The Cosmological Constant and the Vacuum Catastrophe

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General relativity (GR) provides a mathematical framework that can be used to describe not only local spacetime, but the universe as a whole. Such cosmological applications of GR raise new questions about the structure and dynamics of the universe, and one of the terms that can distinguish the different possibilities is the cosmological constant. This paper will describe the role of the cosmological constant in the Lambda-CDM model of the universe, and provide a brief survey of its observational support. With the cosmological constant established, the vacuum catastrophe is presented as a compelling conflict between the two pillars of modern physics: GR and quantum mechanics.

I. INTRODUCTION AND HISTORY

The cosmological constant originates from the Einstein field equations of general relativity. Expressed in tensor notation and in units where $\hbar = c = 1$, the field equations are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

which is often simply referred to as Einstein's equation. G is Newton's gravitational constant and $T_{\mu\nu}$ is called the energy-momentum tensor. It represents the energy/mass content of spacetime such as galaxies and radiation, while the left-hand side describes the curvature of spacetime[1]. The equality of these two ideas has the meaning that mass tells spacetime how to curve and the curvature of spacetime tells mass how to move—at least gravitationally.

But suppose spacetime has an intrinsic curvature. This possibility would be accounted for by including an extra term in the left-hand side of equation (1), which is exactly what Einstein did by introducing the cosmological constant as the Greek character Λ in the following form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{2}$$

Einstein made this modification after he applied GR to the cosmological scale and realized that an intrinsic curvature was necessary in order for a static universe to be possible. A static universe—one which neither expands nor contracts—was believed at the time to be the state of our universe until Edwin Hubble observed that the universe is expanding. This observation caused Einstein to abandon the cosmological constant[2].

However, recent developments indicate that Einstein's modified equation may be the apposite form after all, with the cosmological constant being interpreted as the vacuum energy density of empty space. Such a vacuum energy density enters into Einstein's equation by breaking the energy-momentum tensor down into an ordinary matter part and a vacuum part, $T_{\mu\nu} \to T_{\mu\nu}^{\mathrm{M}} + T_{\mu\nu}^{\mathrm{vac}}$. In arbitrary coordinates, Lorentz invariance implies that[1]

$$T_{\mu\nu}^{\mathrm{vac}} = -\rho_{\mathrm{vac}} g_{\mu\nu}. \tag{3}$$

Einstein's equation thus becomes

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G (T_{\mu\nu}^{\mathrm{M}} - \rho_{\mathrm{vac}} g_{\mu\nu}), \tag{4}$$

and comparing equations (2) and (4) gives

$$\rho_{\mathrm{vac}} = \frac{\Lambda}{8\pi G}. \tag{5}$$

Thus the interpretations of the cosmological constant as either an intrinsic curvature of space or as the energy density of the vacuum are entirely equivalent.

The vacuum energy density interpretation has gained much attention in recent times as a way to explain the surprising observation that the universe appears to be expanding at an accelerating rate. The general idea encapsulating such an acceleration is called dark energy, and the cosmological constant fills the role of dark energy in the Lambda-CDM model of the universe[3]. A brief survey of the Lambda-CDM model provides a useful basis for the rest of this discussion.

II. THE LAMBDA-CDM MODEL

The Lambda-CDM model is a picture of the universe which incorporates cold dark matter (CDM), the cosmological constant, and the Friedmann-Lemaître-Robertson-Walker metric (FLRW) of general relativity. The FLRW metric is the result of adding the assumptions of homogeneity and isotropy of space to GR[1].

In the present context, homogeneity means that the universe looks the same after a coordinate translation, and isotropy means that it looks the same after a coordinate rotation. But of course we know that these assumptions are not true! After all, rotations cause detectible changes in our galaxy reliably enough that the night sky can be used as a navigational tool. But the homogeneity and isotropy assumptions are cosmological statements on the cosmological scale. The assumptions hold down to length scales of about 10 Mpc[3], and a parsec is about 3.26 light-years. To provide a sense of this scale, galaxies range from about 1 to 100 kpc across, while the size of the observable universe is estimated to be on the order of 10 Gpc[4]. What the homogeneity and isotropy assumptions provide is a smoothed-out picture of the universe above 10 Mpc, with the smaller-scale features such as galaxies and solar systems being perturbations to this uniform universe[3].

