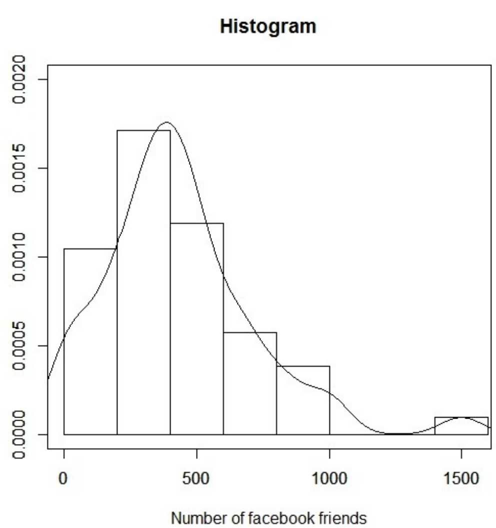
The Normal Density Curve

April 25, 2012

- Smooth Curves.
- Density Curves.
- Normal Density Curve.
- Standard Scores.

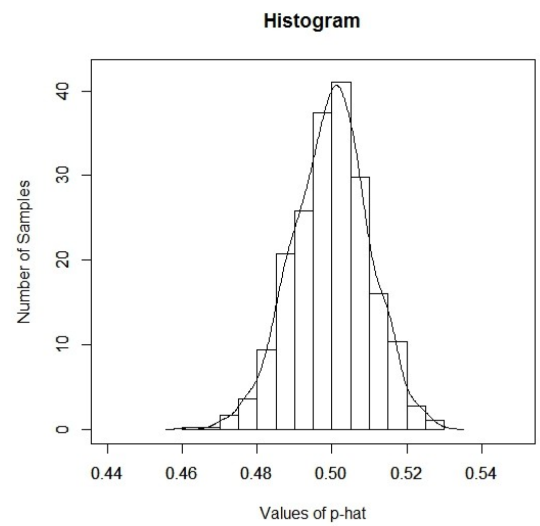
1.0 Smooth Curves

- We can use curves in place of histograms to picture the overall shape of a distribution of data.



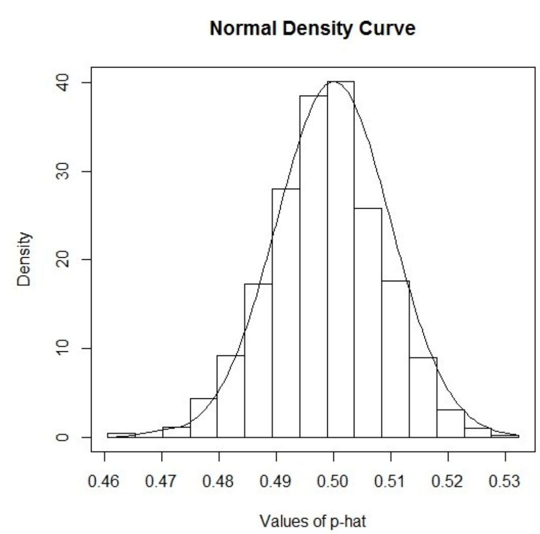
1.0 Smooth Curves

- For the sampling data of 1,000 *p*-hat values, the curve is symmetric, single peaked and bell shaped.



1.0 Smooth Curves

- It is a mathematical fact that the distribution of *p*-hat follows a specific kind of curve called the NORMAL curve.



2.0 Density Curves

Density Curves

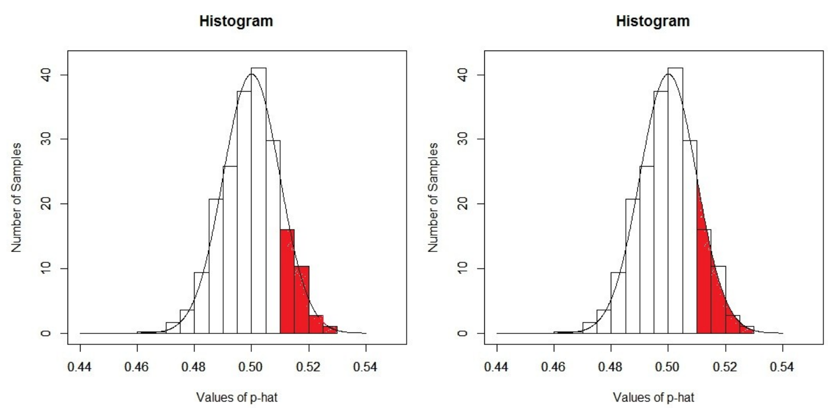
A density curve is a THEORETICAL curve that describes the distribution of a continuous variable.

Density curves versus frequency histograms:

- Frequency histograms are a plot of the data obtained from a sample.
- Frequency histograms show the count of observations in each bin.

- Density curves are the idealized shape of the population distribution.
- Density curves are set up to show the proportion of observations in any region by areas under the curve.

