An algorithm for optimal centralized landing location: rectilinear yarding operations on flat uniform terrain[1]

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Abstract: An algorithm is developed for the optimal location of a single centralized landing on a setting of arbitrary shape under the following conditions: turns are uniformly distributed over a setting that is located on flat, uniform terrain; skidder movement between landing and turn location follows a rectilinear pattern; variable cost of travel between landing and turn location is proportional to the distance traveled. The design objective is to place the landing so that the yarding cost is minimized. It is assumed that the facility may be located at any point on the horizontal plane of the setting and that there are no barriers to travel. An example is given.

Résumé : Un algorithme servant à déterminer l'emplacement optimum d'une jetée à l'intérieur d'une aire de récolte est développé. La méthodologie appliquée nécessite les conditions suivantes : les cycles de débusquage sont uniformément distribués sur terrain plat et homogène; les trajets aller-retour sont rectilignes; les coûts variables imputables aux déplacements sont proportionnels à la distance parcourue. On assume qu'il n'y a aucun obstacle à la traficabilité et que la jetée peut être située n'importe où le long d'un plan horizontal à l'intérieur de l'aire de coupe. La méthode identifie l'emplacement pour lequel le coût du débusquage est minimisé. Un exemple est présenté.

[Traduit par la Rédaction]

Introduction

In a technical note by Greulich (1994) a method was given for the calculation of average yarding distance (AYD) on settings that exhibit a rectilinear yarding pattern to a centralized landing. The specification of setting boundaries and landing location determine AYD. Since expected yarding cost for a setting is a function of AYD, its measure serves as a very useful design parameter. It is the purpose of this technical note to provide an algorithm that optimizes landing placement for the most basic case of rectilinear yarding to a single centralized landing. The problem is first described and mathematically modeled. A numerical algorithm that provides an easily applied and computationally efficient solution procedure is then developed. The note concludes with a numerical example and brief discussion.

Optimal landing location

In this problem formulation it is assumed that all turns on a setting that is located on flat, uniform terrain will be yarded to a single centralized landing. The individual turns are independently and uniformly distributed across the area, A, of the setting. There are no restrictions on the shape of the setting, which for modeling purposes is defined by one or more polygonal regions. Beginning and ending coordinates for line segments used to define the setting boundary are entered in the conventional counterclockwise manner. Skidder movement during the yarding operation follows a barrier-free rectilinear pattern of travel. The two yarding directions, at right angles to one another, are exogenously predetermined; an example would be row thinning of plantations. The coordinate axes are aligned with these yarding directions. During the inhaul (outhaul) process the skidder always moves parallel to the axes and skidder distance from the landing is always decreasing (increasing). The variable cost component of skidder movement during yarding is assumed proportional to the rectilinear travel distance. The objective then is to find the location (x_0, y_0) for the landing that minimizes the expected yarding cost. The problem is stated mathematically as

$$[1] \qquad \underset{(x_0,y_0)}{\mathrm{MIN}} : Z(x_0, y_0) = \mathrm{C} \iint_A (|x - x_0| + |y - y_0|) \frac{\mathrm{d}x\,\mathrm{d}y}{A}$$

The terms on the right side of the above equation may be collected and described as follows. The term $\mathrm{d}x\,\mathrm{d}y/A$ gives the probability that a turn randomly located somewhere within the total area, A, of the setting falls within the infinitesimally small area $\mathrm{d}x\,\mathrm{d}y$. The terms $|x - x_0|$ and $|y - y_0|$ are the absolute values of the distances parallel to the x-axis and y-axis, respectively, that the skidder must travel to reach the landing if a turn is found and retrieved from location (x, y) within the setting. The constant C is the cost per unit distance associated with yarding a turn. The indicated integration over the area A gives the expected rectilinear travel distance to the landing, which, when multiplied by the cost per unit distance traveled, C, yields the expected yarding cost per turn. To minimize this expected cost the partial derivatives of $Z(x_0, y_0)$ with respect to x_0 and y_0 are taken and set equal to zero. These first-order necessary conditions for a minimum with respect to x_0 and y_0 are thereby determined to be

