount DB-453

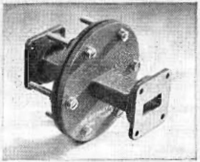

Rotating Joint DB-446


90° Elbow (H Plane) DB-433


Pressurizing Unit DB-452

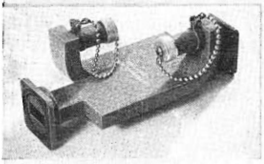

Mitered Elbow (H Plane) DB-439

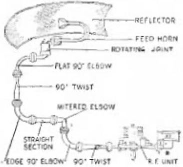

Uni-directional Broad Band Coupler DB-442

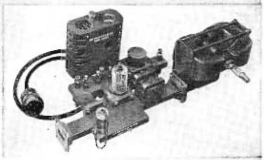

Bulkhead Flange DB-451


Uni-directional Narrow Band Coupler DB-440

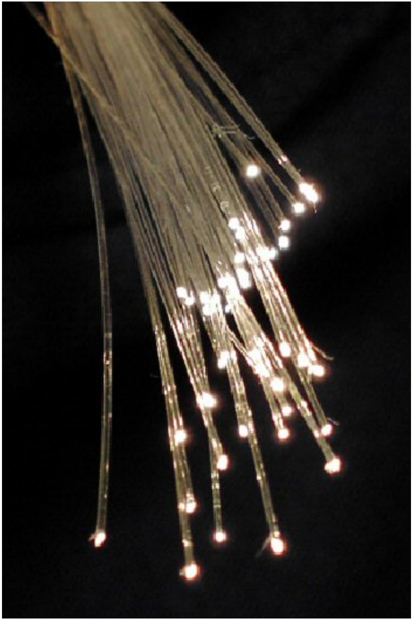

90° Twist DB-435


Bi-directional Narrow Band Coupler DB-441



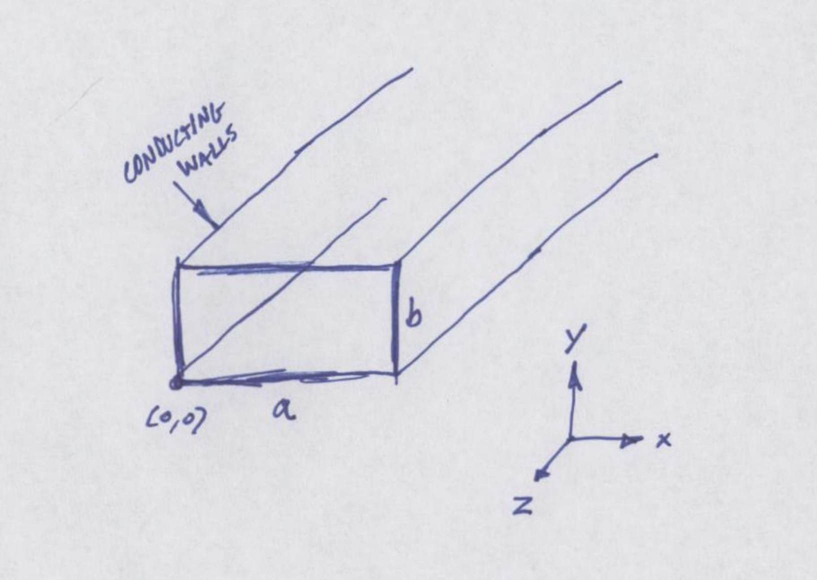

RF Radar Assembly DB-412

Fiber Optics (1960s-1970s)



Part 1b: Maxwell's Equations

Consider a Conducting Waveguide



Maxwell's Equations

$$\nabla \cdot E = \rho/\epsilon \qquad \nabla \cdot H = 0(?)$$

$$\nabla \times E = -\mu\frac{\partial H}{\partial t} \qquad \nabla \times H = \epsilon\frac{\partial E}{\partial t}$$

Remember: General Solution

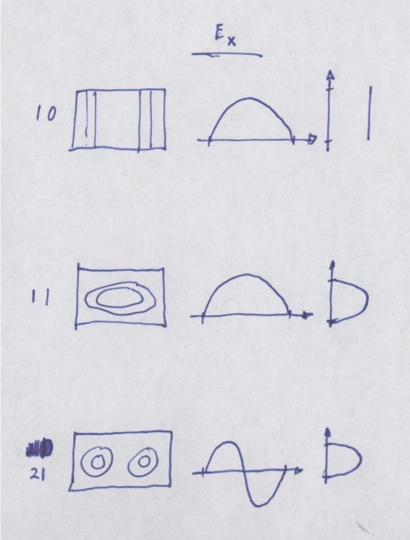
$$E = E(x,y)e^{i(k_z z - \omega t)}$$

$E(x,y)$ is the *mode profile.*

Remember: TE$_{nm}$ (Transverse Electric) Modes

$$
\begin{aligned}
E_x(x,y) &= A_{nm} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \\
E_y(x,y) &= A_{nm} \cos\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \\
E_z(x,y) &= 0
\end{aligned}
$$