2.1 Using a Density Curve

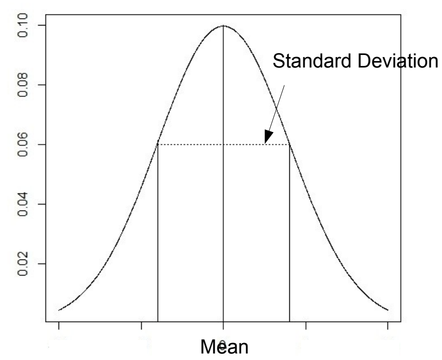


- The area of the shaded bars of the histogram on the left is the percentage of p-hat that were larger than 0.51.
- The shaded area under the NORMAL curve provides a very good approximation.

3.0 The Normal Density Curve

Definition

The NORMAL curves are a family of symmetric, single-peaked bell-shaped density curves.



3.1 Properties of the Normal Curve

- A specific normal curve is completely described by giving its mean and its standard deviation.
- The mean and the median equal each other.
- The standard deviation fixes the spread of the curve.

4.0 Finding Areas Under the Normal Curve

The S.A.T. exams are scaled so that scores should roughly follow the Normal density curve with mean 500 and S.D. 100.

- What proportion of scores fall below 500?
- What proportion of scores fall between 400 and 600?

4.1 Standard Scores

- The first step in finding any area under a Normal density curve is to convert the observation into its standard score.

Definition

The standard score for any observation is

$$\text{standard score} = \frac{\text{observation - mean}}{\text{standard deviation}}$$

- For the S.A.T. data, convert the following into their standard scores: 500, 400, 600.

4.2 Aside

- The area between two standard scores for a Normal density curve is the same, no matter what the mean and standard deviation of the original Normal curve are.
- This area has been tabulated for many different standard scores and can be found in Table B.

4.2 Using Table B

- The second step is to consult Table B for the area.

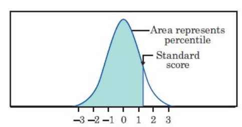


TABLE B Percentiles of the Normal distributions

Standard score	Percentile	Standard score	Percentile	Standard score	Percentile
−3.4	0.03	−1.1	13.57	1.2	88.49
−3.3	0.05	−1.0	15.87	1.3	90.32
−3.2	0.07	−0.9	18.41	1.4	91.92
−3.1	0.10	−0.8	21.19	1.5	93.32
−3.0	0.13	−0.7	24.20	1.6	94.52
−2.9	0.19	−0.6	27.42	1.7	95.54
−2.8	0.26	−0.5	30.85	1.8	96.41
−2.7	0.35	−0.4	34.46	1.9	97.13
−2.6	0.47	−0.3	38.21	2.0	97.73
−2.5	0.62	−0.2	42.07	2.1	98.21
−2.4	0.82	−0.1	46.02	2.2	98.61
−2.3	1.07	0.0	50.00	2.3	98.93
−2.2	1.39	0.1	53.98	2.4	99.18
−2.1	1.79	0.2	57.93	2.5	99.38
−2.0	2.27	0.3	61.79	2.6	99.53
−1.9	2.87	0.4	65.54	2.7	99.65
−1.8	3.59	0.5	69.15	2.8	99.74
−1.7	4.46	0.6	72.58	2.9	99.81
−1.6	5.48	0.7	75.80	3.0	99.87
−1.5	6.68	0.8	78.81	3.1	99.90
−1.4	8.08	0.9	81.59	3.2	99.93
−1.3	9.68	1.0	84.13	3.3	99.95
−1.2	11.51	1.1	86.43	3.4	99.97

4.3 Finding Areas – Examples

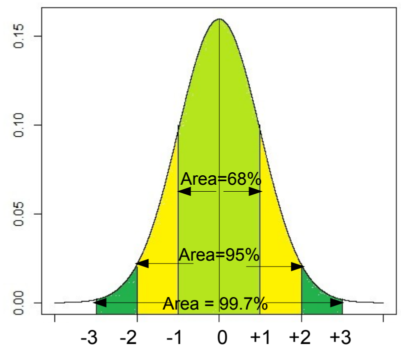
- For the S.A.T. data, what proportion of scores fall between 300 and 700?

- For the S.A.T. data what proportion of scores fall between 200 and 800?

4.4 68-95-99 Rule

For ANY normal density curve

- 68% of the area falls within one S.D. of the mean.
- 95% of the area falls within 2 S.D.s of the mean.
- 99.7% of the area falls within 3 S.D.s of the mean.



5.0 More on Standard Scores

- Standard scores are also called *z*-scores or standard units.
- Standard scores should only be used for density curves where the S.D. is used to describe the spread.
- They can be used to compare values in two different distributions.

The S.A.T.s are scaled to have a Normal distribution with mean 500 and S.D. 100. The A.C.T.s are scaled to have a mean of 18 and S.D. of 6. Student "X" scores 600 on the S.A.T.s. Student "Y" scores 21 on the A.C.T.s. Who does better (relatively speaking)?