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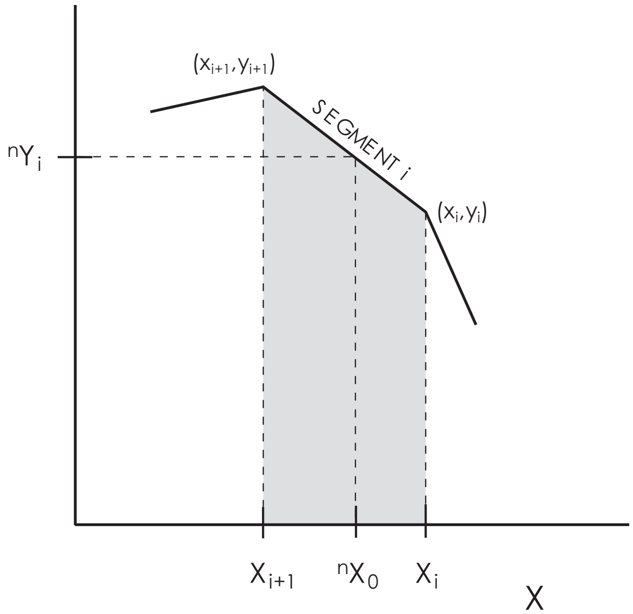
[1] A copy of the FORTRAN source code for optimal landing location may be obtained from the author either via INTERNET at greulich@u.washington.edu or by posting a formatted high-density (1.44 Mbyte) 3.5-in. diskette together with a self-addressed prepaid mailer to the author.

Fig. 1. The trapezoidal area (shaded) associated with line segment i and the intersection point of the vertical line $x = {}^n x_0$ with line segment i.



$$[2a] \quad \iint_A \operatorname{sgn}(x - x_0)\mathrm{d}x\ \mathrm{d}y = 0$$

$$[2b] \quad \iint_A \operatorname{sgn}(y - y_0)\mathrm{d}x\ \mathrm{d}y = 0$$

where sgn(·) is the signum function; defined equal to 1 if the value within parentheses is greater than zero, equal to –1 if less than zero, and equal to zero if zero.

The interpretation of these two necessary conditions is straightforward. The first integral equation states that for x_0 to be the optimal x-coordinate of the landing the vertical line $x = x_0$ must halve area A. In a similar fashion the second integral indicates that the optimal value for y_0 must establish a horizontal line $y = y_0$ that divides A into upper and lower parts of equal area. These two area halving requirements form the basis for the optimization procedure that follows. The two equations are not only similar but functionally independent of one another. Thus the general optimization procedure that will be developed for x_0 based upon eq. 2a may be immediately extended to eq. 2b and optimization with respect to y_0. These necessary conditions for optimality and recognition of their separability have been somewhat less clearly stated elsewhere (Love et al. 1988).

Preliminary results

To begin the analysis the boundary curve of region A is defined by a finite sequence of straight line segments. The usual mathematical convention with respect to the orientation of closed curves is followed: line segments are sequentially numbered 1 through N always keeping the region A to the left as numbering proceeds. The next step in the development is to examine in detail one of the line segments composing the boundary.

A trapezoid may be formed by extending vertical lines to the horizontal axis from the beginning (x_i, y_i) and end (x_{i+1}, y_{i+1}) points of any line segment i (Fig. 1). The signed-area of this trapezoid may be calculated from the coordinate area formula as

$$[3] \quad A_i = \left(\frac{1}{2}\right)[(y_i + y_{i+1})(x_i - x_{i+1})]$$

and the total area of the region enclosed by the N line segments is given by summing respective trapezoidal areas for all N line segments

$$[4a] \quad A = \sum_{i=1}^{N} A_i$$

where, because it is a closed traverse of the area boundary, $(x_{N+1}, y_{N+1}) \equiv (x_1, y_1)$. Substitute eq. 3 into eq. 4a and simplify to obtain

$$[4b] \quad A = \left(\frac{1}{2}\right)\sum_{i=1}^{N}(x_i y_{i+1} - x_{i+1} y_i)$$

The total area of the setting as calculated by this latter formula will have a positive sign automatically attached to it if the counterclockwise traversing convention is followed.