Plots of TE$_{nm}$



Orthonormality Condition

$$\int\int E_i(x, y)E_j(x, y)\, dA = \delta_{ij}$$

(E_i, E_j normalized)

Remember: Cutoff Frequency

$$\omega_c = c\sqrt{\frac{n\pi}{a} + \frac{m\pi}{b}}$$

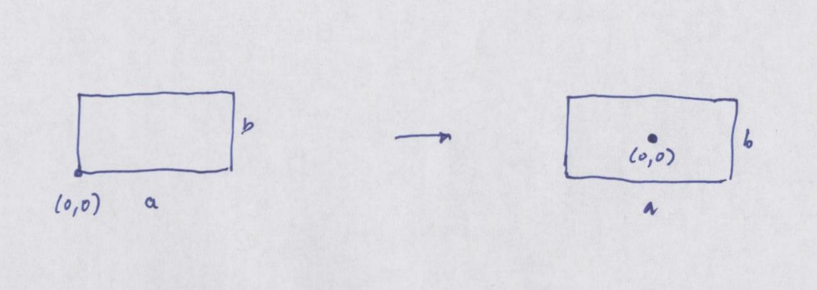
Assume $a = 2b$...

nm	$\omega_c(\sqrt{a}/\pi c)$
10	$1/\sqrt{2}$
01	1
11	$\sqrt{3/2}$
20	1

We Consider TE_{10} Solutions

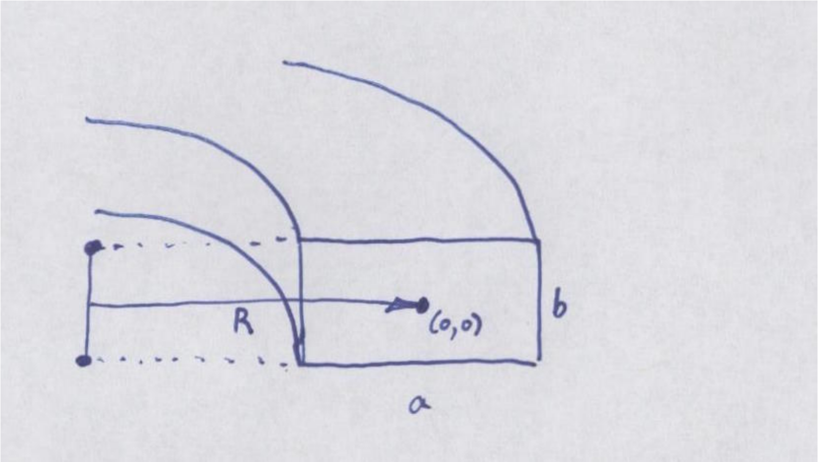
$$\begin{aligned} E_x &= A \sin\left(\frac{n\pi x}{a}\right) \\ E_y &= 0 \\ E_z &= 0 \end{aligned}$$

TE$_{10}$ From Center of Waveguide



$$E_x = A\cos\left(\frac{n\pi x}{a}\right)$$
$$E_y = 0$$
$$E_z = 0$$

Also Want TE_{10} Solution for Waveguide Bend



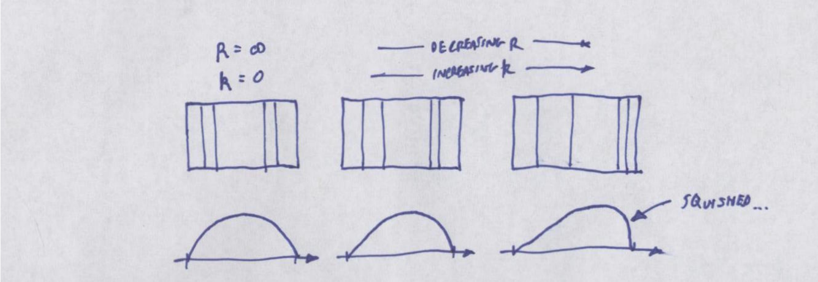
Cylindrical $\mathbf{TE}_{10}$ Solution

$$\begin{aligned} E_x &= AJ_0\left(\alpha(r-R)\right) + BK_0\left(\alpha(r-R)\right) \\ E_y &= 0 \\ E_z &= 0 \end{aligned}$$

Where J_0 and K_0 are Bessel Functions of the 1st and 2nd kind.
A, B, and α satisfy BCs:

- $E_x(\pm a/2) = 0$,
- One anti-node.