The Big Bang model follows directly from the FLRW metric, and the Lambda-CDM model is a particular incorporation of the cosmological constant and cold dark matter into the Big Bang model[3]. But what reasons do we have for adding these exotic ideas to the Big Bang? Dark matter is a fascinating topic which will not be covered here, but we will now examine two independent observations which indicate a non-zero cosmological constant. The first piece of evidence derives from the cosmic microwave background (CMB) while the second is a direct observation of an accelerating expansion. Astronomical observations are very susceptible to systematic errors, so having multiple independent sources of data is not only compelling in its own right, but is essentially necessary in cosmology[5].

III. THE COSMIC MICROWAVE BACKGROUND

The CBM tells us a lot about the early universe, and it turns out that it also tells us about the universe's overall shape. The FLRW metric allows three possible geometries for the universe: spherical, Euclidean, or hyperbolic, which are also referred to as closed, flat, and open, respectively[6]. Since mass curves spacetime, the mass density of the universe determines the geometry. Note that this is another cosmological statement, and that we are talking about the shape of the universe on the cosmological scale. The local perturbations to this overall shape by masses such as the Sun are what we perceive as the force of gravity[3].

Defining the critical density ρ_{crit} to be the density required for a flat geometry allows us to define the density parameter:

$$\Omega_0 \equiv \frac{\text{actual density of the universe}}{\text{density of a flat universe}} = \frac{\rho}{\rho_{\text{crit}}}. \tag{6}$$

Thus $\Omega_0 > 1$ corresponds to a closed universe while $\Omega_0 < 1$ indicates an open universe, as illustrated by Figure 1.

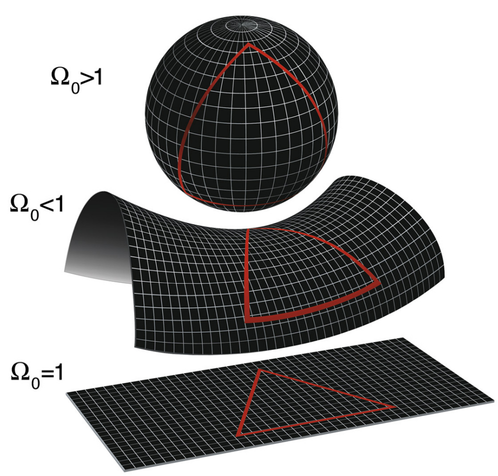

FIG. 1: WMAP image #990006 (`http://map.gsfc.nasa.gov/media/990006/`), courtesy of NASA/WMAP Science Team. This figure shows how a space can have different overall shapes, using two-dimensional spaces for clarity. From the top to the bottom, we have closed, open, and flat spaces, with a triangle superimposed to emphasize how the shape affects the space.

The CMB provides a measurement of Ω_0 via the anisotropies—angular variations from uniformity—of the background temperature. The temperature of the CMB is a nearly uniform 2.73K, but there are tiny variations from this temperature ranging over only half a millikelvin. These variations are visible as spots as in Figure 2, and can be represented by an expansion in spherical harmonics. The angular power spectrum is the contribution to this expansion due to the different values of the spherical harmonic parameter ℓ. Each value of this parameter corresponds to an angular size via the relation[7]

$$\theta = \frac{\pi}{\ell}. \tag{7}$$

Thus increasing values of ℓ correspond to more intricate angular detail in the anisotropies.