Within the algorithm, to be presented below, sequential trial values of x_0, denoted $^n x_0$, $n = 0, 1, 2, ...$ will be evaluated with respect to this total area A. Any value $^n x_0$ that is found to vertically divide the region A into equal left and right areas (i.e., AL = AR; with AL + AR = A) is called a median x value of A. It is also an optimal value for x_0 and as such will be denoted $^* x_0$. There may be more than one median (optimal) value of x, an example of which is provided by a disjoint region consisting of two polygons of equal area, $A/2$. Perpendicular projections of these two areas onto the x-axis define two line segments on that axis. If these two line segments do not overlap then all points within the separating closed interval provide equally good (optimal) x-coordinate values for the landing location.

Arbitrary line segment i (Fig. 1) is examined again. If $^n x_0$ falls between x_i and x_{i+1} for line segment i then the intercept point $^n y_i$ (when it exists; i.e., $x_i \neq x_{i+1}$) of the vertical line $x = {}^n x_0$ and line segment i may be calculated as

$$[5] \quad {}^n y_i = S_i {}^n x_0 + I_i$$

where the slope of the line segment is given by

$$[6] \quad S_i = \frac{y_i - y_{i+1}}{x_i - x_{i+1}}$$

and its intercept with the y axis is

$$[7] \quad I_i = \frac{x_i y_{i+1} - x_{i+1} y_i}{x_i - x_{i+1}}$$

The portion of the total area A that lies to the left and to the right of each trial value $^n x_0$ can now be calculated in the following fashion. Categorize each of the N line segments using four mutually exclusive and exhaustive sets based on the numerical values of x_i and x_{i+1} relative to that of $^n x_0$. Figure 2 illustrates these four sets. With reference to this figure, solid and dotted lines for the boundaries of the regions indicate closed and open regions, respectively; e.g., the boundary for C_4 includes $x_{i+1} = {}^n x_0$ for $x_i \geq {}^n x_0$, while the boundary of C_2 does not. Also note that both axes are broken and pulled apart in

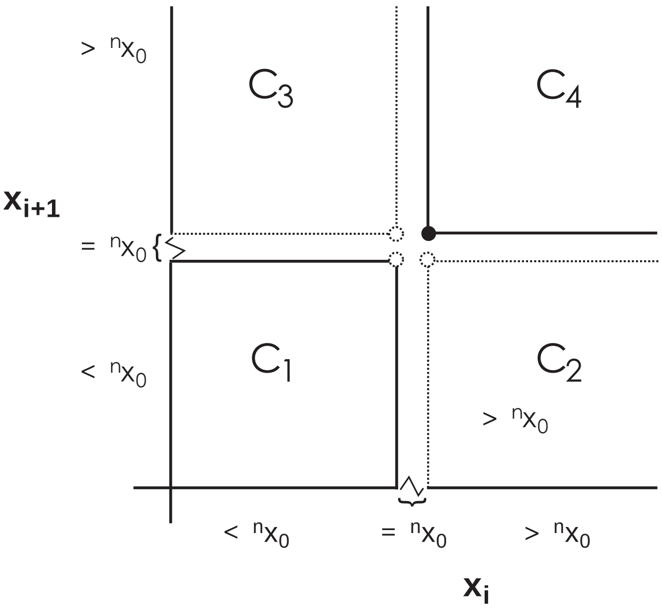


Fig. 2. Partition of the (x_i, x_{i+1}) plane into four mutually exclusive regions.