Plots of Cylindrical TE_{10} Solutions



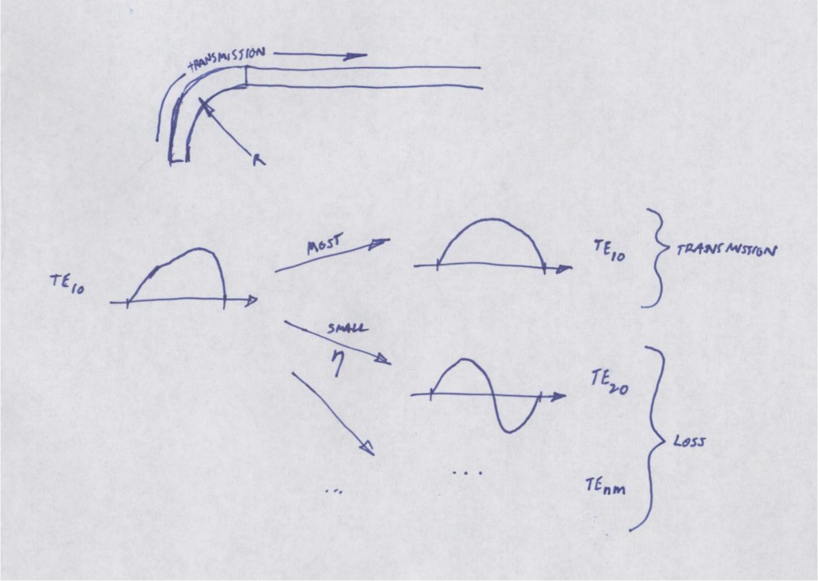
Part 2: Waveguide Bends

Mode Mismatch $\implies$ Loss

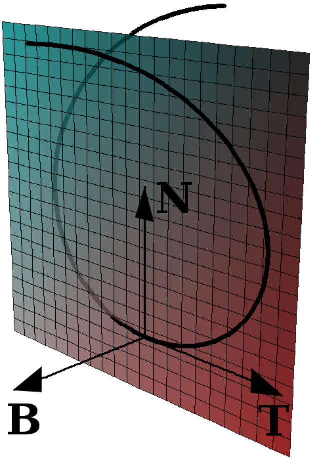
$$\int\int E_{10}^{k_1}(x,y) E_{10}^{k_2}(x,y)\, dA \neq 1$$

(For $k_1 \neq k_2$)

Where Do the Photons Go?

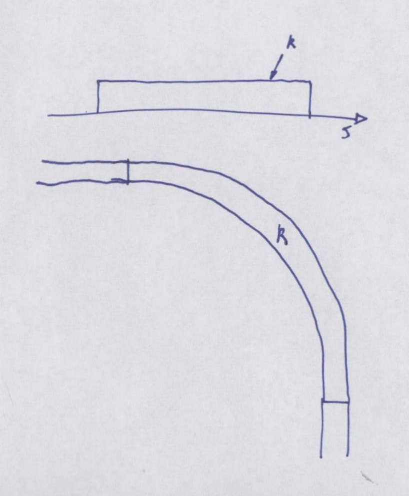


Frenet Notation

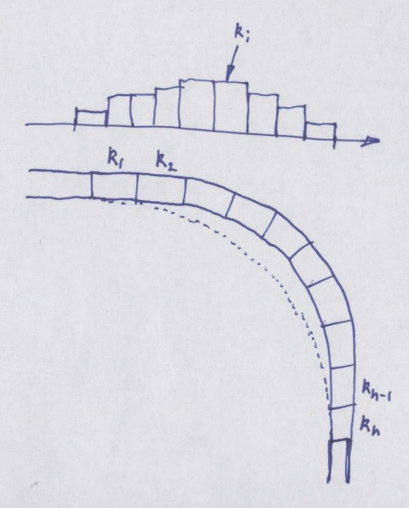


Way to define space curves. $N(s) = k(s)\hat{n}(s)$.