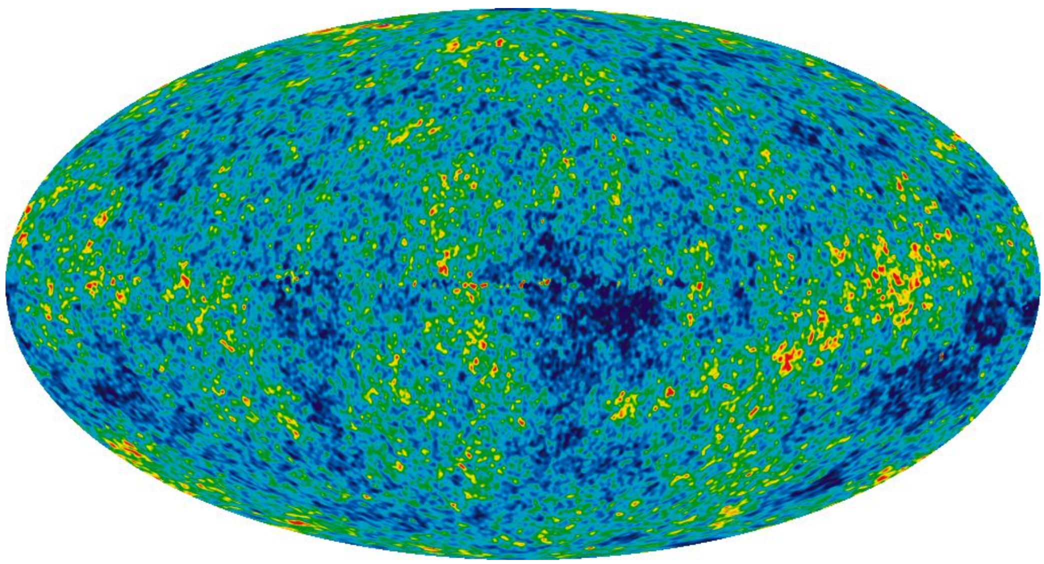

FIG. 2: 7^{th} year WMAP data, WMAP #101080 (`http://map.gsfc.nasa.gov/media/101080`), courtesy of NASA/WMAP Science Team. This image shows the temperature anisotropies of the CMB as a color gradient using a Mollweide projection over the entire sky. The bright red spots are about 0.2 mK higher than the dark blue areas.

The minimum angular size of the largest temperature variations depends primarily on Ω_0, and in fact a flat universe where $\Omega_0 = 1$ will produce a peak in the ℓ spectrum at $\ell_{\text{peak}} = 200$[7]. Not only is this peak clearly visible in observations of the CMB, but a flat universe under the Lambda-CDM model provides an excellent fit to the entire ℓ spectrum[8],[9], as can be seen in Figure 3.

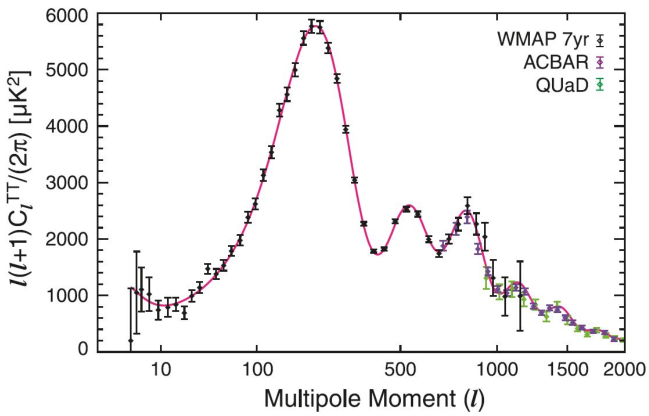

FIG. 3: WMAP power spectrum and a fit to the Lambda-CDM model (Komatsu et al. 2010)

The flatness of the universe as indicated by the CMB is strong support for the cosmological constant because there is not enough matter in the universe to cause its geometry to be flat! Even dark matter together with ordinary matter accounts for only about one fourth of the energy required for flatness[10]. The cosmological constant interpreted as a vacuum energy present throughout all the universe provides the missing energy density, and the value of the cosmological constant needed for a flat universe agrees with the next piece of evidence we shall examine, namely the observation that the expansion of the universe is accelerating.

IV. THE ACCELERATING EXPANSION

To understand how the acceleration of the expansion of the universe can be observed, we first need to know a little about white dwarf supernovae which are known as Type Ia supernovae. A white dwarf is a small, dense star supported from gravitational collapse only by the quantum degeneracy pressure of its constituent electrons, and the critical mass of this equilibrium is the Chandrasekhar limit. A white dwarf can undergo a supernova explosion when it exceeds this limit by accreting mass from a binary companion[11]. White dwarfs become Type Ia supernovae in such a consistent way that the luminosity over time for almost all of these events follows a nearly uniform curve, which makes them excellent standard candles[11].

A standard candle is an astronomical object or event with some intrinsic property possessed by all objects in its class. Such an intrinsic property yields an apparent property that depends on the spacetime location of the candle relative to the location of the observer. The idea is that the location of the candle can be derived by observing its apparent property and considering how the intrinsic property changes over distance. If all people were the same height all the time then they would be standard candles in the sense that we could look across a field and know the distance to anyone we saw by observing their apparent height.