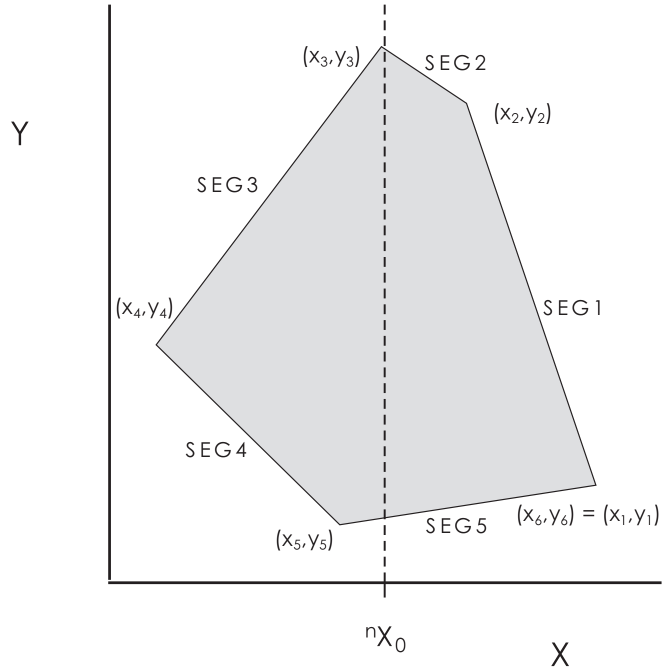


Fig. 3. Illustrative division of an area A into left and right sides by a trial vertical line at $x = {}^nx_0$.

this figure in order to reveal the set relationships along the horizontal and vertical set-interface lines. In using this figure it is first noted that each line segment i has a beginning x-coordinate, x_i, and an ending x-coordinate, x_{i+1}. These two values are compared with the current value of ${}^nx_0$; a procedure that serves to classify each of the N segments. The contribution that each line segment makes to the area lying to the left of ${}^nx_0$ is calculated using a formula that depends upon the specific set into which the segment falls. In a similar fashion its contribution to the area lying to the right of ${}^nx_0$ may also be calculated. The set-specific formulas yielding these contributory, automatically signed areas are listed immediately following each set definition given below. The standard set notation used here facilitates the writing of computerized sorting routines.

$$\text{Set } C_1 : \{[(x_i \le {}^nx_0) \cap (x_{i+1} < {}^nx_0)] \cup [(x_i < {}^nx_0) \cap (x_{i+1} \le {}^nx_0)]\}$$

$$[8a]\quad AL_i = \left(\frac{1}{2}\right)[(y_i + y_{i+l})(x_i - x_{i+1})]$$

$$[8b]\quad AR_i = 0.0$$

$$\text{Set } C_2 : \{[(x_i > {}^nx_0) \cap (x_{i+1} < {}^nx_0)]\}$$

$$[9a]\quad AL_i = \left(\frac{1}{2}\right)[({}^ny_i + y_{i+1})({}^nx_0 - x_{i+1})]$$

$$[9b]\quad AR_i = \left(\frac{1}{2}\right)[(y_i + {}^ny_i)(x_i - {}^nx_0)]$$

$$\text{Set } C_3 : \{[(x_i < {}^nx_0) \cap (x_{i+1} > {}^nx_0)]\}$$

$$[10a]\quad AL_i = \left(\frac{1}{2}\right)[(y_i + {}^ny_i)(x_i - {}^nx_0)]$$

$$[10b]\quad AR_i = \left(\frac{1}{2}\right)[({}^ny_i + y_{i+1})({}^nx_0 - x_{i+1})]$$

$$\text{Set } C_4 : \{[(x_i \ge {}^nx_0) \cap (x_{i+1} \ge {}^nx_0)]\}$$

$$[11a]\quad AL_i = 0.0$$

$$[11b]\quad AR_i = \left(\frac{1}{2}\right)[(y_i + y_{i+1})(x_i - x_{i+1})]$$

Use of these sets and their signed-area formulas can be clarified by the examples provided in Fig. 3. Line segment 1 shown in that figure falls in C_4 and makes a positive contribution to the area falling to the right of ${}^nx_0$. This contribution is calculated by formula [11b]. There is no contribution to the area left of ${}^nx_0$, as shown by formula [11a]. Line segment 2 also falls in C_4, with similar results. Line segment 3 falls in C_1, with a positive contribution to the area left of ${}^nx_0$ and no contribution to the right. Line segment 4 also falls in C_1, with a negative contribution to the area left of ${}^nx_0$ and no contribution to the right. Note that while line segments 3 and 4 both fall in C_1, formula [8a] will automatically attach positive and negative signs, respectively, to the calculated areas. Line segment 5 falls in C_3 and contributes negative area to the left and right of ${}^nx_0$. These two areas are calculated via formulas [10a] and [10b]. These four sets and their corresponding eight signed-area formulas are essential to the optimization process. In the optimization algorithm a value of ${}^nx_0$ is sought that yields

$$[12]\quad \sum_{i=1}^{N} AL_i = \sum_{i=1}^{N} AR_i$$

Before developing that algorithm, however, a new and essential formula must be derived.