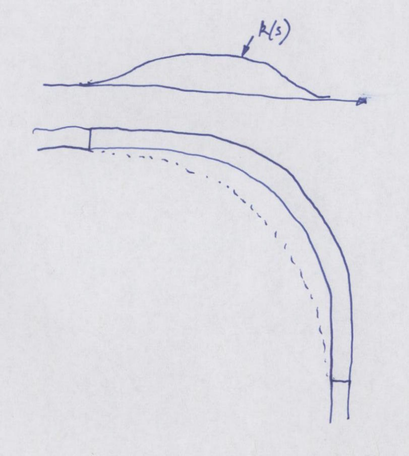
Loss From Circular Bend



Idea: 'Smooth' Curvature



Idea: Continuously 'Smooth' Curvature



Fundamental Theorem of Space Curves

Curve is completely determined (up to rigid transformation) by choice of $k(s)$ and $b(s)$.

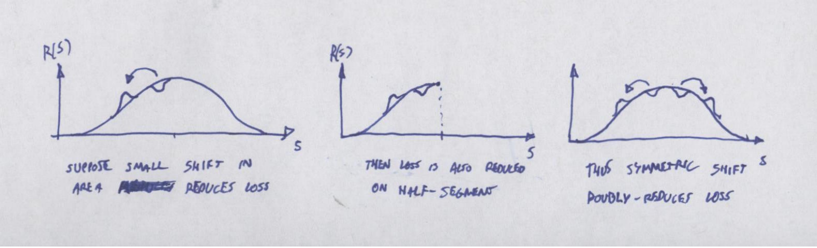
$b(s) = \hat{y} \implies k(s)$ fully determines curve.

Constraint: Begin and End Directions Same

$$\Theta = \int_{-\infty}^{\infty} k(s)\, ds$$

(Θ is the angle between the beginning and ending directions. This works because $k = d\theta/ds$.)

Constraint: Should Be Symmetric



SUPPOSE SMALL SHIFT IN AREA [illegible] REDUCES LOSS

THEN LOSS IS ALSO REDUCED ON HALF-SEGMENT

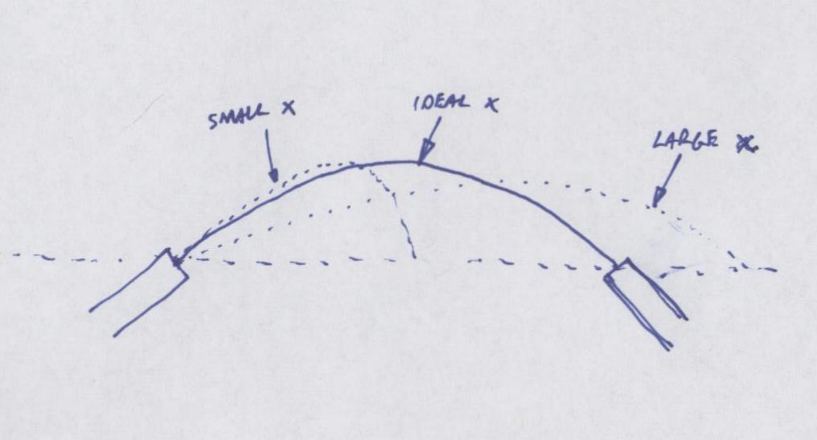
THUS SYMMETRIC SHIFT DOUBLY-REDUCES LOSS

Constraint: Begin and End Locations Same

$$k_{new}(s) = k_{old}(x * s)/x$$

There exists an x such that the beginning and end are at the original vectors. Symmetry and begin-end directions are preserved.

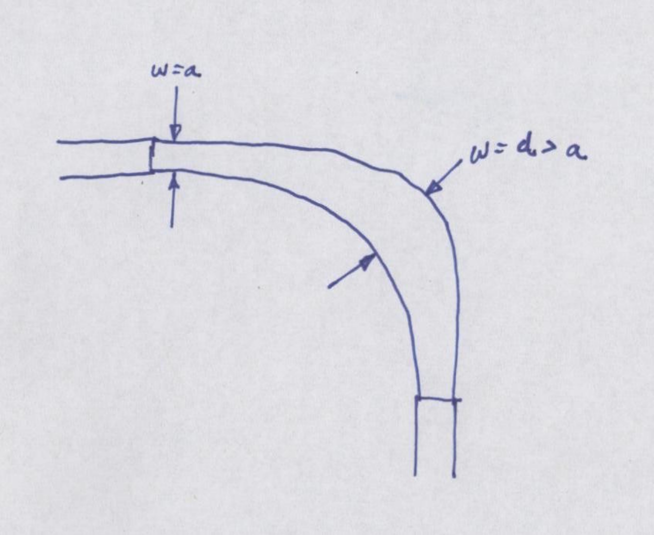
Constraint: Begin and End Locations Same



Future: Actually Calculate it

Have not finished analytic (if possible) computation of optimal curve.

Future: Change Width of Waveguide Too



Future: Dielectric Waveguide Instead of Conducting