A Type Ia supernova tell us about the expansion of the universe when we consider the supernova's redshift together with its distance derived from its apparent brightness as a standard candle. Astronomical redshifts are the direct result of the expansion of the universe: in the time it takes light to travel from one galaxy to another, the space between the galaxies will expand, so the wavelength of the light gets stretched out along with space itself. This cosmological redshift is quite distinct from a Doppler redshift, and the cosmological redshift z is defined as

$$z \equiv \frac{\lambda}{\lambda_0} - 1, \tag{8}$$

where λ is the observed wavelength and λ_0 is the wavelength in the rest frame of the source. If the universe is expanding at an accelerating rate, then Type Ia supernovae will appear further from us than if the expansion were decelerating or even constant. That is, for a given redshift, a Type Ia supernova will have a dimmer apparent brightness in an accelerating expansion than it would if the expansion were constant or decelerating.

In 1998, it was announced that Type Ia supernovae observations do in fact indicate that the expansion of the universe has been accelerating[12], which was verified by several Type Ia surveys over the following decade[13],[14],[15]. The cosmological constant again provides an answer for these counterintuitive observations because a vacuum energy density behaves like a repulsive form of gravity and would drive just such an acceleration[1]. This 'anti-gravity' behavior of Λ is precisely why Einstein introduced it in the first place: to act against the gravitational collapse possible when only the ordinary energy-momentum tensor is present in equation (1).

One might consider it more rational to seek an explanation for the dimness of Type Ia supernovae that does not invoke a new form of energy, yet we are confronted by the astonishing fact that using the supernovae observations to make a measurement of the cosmological constant gives a value that agrees with the amount of dark energy needed for a flat universe in the Lambda-CDM model[10]. Having thus established compelling support for a vacuum energy density in the form of the cosmological constant, we are in a position to appreciate one of the foremost open problems in modern physics.

V. THE VACUUM CATASTROPHE

The standard model of particle physics is formulated in terms of relativistic quantum field theories (QFTs) where spacetime itself is quantized and particles are treated as excitations of the fields. Like all quantum systems, the "zero-point" or ground-state energies of such QFTs are non-zero[16]. These ground-state energies are interpreted as causing an overall quantum vacuum energy density, and in this way the standard model of particle physics provides a theoretical calculation of the cosmological constant: simply add all of the contributions due to the fields of the standard model[17].

But such simple and straightforward interpretations are not always correct, and in fact this calculation of Λ via QFT yields a value that disagrees with the previously discussed astrophysical observations by as much as 120 orders of magnitude[16],[17]! In analogy with the ultraviolet catastrophe of a radiating black body, this discrepancy between theory and observation has been termed the vacuum catastrophe[16]. The formulation of quantum mechanics resolved the ultraviolet catastrophe by providing a theoretical explanation of the observed blackbody spectrum, and it would appear that a corresponding advancement in our present-day theoretical knowledge is needed to resolve the vacuum catastrophe.

But where will this advancement in theory occur? We might suppose that there is a problem with extending GR to cosmological length scales. After all, both the cosmological constant and dark matter are introduced to conform GR to observations[3], which may strike one as a gross degree of curve fitting. Yet GR has such strong observational support that it may be reasonable to accept that most of the matter in the universe interacts almost exclusively through the agency of gravity. The equivalence principle—a fundamental postulate of GR—has been experimentally verified to very high accuracy[18]. Furthermore, it was announced in early 2010 that GR has been verified at the Mpc scale via weak gravitational lensing[19]. If there is something wrong with general relativity, then it is well hidden.

Examining the vacuum catastrophe inevitably emphasizes the lack of unification between particle physics and gravity. The standard model of particle physics does not address gravity at all, while general relativity is notoriously difficult to formulate quantum-mechanically. Nevertheless, a successful formulation of quantum gravity may be required in order to resolve the vacuum catastrophe[1]. While aesthetics regarding matters of theory may provide compelling enough sentiments to pursue such a satisfyingly unifying notion as quantum gravity, I personally find that the stark discrepancy of the vacuum catastrophe causes quantum gravity and indeed quantum field theory and general relativity to be still even more fascinating pursuits.

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