Assume for the moment that the values, $x_1, x_2, ..., x_N$, of the N coordinate pairs, (x_i, y_i), have been ranked in order of ascending value (e.g., for the area shown in Fig. 3 this ranked sequence would be x_4, x_5, x_3, x_2, x_1). Assume also that it has been determined that the optimal value ${}^{*}x_0$ falls somewhere between two adjacently ranked coordinate values x_r and x_s in this sequence. For optimality it must hold that

$$[13]\quad \sum_{j=1}^{4} \sum_{i\in C_j} AL_i = \sum_{j=i}^{4} \sum_{i\in C_j} AR_i$$

To obtain a calculating formula for ${}^{*}x_0$ this fundamental requirement for optimality is expanded and examined. For purposes of identifying the appropriate set C_j for each of the N line segments ${}^{n}x_0$ is assumed to be any point in the open interval (x_r, x_s), where x_r and x_s are two adjacently ranked x-coordinate values as previously described. Expand eq. 13 by substituting eqs. 8–11 and eliminate ${}^{n}y_i$ using eq. 5. Algebraic simplification leads to the following quadratic equation in ${}^{n}x_0$

$$[14]\quad \beta_1 {}^{n}x_0^2 + \beta_2 {}^{n}x_0 + \beta_3 = 0$$

where

$$[15]\quad \beta_1 = 2\left(\sum_{i\in C_2} S_i - \sum_{i\in C_3} S_i\right)$$

$$[16]\quad \beta_2 = 4\left(\sum_{i\in C_2} I_i - \sum_{i\in C_3} I_i\right)$$

and

$$[17]\quad \beta_3 = 2\left(\sum_{i\in C_1} A_i - \sum_{i\in C_4} A_i\right) + 2\left(\sum_{i\in C_2} x_i x_{i+1} S_i - \sum_{i\in C_3} x_i x_{i+1} S_i\right) - \sum_{i\in C_2}[(y_{i+1}+y_i)(x_{i+1}+x_i)] + \sum_{i\in C_3}[(y_{i+1}+y_i)(x_{i+1}+x_i)]$$

In general β_1 of eq. 15 will not equal zero and there will be two real roots to the quadratic equation. Only one of these two values will fall in the interval (x_r, x_s) and that is the optimal value for the x-coordinate of the landing location. This optimal value for ${}^{n}x_0$ is then denoted ${}^{*}x_0$. If β_1 equals zero the solution to the resulting linear equation is the optimal value.

The division of the setting area into equal portions is the almost trivial requirement for optimal placement of the landing. Finding the exact halving point for complex polygonal regions is not, however, a trivial exercise. The application of the coordinate area formula has provided a convenient equation (eq. 14) for the efficient calculation of that point for a planar polygonal region of any shape no matter how complex. It only remains to provide an algorithm for identifying either the two points x_r and x_s that ${}^{*}x_0$ lies between or the one point x_i upon which it falls exactly.

The optimization algorithm

Area to the left of ${}^{n}x_0$ is a monotonic nondecreasing function of ${}^{n}x_0$; i.e., as the value of ${}^{n}x_0$ increases the portion of the total area A lying to the left of the vertical line determined by ${}^{n}x_0$ stays constant or increases; it never decreases. This characteristic and the results of the preceding section suggest a very straightforward ranking algorithm analogous to that employed by Wesolowsky and Love (1971) with the singular difference of eq. 14 presented above that dramatically increases both model scope and ease of application.

Step 1. Rank the values of x_i for all N line segments and specify an initial starting point ${}^{0}x_0$ for the algorithm.

