Expected Values and Standard Errors

Chapter 20 plus Little Extra

March 16, 2012

- The Straight Bet
- Calculating Expected Values
- Law of Large Numbers

1.0 The Straight Bet

- You pay \$0.50 and choose a three-digit number.
- The state chooses a three-digit winning number at random and pays you \$250 if your number is chosen.
- Because there are 1,000 three-digit numbers, you have probability 1/1000 of winning.
- The probability model for your winnings from a ticket is given below:

Outcome	\$0	\$250
Probability	0.999	0.001

- What is the long-run average of your winnings?

1.1 Making Draws From A Box

- Suppose one ticket is drawn AT RANDOM from the following box of numbered tickets:

99.9% $ 0, 0.1% $ 250

- This is the same random process governing a win in StraightBet.
- If I replace the ticket I just drew, and draw again from the box, then this is like placing another bet.
- The sum of the numbers drawn is the sum of my winnings from two draws.
- In the long run you win $250 once every 1000 draws (bets) and $0 on the remaining 999 draws (bets).
- The long run average winnings from one ticket is

$$\$0.25 = 99.9\% \times \$0 + 0.1\% \times \$250$$

1.2 Expected Values

Definition

The expected value of a random phenomenon that has numerical outcomes is the long run average of the outcomes. It can be found by multiplying each outcome by its probability and then adding all the products.
In symbols, if the possible outcomes are $a_1, a_2, \ldots a_k$ and their probabilities are $p_1, p_2, \ldots, p_k$, then the expected value is:

$$\text{expected value} = a_1 p_1 + a_2 p_2 + \cdots + a_k p_k$$

2.0 Calculating Expected Values: The StraightBox Bet

- You pay \$1 and choose a three-digit number.
- You win \$292 if you exactly match the winning number and you win \$42 if your number has the same digits as the winning number, in any order.
- For example, if your number is 123, you win \$292 if the winning number is 123 and \$42 if the winning number is any of 132, 213, 231, 312 and 321.
- The probability model for your winnings from a ticket is

Outcome	\$0	\$42	\$292
Probability	0.994	0.005	0.001

- What is long-run average winnings from a ticket?
- Which game is better for the player? StraightBet or StraightBox Bet?

2.1 Calculating Expected Values: Helper versus Hinderer

If infants have no genuine preference for either toy, how many infants should we "expect" to see choosing the helper?

No. who choose helper	Prob (%)	No. who choose helper	Prob (%)
0	0.001	9	17.456
1	0.024	10	12.219
2	0.183	11	6.665
3	0.854	12	2.777
4	2.777	13	0.854
5	6.665	14	0.183
6	12.219	15	0.024
7	17.456	16	0.001
8	19.631		

2.2 Expected Value of $\hat{p}$

You ask an S.R.S of 1,500 college students whether they applied for admission to any other college.

Suppose, in fact, 35% of all college students applied to colleges besides the ones they are attending.

The sampling distribution of $\hat{p}$ is a Normal curve with mean 0.35 and standard deviation 0.012.

- What is the expected value of $\hat{p}$?

3.0 Law of Large Numbers

Law of Large Numbers

According to the law of large numbers, if a random phenomenon with numerical outcomes is repeated many times independently, the mean of the actually observed outcomes approaches the expected value.

- The law of large numbers is closely related to the idea of probability. In many independent repetitions, the proportion of each possible outcome will be close to its probability, and the average outcome obtained will be close to the expected value.
- The law of large numbers is the true version of the law of averages.
- The law of large numbers explains why gambling is a business. Casinos aren't gambling, you are.

3.1 How Large is a Large Number?

- The law of large numbers doesn't say how many trials are needed to guarantee an average outcome close to the expected value.
- This depends on the variability of the random outcomes.
  - State lottos have extremely variable outcomes. states do not rely on the law of large numbers
  - Casino games have less variability and the long-run average winnings of the house is positive.
- In general, the average of the winnings based on a certain number of draws will be off by a chance error:

  average = expected value for average + chance error.

- The typical size of the chance error is called the STANDARD ERROR.

3.2 Thoughts About the Standard Error

- The standard error for a statistic tells us how far off the values of the statistic will typically be from what we expect for it.

For example, if infants have no genuine preference for the helper, we would expect to see 8 (out of 16) choosing the helper.

The standard error for the number of infants who choose the helper is 2. What does this tell us?

3.3 Standard Error of $\hat{p}$

You ask an S.R.S of 1,500 college students whether they applied for admission to any other college.

Suppose, in fact, 35% of all college students applied to colleges besides the ones they are attending.

The sampling distribution of $\hat{p}$ is a Normal curve with mean 0.35 and standard deviation 0.012.

- What is the standard error of $\hat{p}$?