Step 2. Using eqs. 8–11 calculate Σ AL_i and Σ AR_i at the point ${}^{0}x_0$; if the two areas are equal set ${}^{*}x_0$ equal to ${}^{0}x_0$ and exit, otherwise determine whether the next trial point ${}^{1}x_0$ should be based upon movement along the x-axis to the left or right of ${}^{0}x_0$ given the objective of more nearly equating the two areas left and right of the to-be-selected point. Record the direction of movement either left or right along the axis. (Note here that n still equals 0 in going to the next step.)

Step 3. Find the rank location of ${}^{n}x_0$ among the rank-ordered x_i coordinates; moving in the determined direction along the axis set ${}^{n+1}x_0$ equal to the neighboring left or right x_i value as appropriate.

Step 4. Calculate Σ AL_i and Σ AR_i at the point ${}^{n+1}x_0$; if the two areas are equal set ${}^{*}x_0$ equal to ${}^{n+1}x_0$ and exit, otherwise determine whether ${}^{n+2}x_0$ should be based upon movement to the left or right given the ultimate objective of equating the two areas.

Step 5. If the direction of movement for ${}^{n+2}x_0$ is different from that for ${}^{n+1}x_0$ continue to step 6, otherwise set n equal to $n + 1$ and return to step 3.

Step 6. The optimal value ${}^{*}x_0$ falls between ${}^{n+1}x_0$ and ${}^{n+2}x_0$. Use eq. 14 to solve for ${}^{*}x_0$ (for computerized set classification of the N line segments the midpoint value $0.5({}^{n+1}x_0 + {}^{n+2}x_0)$ may be used). Exit the algorithm.

This same general iterative process can also be used to find ${}^{*}y_0$, thereby simplifying and shortening the computer code.

An excellent initial point for the algorithm is quickly provided by the following calculating formulas for the centroid of a planar polygonal region (Greulich 1995):

$$[18a]\quad {}^{0}x_0 = \left(\frac{1}{6A}\right)\left[\sum_{i=1}^{N}(x_{i+1}+x_i)(x_i y_{i+1} - x_{i+1} y_i)\right]$$

$$[18b]\quad {}^{0}y_0 = \left(\frac{1}{6A}\right)\left[\sum_{i=1}^{N}(y_{i+1}+y_i)(x_i y_{i+1} - x_{i+1} y_i)\right]$$

The use of this centroidal location for default starting coordinates $({}^{0}x_0, {}^{0}y_0)$ is recommended. Solutions are typically obtained with very few iterations of the algorithm when it is initiated at the centroid.

Having obtained the optimal facility location, the corresponding rectilinear AYD may be quickly determined using the previously published calculating procedure for that parameter (Greulich 1994). A numerical example will illustrate the application and results that can be obtained.

Numerical example

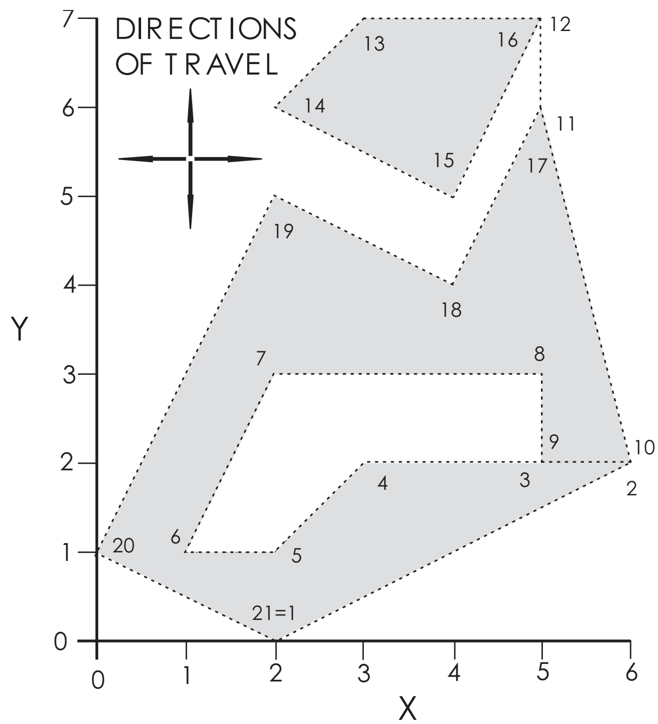
The above formulas and algorithm provide the basis for a computer program. A FORTRAN 77 program was written in accordance with ANSI standard X3.9-1978. The program was compiled using the Microsoft compiler (version 5.0) and run on a 12-MHz Intel 80286/287 Zenith personal computer. An example and the associated computational results are now examined.

The shaded region shown in Fig. 4 represents a hypothetical setting meeting the assumptions of the rectilinear yarding model. It has an area of 18 units^2. The boundary of the region has been delineated (traversed) with 20 line segments

Table 1. (x_i, y_i) coordinates for the boundary line segments of the example area.

Point No. (i)	Coordinate pair (x_i, y_i)
1	(2, 0)
2	(6, 2)
3	(5, 2)
4	(3, 2)
5	(2, 1)
6	(1, 1)
7	(2, 3)
8	(5, 3)
9	(5, 2)
10	(6, 2)
11	(5, 6)
12	(5, 7)
13	(3, 7)
14	(2, 6)
15	(4, 5)
16	(5, 7)
17	(5, 6)
18	(4, 4)
19	(2, 5)
20	(0, 1)
21 ≡ 1	(2, 0)

Fig. 4. Plotted area of a setting used as an illustrative example. The coordinate axes have been aligned with pre-established directions of travel.



(courses). The 20 coordinate pairs (turning points) that define these connected line segments are easily entered as one continuous sequence from an arbitrary starting point. One possible starting point and coordinate pair entry sequence are shown in Table 1. The optimal location for the landing, ($^*x_0$,$^*y_0$), is found to be at (3.282, 3.352) and the rectilinear AYD is 2.722 units. By way of comparison the landing location that offers the minimal straight-line AYD for this region can be found using a previously published algorithm (Greulich 1991). This latter model places the landing at (3.290, 3.449) with an expected yarding distance of 2.156 units. In this particular example the yarding pattern, rectilinear versus straight line, is found to make only a relatively minor difference in optimal landing location. The difference between expected rectilinear and straight-line yarding distances is, however, substantial and clearly has the potential to significantly change the expected cost of yarding this setting. It should not be concluded that the small difference in the two landing locations observed here is typical. Examples can easily be generated that show substantial differences in optimal landing placement as alternately determined by the rectilinear and straight-line yarding models.

The computer program found the optimal landing coordinates and its corresponding rectilinear AYD in 0.22 s of execution time. In larger problems the availability of a good initial point for the algorithm should significantly reduce computational effort. In this example the centroid, located at (3.231, 3.370), was used as the starting point for the algorithm.

Discussion

Application of location optimization models for single centralized landings on settings of irregular shape is greatly facilitated by use of the coordinate area formula (Greulich 1991). The coordinate area formula and its application are well known to forest engineers. This note has shown how its incorporation into the analysis of the rectilinear yarding distance model can provide an optimization algorithm that is easily understood and programmed. Previous optimization models for area destinations with rectilinear distance are very restrictive in the shape of the area that can be accommodated (Wesolowsky and Love 1971) or are not amenable to generalized computational solution procedures (Larson and Odoni 1981). There are, however, some notable limitations to the utility of the present model.

For expository clarity the development has been limited to a setting area that has only one turn density. The extension of the model to include multiple partitions of the setting, each with different turn density, does not present any conceptual difficulty but does require modification of the algorithm. The practical utility of the model would be enhanced by this additional capability.

Another limitation of the current model is the absence of truck road construction and use costs. The consequent assumption in application must be that truck haul related cost per unit distance between spur take-off point and landing is minor compared with yarding cost. More precisely, if marginal cost of truck spur construction and use were to be equated with marginal yarding cost, only an insignificant minor shift of the landing from its now "optimal" location toward the spur road take-off point would occur. Inclusion of these truck road costs is a natural, if somewhat more involved extension of the model.

Development of a constrained optimization model would be of considerable practical advantage. Barriers to direct skidder movement between some locations on the setting and the landing are frequently encountered. In a similar vein user-definable constraints on landing location would also be advantageous. Development of such constrained optimization models would appear to be a promising area for future research.

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